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6. Wave-induced Liquefaction in a Permeable Seabed

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Synopsis

Propagating ocean waves create dynamic water pressures on the surface of seabed. In response to the wave-associated pressures, excess pore pressures are produced in the seabed. When the excess pore pressures become larger than the overburden pressures, liquefaction in seabed may occur.

The mechanism of wave-induced liquefaction in a permeable seabed is studied with particular reference to the excess pore pressure. A concept of the 'oscillatory' excess pore pressure is introduced and the liquefaction criteria are represented in terms of the oscillatory excess pore pressure. According to the criteria, the dominant factors of wave-induced liquefaction are 1) the wave-associated pressure on the seabed surface, 2) the wave-induced pore pressure in the seabed which is referred to as the oscillatory pore pressure in this study and 3) the effective vertical stress in the seabed during calm sea.

Among the above factors, the oscillatory pore pressure is theoretically and experimentally investigated in detail. A governing equation on the oscillatory pore pressure is derived and its validity is verified by the model experiments and field observations. The estimated oscillatory excess pore pressures with the proposed theory compare fairly well with the measured ones. The liquefaction potential in the permeable seabed can be reasonably evaluated by the proposed liquefaction criteria.

On the basis of the theoretical and experimental studies, a procedure for assessing the liquefaction potential is presented and its application to a practical engineering problem is demonstrated.

Key Words: Field Observation, Liquefaction, Model Test, Ocean Soil, Pore Pressure, Wave Propagation (IGC: E7/E8)

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6. 海底砂地盤の波浪による液状化

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要 旨
波浪の伝播によって、海底地盤表面には動的な水圧が作用するが、波浪によって生じる海底面の水圧
に応答して、海底地盤中には過剰間隙水圧が発生する。このとき、過剰間隙水圧が地盤中の有効土被り
圧よりも大きくなると液状化が発生する可能性がある。そこで、本論文では、透水性の良い海底砂地盤
の波浪による液状化的メカニズムを、特に過剰間隙水圧に着目して明らかにしている。

本研究では、”変動” 過剰間隙水圧の概念を導入し、この概念に基づき液状化的発生基準を提示した。
液状化基準によると、液状化の発生に支配的な要因は、1）海底面上の波浪による水圧、2）波浪によ
って引き起こされる地盤中の変動間隙水圧、および3）静穏時の地盤中の有効鉛直応力である。

これらの要因のうち、変動間隙水圧について理論的、実験的に詳細に調べた。変動間隙水圧に関する
基礎方程式を誘導し、その有用性を模型実験および現地観測により証明した。提案された基礎方程式を
用いて予測した変動間隙水圧は、実測値と良く一致した。海底砂地盤の液状化ポテンシャルは、提案し
た液状化基準により合理的に評価される。

理論的、実験的な検討に基づき、液状化ポテンシャルを予測する手法を提示し、その手法の工学的問
題に対する適用性が示されている。

キーワード：現地観測、液状化、模型実験、海底土、間隙水圧、波動伝播（IGC：E7／E8）

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Wave-induced Liquefaction in a Permeable Seabed

1. Introduction

The interaction between propagating ocean waves and seabed has received extensive attention in the field of offshore engineering. The wave-induced liquefaction in porous seabed is one of the significant research subjects, since the liquefaction, if it happens, may cause severe damage to offshore structures such as breakwaters, platforms, pipelines and anchors. Furthermore, wave-induced liquefaction is said to be closely related to the scour, littoral drift and settlement of concrete blocks.

Yamamoto (1977), Madsen (1978), Nago (1981), Finn et al. (1983) and Okusa (1985) theoretically analyzed the wave associated stresses, strains and pore pressures in the porous seabed on the basis of Biot’s pro-elastic theory. In their works, the concept of the excess pore pressure, which is one of the comprehensive parameters in considering liquefaction, is not clearly addressed. Lee and Focht (1975) and Rahman et al. (1978) examined the wave-induced liquefaction taking account of the excess pore pressure build-up due to the cyclic shear stresses induced by ocean waves.

The review of the previous works tells us that the mechanism of wave-induced liquefaction is classified into two categories by means of the ways of excess pore pressure generation (Zen and Yamazaki, 1991a). One type of liquefaction, such as earthquake-induced liquefaction, is caused by the ‘residual’ or ‘progressive’ nature of excess pore pressures which are generated by the cyclic shear stresses in the seabed. Clausen et al. (1975) and Lee (1976) measured this type of excess pore pressure in the Ekofisk Tank foundation in the North Sea. Umehara et al. (1979) performed model experiments and examined the liquefaction potential in sand layers beneath a damaged breakwater. Zen et al. (1986a) analyzed the damaged breakwater and pointed out the possibility of wave-induced liquefaction in the seabed foundation of the breakwater.

The other type of liquefaction, which is the main subject in this study, is due to the ‘oscillatory’ or ‘transient’ nature of excess pore pressures which are produced by the spacial difference of pore pressures in the seabed. The wave-associated pressures are found to propagate into deposits with some damping and phase lag. The model experiments done by Inoue (1975), Nago (1981), Tsui and Helfrich (1983) and Zen et al. (1986b) clearly present such propagation characteristics. The field measurements made by Okusa et al. (1984) and Maeno and Hasegawa (1985) also confirmed the damping and lag effect. The pore pressure distributions brought by the damping and phase lag in the propagation process of pressures are considered the reason for the wave-induced liquefaction.

The previous experimental works and field observations were, however, mainly focused on the pore pressure responses in the seabed deposits, so that the mechanism of the wave-induced liquefaction was not definitely addressed in the context of the excess pore pressure. So, in this paper, the mechanism of the liquefaction is theoretically and experimentally studied. Then, the field observation is executed to confirm the liquefaction phenomenon in the real seabed. On the basis of the theoretical and experimental studies, a procedure to evaluate the liquefaction potential is proposed. The application of the proposed procedure to a practical engineering problem is demonstrated.
2. Theoretical Background

2.1 Wave Forces

When ocean waves propagate over a seabed, they impose periodical wave pressures on the surface of the seabed. The wave-associated pressures on the seabed surface can be evaluated using the linear wave theory, in which a small amplitude of waves is assumed. This assumption has been used in most of the previous works which treated the interaction between ocean waves and seabed (Henkel, 1970; Bjerrum, 1973 among others). The linear wave theory gives the wave-associated pressure, \( p \), as:

\[
p(x, t) = p_o \sin \left( \frac{2\pi}{L} x - \frac{2\pi}{T} t \right)
\]

where the amplitude of the bottom pressure, \( p_o \), is,

\[
p_o = \frac{\gamma_w H}{2} \left( \frac{1}{\cosh \left( \frac{2\pi h}{L} \right)} \right)
\]

\( \gamma_w \); the unit weight of water, \( h \); the water depth, \( H \); the wave height, \( T \); the wave period, \( L \); the wave length, \( t \); time and \( x \); the coordinate axis in the direction of the progress of waves.

2.2 Excess Pore Pressure and Effective Stress

In this study, the oscillation of wave-associated pressure and the motion of seabed are assumed one dimensional. Also, the cyclic shear stresses induced by waves are omitted. These assumptions may be acceptable if the wave length is large enough to disregard the horizontal water flow on and in the seabed.

The initial vertical total stress, \( \sigma_v(z, 0) \), at arbitrary depth, \( z \), of the deposit is expressed by:

\[
\sigma_v(z, 0) = \sigma_v^0(z, 0) + u(z, 0)
\]

where, \( \sigma_v^0(z, 0) \) and \( u(z, 0) \) are the vertical effective stress and the hydro-static pressure at the still water level, respectively. When the water level is subjected to a change from the initial still water level, it induces the total stress change in the deposit. Provided that the pore pressure change from the initial state of the hydro-static pressure, \( p(z, t) \), is not equal to the pressure change imposed on the seabed surface, \( p(0, t) \), the vertical total stress, \( \sigma_v(z, t) \), and the pore pressure, \( u(z, t) \), in the deposit are given by the following equations, respectively:

\[
\sigma_v(z, t) = \sigma_v^0(z, 0) + p(0, t)
\]

\[
u(z, t) = u(z, 0) + p(z, t)
\]

Both pressures are positive in the direction to which they increase from the initial hydro-static pressure. Therefore, the vertical effective stress in the deposit, \( \sigma_v^0(z, t) \), can be derived by subtracting Eq. 5 from Eq. 4 and substituting Eq. 3:
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\[ \sigma'_v(z, t) = \sigma'_v(z, 0) + p(0, t) - p(z, t) \] (6)

The effective stress change, \( \Delta \sigma'_v(z, t) \), is obtained from Eq. 6:

\[ \Delta \sigma'_v(z, t) = p(0, t) - p(z, t) \] (7)

Since the pore pressure change, \( p(z, t) \), is to be equal to the water pressure change on the surface of the seabed, \( p(0, t) \), under steady conditions, the difference can be thought to represent an 'excess' component of the pore pressure in the deposit. The excess pore pressure, \( u_e(z, t) \), is defined as:

\[ u_e(z, t) = - \{ p(0, t) - p(z, t) \} \] (8)

The effective stress change, \( \Delta \sigma'_v(z, t) \), is caused by this excess pore pressure, \( u_e(z, t) \).

2.3 Liquefaction Criteria

In order to apply the concept stated above to the seabed, the schematic drawings of the pore pressure and effective vertical stress distributions are illustrated in Fig. 1. The solid curves in Fig. 1(a) indicate the pore pressure beneath a wave trough and a wave crest. The excess pore pressure expressed by Eq. 8 is transient in nature, because the \( p(0, t) \) and \( p(z, t) \) are oscillatory and periodical in real ocean environment. Consequently, the effective vertical stress expressed by Eq. 6 varies periodically in accordance with the change of the \( \{ p(0, t) - p(z, t) \} \). If it attains zero or less at certain depths, the soil skeleton will become a liquefied state there. Thus, the criterion for the wave-induced liquefaction is easily derived from Eq. 6 by setting the vertical effective stress, \( \sigma'_v(z, t) \), equal to zero or less:

\[ \sigma'_v(z, 0) \leq - \{ p(0, t) - p(z, t) \} = u_e(z, t) \] (9)

The solid curves in Fig. 1(b) show the vertical effective stress distribution drawn by replacing the \( \sigma'_v(z, 0) \), with \( \gamma'z \), where \( \gamma' \) is the submerged unit weight of deposit. The line numbered (1) and (2) in Fig. 1(b) correspond to ones numbered (1) and (2) in Fig. 1(a), respectively. In Fig. 1(b), the liquefied zone shown by the slant lines where the vertical

\[ \sigma'(z, 0) = \gamma'z \]

\[ \sigma'(z, 0) = \gamma'z + [p(0, t) - p(z, t)] \]

Fig. 1 Concept of wave-induced liquefaction and densification; (a) Oscillatory excess pore pressure, (b) Effective vertical stress

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effective stress becomes zero or less appears near the seabed surface, under the wave trough. As the excess pore pressure is positive in this situation, the transient upward seepage flow is generated toward the seabed surface. From another point of view, the liquefaction, namely quick sand, is considered to be induced by the seepage force exerted on the soil skeleton in the seabed.

When the $\sigma'_v(z,0)$, $p(0,t)$ and $p(z,t)$ are known, the liquefaction potential can be easily evaluated by using Eq. 9. As the $\sigma'_v(z,0)$ is calculated from the submerged unit weight of deposit and the $p(0,t)$ is estimated approximately by the liner wave theory by using Eq. 1, the only factor to be known is the wave-associated pore pressure, $p(z,t)$, the evaluation of which will be explained in detail in section 2.4.

Whereas, if the vertical effective stress change, $\Delta \sigma'_v(z,t)$, reaches positive values, say it exceeds the initial vertical effective stress, $\sigma'_v(z,0)$, as shown by the line numbered 2 in Fig. 1(b), the wave-induced stress exerts a force on the soil skeleton to possibly densify the seabed.

As stated above, the wave-induced liquefaction is governed by the initial vertical effective stress at calm sea, $\sigma'_v(z,0)$, and the oscillatory excess pore pressure, $-(p(0,t) - p(z,t))$. So, the $\sigma'_v(z,0)$, $p(0,t)$ and $p(z,t)$ are expressed for convenience as; $\sigma'_v(z,0) = \sigma'_sw$, $p(0,t) = p_s$ and $p(z,t) = p_m$, hereafter.

2.4 Oscillatory Pore Pressure

(1) Governing Equation to Irregular Waves

The propagation of ocean waves exerts differential pressures on the surface of the seabed. When the wave length is large enough compared with the thickness of permeable seabed, it may be assumed that the wave-associated pressures distribute uniformly on the surface of the seabed. Then, the oscillation of pressures and the flow of pore water are treated under one dimensional conditions.

Considering a soil element at depth, $z$, in the seabed and assuming the flow of pore water in the soil element is governed by the steady state form of Darcy's law, Eq. 10 is derived from the conservation of mass of pore water:

\[
\frac{k}{\gamma_w} \frac{\partial^2 p_m}{\partial z^2} = -m_v \frac{\partial \sigma'_v}{\partial t} + n \frac{m_w}{m_v} \frac{\partial p_m}{\partial t} \tag{10}
\]

where, $k$: the coefficient of permeability, $m_v$: the coefficient of volume compressibility, $n$: the porosity, $m_w$: the compressibility of pore water, $\gamma_w$: the unit weight of pore water and $t$: time. The $k$ and $m_v$ are assumed constant irrespective of time and space. As the effective vertical stress is expressed by Eq. 6 and $\partial \sigma'_w / \partial t$ is equal to 0, we obtain,

\[
\frac{\partial \sigma'_v}{\partial t} = \frac{\partial (p_b - p_m)}{\partial t} \tag{11}
\]

which may be introduced into Eq. 10 to yield:

\[
\frac{k}{\gamma_w m_v} \frac{\partial^2 p_m}{\partial z^2} = \left(1 + \frac{n m_w}{m_v}\right) \frac{\partial p_m}{\partial t} - \frac{\partial p_b}{\partial t} \tag{12}
\]
Replacing,

\[ C_v = \frac{k}{\gamma_w \mu_v} \]  \hspace{1cm} (13)

\[ \alpha = 1 + \frac{n}{m_v} \frac{m_w}{m_v} \]  \hspace{1cm} (14)

Eq. 12 may be written in the form:

\[ C_v \frac{\partial^2 p_m}{\partial z^2} = \alpha \frac{\partial p_m}{\partial t} - \frac{\partial p_b}{\partial t} \]  \hspace{1cm} (15)

where the \( C_v \) is the coefficient of consolidation and the \( \alpha \) is a parameter which controls the water pressure propagation. In that sense, the \( \alpha \) is referred to as the coefficient of propagation, hereafter. The compressibility of pore water is affected by the amount of the dissolved air in water and the attached air to the soil particles. It is known that the effect of such air is represented by the degree of saturation, \( S_r \), and the pore pressure by means of absolute value, \( p_{mg} \). The \( p_{mg} \) is expressed by the sum of the hydro-static and dynamic pressures. Okusa (1985) presented the compressibility of pore water by the following equation:

\[ m_w = m_w + \frac{1 - S_r}{p_{mg}} \]  \hspace{1cm} (16)

where the \( m_w \) is the compressibility of pore water at fully saturated state and is the orders of \( 10^4 \) cm³/kgf. Introducing Eq. 16 into Eq. 14, we obtain:

\[ \alpha = 1 + \frac{n}{m_v} \left( m_w + \frac{1 - S_r}{p_{mg}} \right) \]  \hspace{1cm} (17)

**Equation 15** represents the governing equation of the oscillatory pore pressure in the seabed, applicable to any complex wave conditions such as irregular waves in real ocean.

As the oscillatory pore pressure is equal to the wave-associated pressure at the surface of the seabed, the boundary condition is written:

\[ p_m = p_b \] \hspace{1cm} at \hspace{0.5cm} z = 0 \hspace{1cm} (18)

If the porous seabed is of finite thickness, \( l \), and the underlying soil is assumed impermeable, no water flow normal to the horizontal boundary arises. Thus, we obtain:

\[ \frac{\partial p_m}{\partial z} = 0 \] \hspace{1cm} at \hspace{0.5cm} z = l \hspace{1cm} (19)

Initial condition is written as:

\[ p_m = 0 \] \hspace{1cm} at \hspace{0.5cm} t = 0 \hspace{1cm} (20)
It is noted that the \( p_w \) does not mean the 'excess' pore pressure but denotes the oscillatory nature of pore pressure. The governing equation on the excess pore pressure is derived from Eq. 15 by using Eq. 8:
\[
C_v \frac{\partial^2 u_e}{\partial z^2} = a \frac{\partial u_e}{\partial t} + (a - 1) \frac{\partial p_b}{\partial t}
\]  
(21)

When the compressibility of pore water is negligible, say the coefficient, \( \alpha \), is equal to 1.0, Eq. 21 becomes identical with the consolidation equation.

(2) **Governing Equation of Linear Waves**

When a small amplitude wave is assumed, the wave-associated pressure on the surface of the seabed, \( p_b \), is expressed by Eq. 1. Then, Eq. 15 is rewritten by replacing:
\[
\overline{P} = \frac{p_m}{p_0}
\]  
(22)
\[
\overline{Z} = \frac{z}{l}
\]  
(23)
\[
\overline{T} = \frac{t}{T}
\]  
(24)
\[
C = \frac{C_v T}{l^2}
\]  
(25)
\[
C \frac{\partial^2 \overline{P}}{\partial \overline{Z}^2} = \alpha \frac{\partial \overline{P}}{\partial \overline{T}} - 2 \pi \cos (2 \pi \overline{T}) - 2 \pi \overline{P}
\]  
(26)

The boundary conditions are:
\[
\overline{P} = \sin (2 \pi \overline{T}) \quad \text{at} \quad \overline{Z} = 0
\]  
(27)
\[
\frac{\partial \overline{P}}{\partial \overline{Z}} = 0 \quad \text{at} \quad \overline{Z} = 1
\]  
(28)

The initial condition is:
\[
\overline{P} = 0 \quad \text{at} \quad \overline{T} = 0
\]  
(29)

(3) **Finite Difference Solution**

In order to solve Eq. 26, it is replaced by the finite difference equation:
\[
\frac{\overline{P} (\overline{Z} + \Delta \overline{Z}, \overline{T}) - 2 \overline{P} (\overline{Z}, \overline{T}) + \overline{P} (\overline{Z} - \Delta \overline{Z}, \overline{T})}{(\Delta \overline{Z})^2}
\]  
(26)
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\[ \alpha \frac{[P(Z, T + \Delta T) - P(Z, T)]}{\Delta T} - 2\pi \cos(2\pi \bar{T}) \]  

(30)

The boundary conditions and the initial condition are:

\[ \bar{P}(0, T) = \sin(2\pi T) \]  

(31)

\[ \bar{P}(1, T) = \bar{P}(1 + \Delta Z, T) \]  

(32)

\[ \bar{P}(Z, 0) = 0 \]  

(33)

The differential meshes in space and time used in the computation are respectively \( \Delta Z = 0.1 \) and \( \Delta T = 0.001 \). These mesh intervals are determined from the ones that give the stable and convergent solutions. The computation is continued for more than 3 cycles of waves, until a steady state of the oscillatory pore pressures is obtained. An example of the solution is illustrated in Figs. 2 and 3. It is clearly shown in Fig. 2 that the wave-associated pressure, \( P_b \), propagates into the seabed with damping and phase lag. The changes in the effective stresses, \( (p_b - p_m) \), are presented in Fig. 3, in which the \( (p_b - p_m) \) is

---

**Fig. 2** Response of pore pressure
normalized by the amplitude of the wave-associated pressure, $p_w$. The vertical effective stresses oscillate continuously on both negative and positive sides even in one period of wave loading. When the wave-associated stresses exceed the vertical effective stress at calm, the liquefaction occurs.

3. Laboratory Experiment

3.1 Outline of Experiment

Model tests were performed, under one dimensional wave loading conditions, with a newly developed apparatus shown in Photo. 1 and Fig. 4. The apparatus is made up of 1) a cylinder for constituting the model seabed, 2) a pneumatic sine loading unit to apply the hydro-static and dynamic pressures, 3) measuring sensors and 4) a recording system. Air-dried Toyoura sand was used for the tests. The grain size accumulation curve and the physical properties are shown in Fig. 5 and Table 1, respectively. The sample was poured into the cells in the process of the fabrication of the cylinder. The total thickness of the deposit constituted in the cylinder was 0.28 m or 1.90 m. The initial relative density was set approximately at 50%. The carbon dioxide gas was circulated before filling the water. The hydro-static pressure of 1.0 kgf/cm² was applied to the deposit and it was left for more than 10 hours. Then, the dynamic pressures were applied by the so-called stage test.

The sinusoidal wave forms of the dynamic pressures were repeated up to 500 cycles in each loading stage. The double amplitudes of the hydro-dynamic pressures were between 0.197 kgf/cm² and 0.818 kgf/cm² and the wave periods were between 3 s and 15 s. The oscillatory pore pressures were measured at the depth shown by P1, P3, P5, P7, P9, P11, P13 and PL in Fig. 4. After the wave loading had been completed, the density of the deposit and the degree of saturation were measured by cutting out the deposit by the cell. More details of the experiments are presented elsewhere (Zen et al., 1987).
Wave-induced Liquefaction in a Permeable Seabed

**Photo 1** Test apparatus

**Fig. 4** Schematic drawing of test apparatus

**Fig. 5** Grain size acculation curve

**Table 1** Physical properties of sands

<table>
<thead>
<tr>
<th></th>
<th>Samples</th>
<th>Toyoura sand</th>
<th>Hazaki sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity $G_s$</td>
<td></td>
<td>2.674</td>
<td>2.689</td>
</tr>
<tr>
<td>Maximum dry unit weight $\gamma_{d_{\text{max}}}$ (gf/cm$^2$)</td>
<td></td>
<td>1.656</td>
<td>1.600</td>
</tr>
<tr>
<td>Minimum dry unit weight $\gamma_{d_{\text{min}}}$ (gf/cm$^2$)</td>
<td></td>
<td>1.345</td>
<td>1.233</td>
</tr>
<tr>
<td>Maximum void ratio $e_{\text{max}}$</td>
<td></td>
<td>0.988</td>
<td>1.181</td>
</tr>
<tr>
<td>Minimum void ratio $e_{\text{min}}$</td>
<td></td>
<td>0.615</td>
<td>0.681</td>
</tr>
<tr>
<td>Mean grain size $D_{50}$ (mm)</td>
<td></td>
<td>0.181</td>
<td>0.16–0.17</td>
</tr>
<tr>
<td>Uniformity coefficient $U_e$</td>
<td></td>
<td>1.79</td>
<td>1.5–1.6</td>
</tr>
</tbody>
</table>
The permeability tests were performed to measure the coefficient of permeability, \( k \). To measure the coefficient of volume compressibility, \( m_v \), cyclic compression tests were executed with a steel mold. The inner diameter and the height of the mold are 305 mm and 350 mm, respectively. The loading, unloading and reloading were repeated up to 5 cycles by applying the weight step by step. The vertical settlements were measured at each loading step. The \( m_v \) was calculated from the vertical stress and strain relationships. The coefficient of permeability, \( k \), and the coefficient of volume compressibility, \( m_v \), were used to obtain the coefficient of consolidation, \( C_v \).

The coefficient of propagation, \( \alpha \), was basically determined by introducing the n/\( m_v \), \( p_m \) and \( S_r \) into Eq. 17. It is noted that much attention should be paid to the measurement of the \( S_r \) because the \( \alpha \) is sensitive to the \( S_r \). Other practical ways to determine the \( \alpha \) are presented elsewhere (Zen et al., 1987; 1989).

### 3.2 Pore Pressure Response

**Figure 6** is an example of the wave-associated pressures recorded at the depths corresponding to the transducers P0 to P13 in **Fig. 3**. The \( \bar{p}_a \) is the amplitude of the applied water pressure on the surface of deposit. It is clearly shown in **Fig. 6** that the oscillatory pore pressure at the time, 2.25 s, say at the 270 degree phase, attenuates from \( p_{mg} = -0.243 \text{ kgf/cm}^2 \) to \( p_{mg} = -0.190 \text{ kgf/cm}^2 \) with the depths. In addition, it is found that their peak values shift to the right indicating greater phase lags at the lower depths. The attenuation is more definitely illustrated in **Fig. 7** in which the oscillatory pore pressure, \( p_m \), is normalized by the amplitude of the applied water pressure, \( p_a \). At the bottom of deposit, only 80\% of the \( p_a \) is propagated. The phase lags of the pore pressures, \( \theta \), are drawn in **Fig. 8**. The phase lags attain 20 degrees at the bottom of deposit. **Figure 9** shows the relationship between the pore pressure ratio, \( p_m/p_a \), and the wave period, \( T \), at the non-dimensional depth of \( z/l = 0.45 \), where the \( l \) is the thickness of permeable deposits. The plots in **Fig. 9** include other investigators’ data. From **Figs. 7** to **9**, we can understand that the water pressure on the seabed surface propagates into the seabed with some damping and phase lag. This means that the assumption used in the derivation of Eq. 5 in section 2.2 is reasonable.

### 3.3 Liquefaction due to Oscillatory Excess Pore Pressure

The observed oscillatory pore pressures at the number of waves, \( N = 10 \) cycles, are illustrated in **Fig. 10** together with the theoretically calculated ones. In the measurements, the following wave conditions were applied; the double amplitude of the applied pressure, \( 2p_a = 0.247 \text{ kgf/cm}^2 \) and the wave period, \( T = 7 \text{ s} \). The thickness of the layer was 1.90 m. The coefficient of drainage, \( C = 5.57 \), measured with the element tests and the coefficient of propagation, \( \alpha = 2.3 \), determined from Eq. 17 were used for the calculation as input data. The measured dynamic pressures at the deposit surface, \( p_a \), were introduced into the computation of the \( p_m \) as the boundary conditions.

**Figure 10(a)** shows the pressure ratios at the non-dimensional depths, \( z/l = 0.0 \), \( z/l = 0.45 \) and \( z/l = 1.0 \). **Figure 10(b)** indicates the oscillatory excess pore pressures, \( u_e \), at the typical phases, \( \omega t \), between 0 and 360 degrees. **Figure 10(c)** denotes the vertical effective stress variations. It is found from **Fig. 10** that the results calculated by using **Eqs. 6, 8 and 15** compare very well with the observed ones. Therefore, the theory proposed in chapter 2 can be said applicable to assess the liquefaction potential.

According to the liquefaction criteria expressed by **Eq. 9**, the liquefaction will take place where the \( u_e \) exceeds the \( \sigma'_{ve} \). The \( u_e \) is compared with the \( \sigma'_{ve} \) drawn by the dotted line in **Fig. 10(b)** but the depth where the \( u_e \) exceeds the \( \sigma'_{ve} \) is not observed. This means
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Fig. 6 Measured pore pressures

Fig. 7 Pressure ratio

Fig. 8 Phase lag

Fig. 9 Pressure ratio and wave period (after: a) Tsui and Helfrich (1983), b) Yamamoto (1977), c) Inoue (1975), d) Maeno and Hasegawa 91985), e) Okusa et al. (1984))
Test No. 3, Stage No. 1, \( \alpha = 2.3, C = 5.57 \)

Observed \quad Calculated

\( \frac{p}{p_0} \) vs. \( \omega t \)

(a)

\( u_e = -(p_s - p_m) \) (kgf/cm²)

(b)

\( \sigma_v = \gamma z + (p_s - p_m) \) (kgf/cm²)

(c)

Fig. 10 Comparisons between theory and measurement in not-liquefied deposit; (a) Pressure ratio, (b) Oscillatory excess pore pressure, (c) Vertical effective stress
that no liquefaction is induced in the deposit in this test case. It is, however, noteworthy that the vertical effective stresses vary periodically as shown in Fig. 10(c). Especially, the vertical effective stresses become smaller than the initial vertical effective stresses at the phases between 140 and 290 degrees. This fact is very important from engineering points of view since it will lead to the reduction of the shear strength under the initial state without waves.

The same comparisons as in Fig. 10 are presented in Fig. 11. In this case, the double amplitude of the applied wave pressure, $2P_o$, is increased to 0.756 kgf/cm². The coefficient of propagation, $\alpha = 1.7$, is used for the calculation. In Fig. 11(b), it is found that the excess pore pressures exceed the initial vertical effective stresses above a certain depth. The liquefaction criteria represented by Eq. 9 tells us that liquefaction occurs in such a case. The liquefied depth is more clearly illustrated in Fig. 11(c), in which the maximum depth of the liquefaction, 0.75 m, appears at the phase of 230 degrees.

The observed distributions of the oscillatory excess pore pressures in Fig. 11(c) reveal some peculiar shapes at the depth above 0.75 m after the liquefaction occurs, and they are quite different from the calculated ones. The assumption of Darcy’s law introduced in Eq. 15 becomes impertinent after liquefaction, since the suspended soil particles behave like a fluid. Post-liquefaction phenomena should be treated with some other approaches.

It is demonstrated in Fig. 11(c) that the water pressure oscillation induces the compressive stresses to possibly densify the deposit. Namely, the vertical effective stresses which exceed the initial vertical effective stresses are observed at the phases over 300 degrees. As already presented elsewhere (Zen et al., 1987), the densification in sand deposits may be attributed to these stresses repeatedly and alternately excited by waves.

3.4 Liquefaction due to Seepage Flow

The spatial differences of the oscillatory excess pore pressure in the seabed will exert seepage forces on the soil skeleton. As the increment of the vertical effective stress is represented by:

$$\frac{\partial \sigma^\prime_v}{\partial z} = j + \gamma'$$

(34)

the vertical effective stress is derived by integrating Eq. 34;

$$\sigma^\prime_v = \int_0^z j \, dz + \gamma'z$$

(35)

where $j$: the seepage force and $\gamma'$: the submerged unit weight of deposits. Equation 35 is equivalent to Eq. 6 when the $\sigma^\prime_v$ is identical with $\gamma'z$.

The seepage force, $j$, is derived from Eq. 6 by taking $\partial \sigma^\prime_{\omega}/\partial z = \gamma'$ and $\partial P_o/\partial z = 0$ into account:

$$j = -\frac{\partial P_m}{\partial z}$$

(36)
Fig. 11 Comparisons between theory and measurement in liquefied deposit; (a) Pressure ratio, (b) Oscillatory excess pore pressure, (c) Vertical effective stress
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The hydraulic gradient, $i$, and the flow velocity, $v$, are respectively given by the following equations:

$$i = -\frac{j}{\gamma_w}$$

$$v = -\frac{kj}{\gamma_w}$$ (37)

Figure 12 shows the seepage forces at the phase of 270 degrees calculated from the measured distribution of the $p_m$ by using Eq. 36. The negative values of the $j$, $i$ and $v$ mean that the upward flow of pore water is excited in the deposit. The upward seepage force under wave trough becomes maximum at the surface of the deposit and the hydraulic gradient attains more than 1.0. Considering the equilibrium of force, the critical hydraulic gradient, $i_c$, against the quick sand is expressed by:

$$i_c = -\frac{G_s - 1}{1 + e}$$ (39)

where $G_s$; the specific gravity and $e$; the void ratio. Against the usual ranges of values of the $G_s$ and $e$, the $i_c$ becomes nearly equal to 1.0. Thus, Fig. 12 definitely indicates that the quick sand, that is liquefaction, occurs in the deposit. Actually, the suspension of soil particles was observed in the model experiment.

The vertical effective stresses which can be calculated by using Eq. 35 and Eq. 6 are also illustrated respectively with the open circles and the solid line in Fig. 12. The $\sigma'_v$ in Eq. 35 is calculated by integrating the seepage force shown in Fig. 12 along the depth, while the $\sigma'_v$ in Eq. 6 is calculated from the measured excess pore pressure represented by Eq. 8. In spite of the different calculation, the comparison of the open circles with the solid line compare well. Judging from Fig. 12, the liquefaction, in other words quick sand, occurs at the depth shallower than 0.54 m.

Madsen (1974) concluded that the flow induced in a porous seabed by waves generally is of minor importance in the context of transport mechanics of soils. Carstens

![Fig. 12 Seepage force and liquefaction (model test)]
et al. (1976) demonstrated that the effects of pressure gradients, except the critical gradient, may be ignored in considering the scouring. As indicated in the previous section, however, liquefaction is considered to easily occur in real ocean environment. When liquefaction happens, the suspended state of soil particles will be transported as a fluid together with the bottom current. Furthermore, even if the liquefied depth is limited to very near the seafloor surface, a repeat of the liquefaction by a large number of cyclic wave loading may result in critical scouring which causes significant damage to offshore structures.

4. Field Observation

4.1 Outline of Observation

(1) Observation Site and Soil Profile

The Hazaki Oceanographical Research Facility (HORF) of the Port and Harbour Research Institute, Ministry of Transport, is utilized for the field observation (Zen and Yamazaki, 1991). The HORF is located at Hazaki in Ibaraki prefecture facing the Pacific Ocean as shown in Fig. 13. It extends 427 m offshore from the shore line thereby enabling the observation of waves, currents and other marine phenomena even in rough seas. A bird's-eye view of the HORF is shown in Photo 2. The height of deck is 7.5 m from the low water level and the width is 3.3 m. The field observation was performed at the tip of the HORF where the water depth is between 4 m and 6 m.

![Fig. 13 Observation site](image)

![Photo 2 Bird's-eye view of HORF](image)

Figure 14 shows the soil profile along the HORF. The seabed is composed of four layers indicated by A_1, A_2, D_1, and D_2. The A_1 and D_1 mean respectively the alluvial and diluvial sands. The sand deposit at the top of the layers, A_1, whose N-values are between 6 and 8, is the objective layer of the measurement. The physical properties of the sand taken from this layer are tabulated in Table 1. The grain size accumulation curve is in Fig. 5. The unit weight of soil is measured by taking undisturbed samples from the surface of the deposit. The measured unit weights were between 1.490 g/cm³ and 1.580 g/cm³. The average value of 1.543 g/cm³ corresponds to 0.969 g/cm³ of the submerged unit weight, γ'_s.

(2) Installation of Equipment

All sensors were fixed to a steel pile driven into the seabed as illustrated in Fig. 15. Five pore pressure gauges integrated in a stainless steel pipe with porous metals
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Fig. 14 Soil profile

Fig. 15 Installation of equipment
were penetrated into the seabed with a water jet, then fixed to the pile with a horizontal bar extended 1 m from the pile. Four earth pressure gauges, two for horizontal direction and two for vertical, were also fixed to the stainless steel pipe. The gauges for the horizontal earth pressure were installed, facing offshore, with an inclination of 40 degrees from the magnetic north to the east in the horizontal plane. The lower gauge for the vertical earth pressure was inclined 90 degrees against the upper gauge to avoid the possible effect of the upper gauge. The sand recorder was installed at the opposite side of the pipe. Since a lot of sediment movement at the breaker zone was expected, which would consequently change the water depth during stormy days, water depth change was monitored with the sand recorder. The equipment was reinstalled when a significant change in the seabed was observed. A current meter was also installed near the surface of the seabed. The ultrasonic wave gauge on the deck was utilized for the measurement of the wave height and wave period. The sensors specifications are listed in Table 2.

<table>
<thead>
<tr>
<th>Sensor/equipment</th>
<th>Type</th>
<th>Capacity</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pore pressure gauge</td>
<td>Digital Equipment type: AB-50</td>
<td>3.5 kgf/cm²</td>
<td>0.5% max of full scale</td>
</tr>
<tr>
<td>Earth pressure gauge</td>
<td>Kyowa Electronic Instruments type: BE-2 KM</td>
<td>2.0 kgf/cm²</td>
<td>0.5% max of full scale</td>
</tr>
<tr>
<td>Sand recorder</td>
<td>Sanyo Sokki type: SPM-V</td>
<td>3 m range</td>
<td>5 cm</td>
</tr>
<tr>
<td>Ultrasonic wave gauge</td>
<td>Keisoku Giken type: UHT 2-10</td>
<td>10 m max</td>
<td>5 cm</td>
</tr>
<tr>
<td>Inclinometer</td>
<td>Arec Electronics type: ACO-200</td>
<td>±70 degree</td>
<td>±1 degree</td>
</tr>
<tr>
<td>Electro-magnetic current meter</td>
<td>Arec Electronics type: ACM-200 PC</td>
<td>600 cm/s</td>
<td>0.5 cm/s</td>
</tr>
</tbody>
</table>

(3) Data Collection
The field observation was conducted intermittently for two years between 1988 and 1989. Three series of data collections referred to as Series Nos. 1 to 3 were performed. Series Nos. 1 and 2 were collected respectively from 18:00 on May 9, 1988 and from 20:00 on May 11, 1988. Series No. 3 were observed from 2:00 on April 24, 1989. The earth pressures were measured only in Series No. 3. Recording in each series was started at least several days after the installation of equipment to eliminate the effect of seabed disturbances. The data were collected for 20 minutes every two hours and continued for more than 24 hours during storm waves. The collected data were sent to the laboratory at the foot of the HORF and were stored in a data recorder and a personal computer.

4.2 Wave Characteristics
The wave characteristics observed at the site during April, 1988 and May, 1988 are shown in Fig. 16. Figure 16(a) is the significant wave height, $H_{1/3}$, read from the raw
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![Graphs showing wave characteristics](image)

**Fig. 16** Wave characteristics; (a) Significant wave height, (b) Significant wave period, (c) Significant wave height and wave period

data with the zero-down cross method. The maximum significant wave height was about 2.5 m in this term. The significant wave period, $T_{1/3}$, is shown in Fig. 16(b). The dominant period is between 7s and 9s. Figure 16(c) indicates the relationship between the $H_{1/3}$ and the $T_{1/3}$. The $T_{1/3}$ tends to become peak values in a certain range of waves when the $H_{1/3}$ exceeds 1 m. This may be attributed to the breaking of higher waves when they pass through the observation site.

### 4.3 Oscillatory Excess Pore Pressure

An example of the raw data on the wave-associated bottom pressure, $p_b$, the oscillatory pore pressure, $p_m$ and the sea surface elevation, $\eta$, is presented in Fig. 17. In Fig. 17, the hydro-static pressures are omitted taking account of the tidal level changes and only the dynamic components of the pressures are illustrated. The depths in Fig. 17 show the distance measured from the surface of the seabed. High frequent components that appear on the trains of pressures are probably caused by the turbulence due to the breaking of waves.

The typical records of about one minute are shown in Fig. 18. The water depth from the datum level (D.L.) measured by the sand recorder is also drawn in Figs. 18. Each wave is numbered for convenience such as Wave: Nos. 1 through 7. It is found from Fig. 18 that the oscillatory pore pressure, $p_m$, is sensitive to the wave-associated bottom pressure, $p_b$, although the high frequent components of the $\eta$ disappear on the train of the $p_b$.

The top pressure gauge used for the measurement of the $p_b$ was not always situated on the surface of the seabed because of the changes of water depth during storm waves. So, the effect of the difference in depth on the $p_b$ was compensated for by assuming the small amplitude waves.
Fig. 17 Example of raw data

Fig. 18 Oscillatory pore pressure (Series No. 1)
The oscillatory excess pore pressure, \( u_e \), is calculated using Eq. 8 from the measured \( p_b \) and \( p_m \). Figure 19 shows the distribution of the excess pore pressure at the typical phases of a wave denoted by the numbers 1 to 5 on the \( p_b \) curves in Fig. 19. The excess pore pressure varies periodically on both positive and negative values even in one wave period. Thus, if the positive excess pore pressure becomes larger than the vertical effective stress at calm, it causes liquefaction in the seabed. Whereas, the negative excess pore pressure may lead to the increase of the vertical effective stress following the densification in the seabed.

![Figure 19](image)

**Fig. 19** Distribution of oscillatory excess pore pressure; (a) Sinusoidal form of a wave, (b) Irregular form of a wave

When compared the positive excess pore pressure under wave trough in Fig. 19(a) with that in Fig. 19(b), the latter is considerably smaller than the former. Furthermore, the distribution curves in the latter are not symmetrical against the vertical axis. This is mainly due to the characteristics of wave forms of the \( p_b \) exerted on the seabed by breaking waves. The flat wave forms after the breaking of waves lead to the very small rate of changes of the \( p_b \). Consequently, a slightly positive excess pore pressure is generated in the seabed. For such waves as observed at the breaker zone, the existing linear wave theory can no longer be applied.

### 4.4 Effective Stress Variation

The wave-induced stresses in the seabed are measured in Series No. 3 with earth pressure gauges. The variations of the vertical and horizontal stresses are denoted respectively by the \( \Delta \sigma_v \) and \( \Delta \sigma_h \) in Fig. 20. The \( \Delta \sigma_h \) is negative in the horizontal direction rotated 40 degrees from the magnetic north to the east. As the records include the oscillatory pore pressures, they represent the total stress variations in the seabed. The trains of the stresses show similar response with each other except the \( \Delta \sigma_h \). When comparing the \( \Delta \sigma_v \) with the \( \Delta \sigma_h \), it is found that the high frequent fluctuations highlighted by the dotted circles appear only on the train of the \( \Delta \sigma_v \) at the time around 146.8 s, 164.1 s and 187 s. This suggests that some drastic changes may happen in the soil elements composing the seabed. Provided that the liquefaction occurs in the seabed, the soil elements behave like a fluid in response to the wave-associated pressures. The
high frequency fluctuations may be considered to have been brought by the hydrodynamic pressures due to the liquefied state of the soil elements.

In order to obtain the effective stress variations, the oscillatory pore pressures are subtracted from the wave-associated total stresses in Fig. 20. In the subtraction, the oscillatory pore pressures are compensated for with interpolation so as to correspond to the values at the depths of each earth pressure gauge. The results illustrated in Fig. 21 clearly demonstrate that the effective stresses vary periodically in the seabed.

It is noted that the horizontal effective stress variation, \( \Delta \sigma_{\text{h}} \), is 4 times as large as other stress variations, \( \Delta \sigma_{\text{v1}} \), \( \Delta \sigma_{\text{v2}} \), and \( \Delta \sigma_{\text{h2}} \). All the censors were checked after the observation and they were found to work well. The large value of the \( \Delta \sigma_{\text{h}} \) is thought that the phase lag, leading to the reverse sign in the earth pressure variation, \( \Delta \sigma_{\text{h}} \), against the oscillatory pore pressure, \( p_{\text{m}} \), amplified the calculated value of the horizontal effective stress, \( \Delta \sigma_{\text{h}} \), in the subtraction process of the calculation. The reason for the phase lag is, however, still unknown.

The wave-induced vertical effective stresses in the seabed observed and evaluated from the two different measurements are illustrated in Fig. 22. The dotted lines are obtained from the direct measurements of the \( \Delta \sigma_{\text{v1}} \) and \( \Delta \sigma_{\text{v2}} \) previously presented in Fig. 21. The noise-like fluctuations on the trains in Fig. 21 are filtered out in Fig. 22. The solid lines are evaluated ones using Eq. 7 from the measured \( p_s \) and \( p_m \). The wave-induced vertical effective stresses calculated using Eq. 7 compare fairly well with the directly measured ones. Therefore, it may be concluded that the wave-induced vertical effective stress in the seabed is estimated reasonably with Eq. 7.

It is noted that the solid line enclosed by the dotted circles in Fig. 22 reveals a convex form upward unlike the dotted curve. The time when the circles are drawn seems to correspond to that shown in Fig. 20. In addition, this is observed only on the train at
the depth of -0.13 m. In order to examine this in detail, the distribution of the wave-induced vertical effective stresses at the typical phases within one wave period are drawn in Fig. 23. The open plots are measured values and the solid plots connected with the solid straight lines are the estimated ones from Eq. 7. The numbers in Fig. 23 correspond to the phases numbered on the $p_s$ curves in Fig. 23. It is found that the open plots show remarkably poor agreements with the solid ones at the shallower depth, while they compare fairly well with each other at the larger depth. This suggest that some 'peculiar' phenomena might have occurred at the shallower depth. The meaning of the peculiar phenomena will be discussed in the next section.
Fig. 23  Distribution of wave-induced vertical effective stress; (a) Wave: No. 28 in Series No. 3, (b) Wave: No. 30 in Series No. 3

4.5 Occurrence of Liquefaction

The maximum oscillatory excess pore pressures, $u_{\text{max}}$, are presented in Fig. 24 to compare with the vertical effective stress at calm, $\sigma'_{\text{ver}}$. According to the liquefaction criterion expressed by Eq. 9, liquefaction is considered to occur at the depths where the $u_{\text{max}}$ exceeds the $\sigma'_{\text{ver}}$. Fig. 24(a) is an example of the results in which no liquefaction is observed because the oscillatory excess pore pressures are less than the $\sigma'_{\text{ver}}$. Whereas, in Fig. 24(b) the oscillatory excess pore pressures to Wave: Nos. 27, 28, 29 and 30 exceed the $\sigma'_{\text{ver}}$. This demonstrates that liquefaction takes place near the surface of the seabed.

In Fig. 24(b), it is quite interesting to note that Wave: Nos. 28 to 30 correspond to the dotted circles in Figs. 20 and 22 where some peculiar phenomena have been previously pointed out. Taking account of the liquefaction at the shallow depths, the high frequency waves on the train of the $\Delta \sigma'_r$, in Fig. 20 can be explained. Namely, after liquefaction, the liquefied deposit behaves like a fluid, exerting a hydro-dynamic pressure

Fig. 24  Liquefaction in seabed; (a) Not-induced, (b) Induced
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on the earth pressure gauge. Furthermore, under the liquefied state of soils, the excess pore pressure can not be defined any more. Equation 7 does not hold true for such a soil element as in a suspended state of soil particles. The reason for the disagreement in Fig. 23 at the shallow depths may be attributed to the occurrence of liquefaction.

4.6 Verification of Proposed Theory

(1) Comparison between Theory and Observation

The oscillatory pore pressure in the seabed, $p_m$, is one of the dominant factors in evaluating the liquefaction potential. In order to examine the validity of the governing equation expressed by Eq. 15, the theoretical results are compared with the observed data. The input data for the analysis are tabulated in Table 3. The coefficient of consolidation, $C_u$, was calculated from the permeability, $k$, and the coefficient of volume compressibility, $m_v$, which were measured in the laboratory with Hazaki sand taken from the observation site. The coefficient of propagation, $\alpha$, is determined with two methods (Zen and Yamazaki 1990a).

<table>
<thead>
<tr>
<th>Table 3 Input data for analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Submerged unit weight $\gamma'$ (g/cm$^2$)</td>
</tr>
<tr>
<td>Coefficient of permeability $k$ (cm/s)</td>
</tr>
<tr>
<td>Coefficient of volume compressibility $m_v$ (cm$^2$/kgf)</td>
</tr>
<tr>
<td>Coefficient of consolidation $C_u$ (cm/s)</td>
</tr>
<tr>
<td>Degree of saturation $S_r$ (%)</td>
</tr>
<tr>
<td>Thickness of permeable layer $l$ (cm)</td>
</tr>
<tr>
<td>Coefficient of propagation $\alpha$</td>
</tr>
</tbody>
</table>

In the solution of Eq. 15, the finite difference technique is adopted (Zen and Yamazaki, 1990b). As the wave-associated bottom pressure has irregular forms, the observed $p_b$ is directly introduced into Eq. 15 as the surface boundary condition. The thickness of permeable layer, $l$, is determined by calculating the minimum depth below which the effect of the thickness on the $p_m$ distribution disappears. The thickness thus obtained was 230 cm.

Figure 25 shows the comparisons of the trains of the oscillatory pore pressures and vertical effective stress ratio between the theory and observation. Figure 26 shows the comparison of the distribution curves of the oscillatory pore pressures between the theory and observation at the typical phases of the period of a wave. The theoretical results shown in Figs. 25 and 26 clearly indicate that the proposed equation can successfully estimate the oscillatory pore pressures and effective stress variations in the seabed.

(2) Transient Seepage Flow and Liquefaction

The wave-induced liquefaction in the seabed is closely related to the transient seepage flow. The seepage force, $j$, the hydraulic gradient, $i$, and the flow velocity, $v$, are respectively given by Eqs. 36 to 38. Therefore, the $j$, $i$ and $v$ are easily calculated from the distribution curves of the oscillatory pore pressure, $p_m$. The solid circles shown in Fig. 27 are calculated from the curve numbered $d$ in Fig. 26 which reveals the most rapid reduction of the $p_b$ (see the wave form of the $p_b$ in Fig. 26). The hydraulic gradient and seepage force become remarkably large near the seabed surface. Especially the hydraulic
Fig. 25 Trains of oscillatory pore pressure and effective stress ratio

Fig. 26 Distribution of oscillatory pore pressure

Fig. 27 Seepage flow and liquefaction (field measurement)
gradient attains more than 1.0 in absolute value near the surface of the seabed. This upward seepage flow is considered to cause the quick sand, that is the liquefaction, in the seabed. The liquefaction near the seabed surface creates a large potential for the transportation of suspended sand particles. This has been actually confirmed by the field measurement done by Tsuruya and Korezumi (1990). Meanwhile, the flow velocity in Fig. 27 appears too small to bring about the boiling of the liquefied sand deposit. Ohkawa et al. (1990), however, demonstrated that the velocity causing the boiling of liquefied deposits is two orders larger than the velocity of the seepage flow evaluated from Darcy's law. The actual flow velocity of the suspension in the liquefied zone might be far more larger than that evaluated from Darcy's law.

5. Procedure for Assessing Liquefaction Potential and Its Application

5.1 Two Types of Liquefaction

Wave-induced liquefaction is classified into two categories by the mechanism of the excess pore pressure build-up (Zen and Yamazaki, 1990a). The differences of the excess pore pressure are schematically illustrated in Fig. 28. Liquefaction due to the oscillatory excess pore pressures occurs transiently and appears periodically so many times during a storm wave, responding to each wave. Whereas, liquefaction due to the residual nature of excess pore pressures (referred to as the residual excess pore pressure in Fig. 28), happens at once after certain numbers of cyclic wave loading. This type of liquefaction is similar to that induced by earthquakes in the mechanism of the excess pore pressure build-up. The total excess pore pressure to cause liquefaction should be the superposition of both the oscillatory and residual excess pore pressures. In this study, however, only liquefaction caused by oscillatory excess pore pressure is treated.

![Fig. 28 Two types of liquefaction](image)

5.2 Influential Factors

Referring to Eqs. 1, 2 and 10, the influential factors on the oscillatory pore pressure, \( p_m \), are expressed by a function, \( F \):

\[
\begin{align*}
    p_m &= F \left( H, L, T, m_w, k, n, \gamma_w, m_w, h, z, l, t, S_r, N \right) \\
    & \quad \text{(40)}
\end{align*}
\]

Among these factors, the \( H, L, T \) and \( N \) are the factors depending on the wave
characteristics. The \( m_0 \) and \( n \) are ones related to the soil properties, and the \( \gamma_0 \) and \( m_w \) depend on the properties of pore water. The \( h, l \) and \( z \) are the geometrical factors. The \( k \) and \( S_0 \) are related to the properties both of soils and pore water. Although the number of waves, \( N \), is not included in Eq. 10, it is an important factor to be considered.

When a small amplitude wave is assumed and the non-dimensional pressure ratio, \( \bar{P} \), is introduced, Eq. 40 can be expressed by the following function, \( G \):

\[
\bar{P} = G \left( \bar{T}, \bar{Z}, C, \alpha, N \right)
\]  

When the \( \bar{T} \) and \( \bar{Z} \) are given, the influential factors on the \( \bar{P} \) are represented in terms of the \( C \), \( \alpha \) and \( N \). According to the laboratory model tests, the effect of the \( N \) on the \( \bar{P} \) was so small as to be disregarded (Zen et al., 1987).

5.3 Evaluation Procedure

The liquefaction criteria expressed by Eq. 9 includes three parameters; the wave-associated pressure at the seabed surface, the oscillatory pore pressure and the effective vertical stress at calm sea. These parameters can be assessed by the procedure presented in Fig. 29. As the waves are irregular in their forms in real ocean, it is quite difficult to prospect the forms in advance. So, the small amplitude of waves is assumed as a first approximation. The wave height, wave period and wave length during a storm are estimated statistically every one or two hours with data collected by observation or by wave hindcasting. Introducing wave characteristics and the water depth into Eqs. 1 and 2, the \( p_b \) is evaluated. The \( p_b \) is used for the calculation of the \( p_m \) as a boundary pressure condition. The coefficient of drainage, \( C \), and the coefficient of propagation, \( \alpha \), are determined from the laboratory tests performed by using the sample taken from the site. The \( p_m \) is calculated using Eq. 15 or Eq. 26 by giving the \( p_b \), \( C \), \( \alpha \) and \( l \). When the \( p_b \) and \( p_m \) are obtained, the oscillatory excess pore pressure, \( u_e \), is calculated with Eq. 8. The liquefaction is assessed by Eq. 9 by comparing the oscillatory excess pore pressure with the effective vertical stress at calm sea.

5.4 Application to Practical Problem

An extensive local scour at the edge of a detached breakwater foundation has been observed. A large number of concrete blocks, which had composed the detached breakwater in Niigata coast, were found widely and deeply spread in the seabed. These phenomena are supposed closely related to the wave-induced liquefaction of seabed sands, because once liquefaction occurs in the seabed, the seabed loses its shear strength and suspended sand particles are very easily transported by currents.

The procedure presented in the previous section was applied to the evaluation of the liquefaction potential in a permeable seabed beneath detached breakwaters shown in Fig. 30. The \( p_b \) is estimated from the wave characteristics observed near the site using the linear wave theory. The \( p_m \) is calculated from Eq. 15 and the liquefaction depth is assessed by using Eq. 9. The input data for the analysis are: the wave height, \( H_{\text{max}} \), is 7.2 m, the wave period, \( T \), is 11.8 s, the coefficient of propagation, \( \alpha \), is 2.0, the coefficient of consolidation, \( C_u \), is 0.71 m²/s, the submerged unit weight of deposits is 0.9 t/m³ and the thickness of permeable seabed is 20.5 m.

Figure 31 shows the oscillatory excess pore pressures and the effective vertical stresses beneath the toe and center of the breakwater. It is obvious from Fig. 31 that the liquefaction occurs at the toe. No liquefaction, however, takes place in the seabed beneath the center of the breakwater because of the large effective vertical
Wave-induced Liquefaction in a Permeable Seabed

Figure 29 Flow chart
stresses. Figure 32 denotes the liquefied zone in the seabed in front of the breakwater. The liquefied depth at the toe attains 1.25 m in maximum under wave trough. Figure 33 shows the circular failure of the rubble mound after the liquefaction. The safety factor during calm sea was more than 1.3, but it decreases to 0.90 due to the liquefaction in seabed sands. The submergence of concrete blocks into the seabed reported so far can be attributed to such a liquefaction-related instability of seabed.

The liquefied zone shown in Fig. 32 is computed for only one wave. Provided that the liquefied sand particles are completely transported after the liquefaction and no sand particles are supplied there, the scoured area gradually spreads widely and deeply in the seabed until no liquefaction occurs in the seabed. Thus, the wave-induced liquefaction is considered one of the significant reasons related to the scouring of seabed. The accuracy of the existing estimation methods of scouring zone should be improved by properly evaluating the transport mechanics of liquefied sand particles.
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Fig. 32 Liquefied zone

6. Conclusions

The mechanism of the wave-induced liquefaction in a permeable seabed was examined in terms of the oscillatory excess pore pressure. The liquefaction criteria were theoretically derived and the validity was confirmed by model experiments. Furthermore, the wave-induced liquefaction was observed at a breaker zone in real ocean environment. In spite of the complex irregularity of wave forms, the results of the field measurement were basically the same as those observed in the laboratory experiment. On the basis of theoretical and experimental studies, a procedure for assessing the liquefaction potential was presented and its application to a practical problem was demonstrated.

The major conclusions drawn from the study are as follows:

(1) The wave-induced effective stress in the seabed varies periodically on both positive and negative values following the passage of ocean waves. The variations are attributed to the oscillatory excess pore pressures generated by waves.

(2) The variation of the vertical effective stress is equivalent to the reverse sign of the oscillatory excess pore pressure. It is expressed by the difference between the wave-associated pressure on the seabed surface and the oscillatory pore pressure in the seabed.

(3) The liquefaction potential in the seabed can be evaluated by using the proposed liquefaction criteria, which are composed of the wave-associated pore pressure on the seabed surface, the oscillatory pore pressure and the effective vertical stress in the seabed at calm sea.

(4) The oscillatory pore pressure induced by the irregular waves can be successfully estimated by the equation derived in this study.

(5) The wave-induced liquefaction is a kind of quick sand caused by the transient seepage flow due to the rapid lowering of the sea surface during the passage of wave trough.

(6) By means of a case study it is demonstrated that the liquefaction should be taken into account in estimating the scouring of seabed and stability of breakwaters.

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References

Wave-induced Liquefaction in a Permeable Seabed


List of Symbols

\[ C = \text{coefficient of drainage} \]

\[ H = \text{wave height} \]

\[ H_{\text{sig}} = \text{significant wave height} \]

\[ h = \text{water depth} \]
\( i \) = hydraulic gradient
\( j \) = seepage force
\( L \) = wavelength
\( l \) = thickness of permeable layer
\( m_w \) = compressibility of pore water
\( m_{sw} \) = compressibility of pore water at fully saturated state
\( N \) = number of waves
\( n \) = porosity
\( \bar{P} \) = pressure ratio \((=p_m/p_a)\)
\( p_b \) = wave-associated water pressure at the surface of seabed
\( p_m \) = oscillatory pore pressure in seabed
\( p_{mg} \) = pore pressure in seabed in terms of absolute pressure
\( p_o \) = amplitude of \( p_b \)
\( \bar{p}_o \) = amplitude of applied dynamic pressure
\( T \) = wave period
\( \bar{T} \) = non-dimensional time \((=t/T)\)
\( T_{1/2} \) = significant wave period
\( u_e \) = excess pore pressure
\( v \) = velocity of seepage flow
\( \bar{Z} \) = non-dimensional depth \((=z/l)\)
\( \alpha \) = coefficient of propagation
\( \theta \) = phase lag
\( \sigma_v' \) = vertical effective stress
\( \sigma_{v,0} \) = vertical effective stress at calm sea
\( \Delta \sigma_h \) = horizontal total stress variation
\( \Delta \sigma_{h}' \) = horizontal effective stress variation
\( \Delta \sigma_v \) = vertical total stress variation
\( \Delta \sigma_{v}' \) = vertical effective stress variation