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目　次 (CONTENTS)

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目　次 (CONTENTS)

REPORT OF PORT AND
HARBOUR RESEARCH INSTITUTE
Vol.31, No.2 (June. 1992)

1. Numerical Simulation of the Shoaling of Irregular Waves Using a New Boussinesq Model

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Tomotsuka Takayama**
Naota Ikeda***

Synopsis

A new set of Boussinesq type equations is used in this paper to numerically simulate the shoaling of surface waves from deep to shallow water. The new equations, derived by Nwogu, one of the authors, are able to model larger coastal areas because of its improved linear dispersion characteristics. The Boussinesq model is able to simulate the nonlinear interactions that occur as a wave shoals, including the generation of lower and higher frequency waves. The effect of wave breaking and bottom friction can also be incorporated into the equations. Numerical results are presented for the shoaling of solitary, regular and irregular waves. Comparisons are also made with field data obtained from the Hazaki Oceanographical Research Facility(HORF). The results demonstrate that the new Boussinesq model is a practical tool for simulating the nonlinear shoaling of ocean waves from "deep" to shallow water.

Key Words : Numerical simulation, Boussinesq type equations, Wave shoaling, Irregular waves

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1．新ブシネスクモデルによる不規則波の浅水変形計算

Okey Nwogu*・高山知司**・池田直太***

要旨

本論文では，新ブシネスク型方程式を用いて深海から浅海までの波の浅水変形計算を行ったものである。新ブシネスク型方程式とは，著者の一人であるNwoguが開発したもので，線形分散特性を修正することによって広い海域での計算を可能にしたものである。ブシネスクモデルは，浅水変形における波の非線形干渉を計算できるとともに，砕波や底面摩擦の効果も取り入れることができる。孤立波や規則波，不規則波について浅水変形計算を実施した。波崎海洋研究施設の沖合いで観測された現地波浪データを用いて，波の浅水変形について数値計算を行い，観測値と計算値のスペクトル形状について比較した。その結果，本手法は，深海から浅海までの波の浅水変形計算に適用できることがわかった。

キーワード：数値計算，ブシネスク型方程式，浅水変形，不規則波

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Contents

1 INTRODUCTION

As ocean waves propagate from deep to shallow water, they undergo a significant transformation with changes in height, length, shape and direction. The transformation process is a complex one that involves shoaling, refraction, diffraction, reflection, wave breaking and the generation of currents. In this paper, we shall only examine the two-dimensional shoaling problem, i.e. the propagation of normally incident waves on a beach with parallel contours.

The simplest model of the shoaling process for periodic waves is the linear or Airy theory model. Assuming a relatively mild bottom slope, the conservation of energy flux can be applied locally at different depths to determine the variation of wave height with water depth. However, linear theory cannot predict the asymmetric wave profile and set-down of the mean water level that occur in shallow water prior to wave breaking. Thus, several authors have applied higher order wave theories to the shoaling problem. These include Svendsen and Brink-Kjaer(1972) who matched linear theory in deep water to cnoidal theory in shallow water; Sakai and Battjes(1980) who calculated shoaling wave parameters from Cokelet's(1977) theory; Stiassnie and Peregrine(1980) who matched Cokelet's theory to the solitary wave solution of Longuet-Higgins and Fenton(1974); and Sobey and Bando(1991) who extended Reinecker and Fenton's(1981) stream function theory to include set-down.

While considerable success has been achieved in developing theories for the shoaling of periodic waves, there is as yet no practical theoretical model for simulating the nonlinear transformation of irregular waves from deep to shallow water. In the absence of a complete solution of the nonlinear initial-boundary value shoaling problem, models based on either linear theory or Boussinesq equations have typically been used. The Boussinesq equations include the lowest order effects of frequency dispersion and nonlinearity. They can thus account for the transfer of energy between different frequency components, changes in the shape of the individual waves, and the evolution of the wave groups, in the shoaling of an irregular wave train (e.g. Freilich and Guza,1984). A major limitation of the commonly used form of the Boussinesq equations is that they are applicable to relatively shallow water depths. In order to keep errors in the phase velocity less than 5%, the water depth has to less than about one-fifth of the equivalent deep water wavelength (McCowan, 1987).

Recently, Nwogu(1992) derived an alternative form of the Boussinesq equations that is applicable to a wider range of water depths. The new equations employ the velocity at an arbitrary distance from the still water level as the velocity variable instead of the commonly used depth-averaged velocity. This significantly improves the linear dispersion properties of the Boussinesq equations in intermediate and deep water. A finite difference method is used to solve the new set of equations. Numerical examples are presented for the one-dimensional propagation of solitary, regular and irregular waves on a constant slope beach. The results of the numerical model are also compared with field data on the propagation of typhoons obtained from the Hazaki Oceanographical Research Facility (HORF).

2 GOVERNING EQUATIONS

2.1 Standard Form of Boussinesq Equations

The Boussinesq equations represent the depth integrated equations for the conservation of mass and momentum for an inviscid and incompressible fluid. The fluid motion is assumed to be irrotational and the seabed impermeable. The vertical velocity is assumed to vary linearly over the depth. This allows the governing Euler equations to be integrated over the depth, reducing the three-dimensional problem to a two-dimensional one. The latter assumption also results in a quadratic variation of the horizontal velocity and hydrodynamic pressure over depth. The most commonly used form of the Boussinesq equations for a sloping seabed is that derived by Peregrine(1967). The equations can be written as:

$$\left.\begin{aligned} \eta_t+\nabla\cdot[(h+\eta)\bar{u}] &= 0 \\ \bar{u}_t+g\nabla\eta+(\bar{u}\cdot\nabla)\bar{u}+\left[\frac{h^2}{6}\nabla(\nabla\cdot\bar{u}_t)-\frac{h}{2}\nabla[\nabla\cdot(h\bar{u}_t)]\right] &= 0 \end{aligned}\right\} \tag{1}$$

where $\eta(x,y,t)$ is the water surface elevation, $\bar{u}(x,y,t)$ is the depth-averaged velocity, $h(x,y)$ is the water depth, g is the gravitational acceleration, and $\nabla=(\partial/\partial x,\partial/\partial y)$. The equations include weakly nonlinear terms($\nabla\cdot(\eta\bar{u})$,($\bar{u}\cdot\nabla)\bar{u}$) and frequency dispersion terms of the form $\nabla(\nabla\cdot\bar{u}_t)$. The frequency dispersion terms allow waves of different frequencies to travel at different speeds. The equations can simulate the nonlinear shoaling of irregular, multi-directional waves in shallow water provided that the waves are not too steep and the changes in water depth are of the same order of magnitude as the water depth. These restrictions can be stated mathematically as:

$$O(a/h)=O((h/\ell)^2)\ll 1\ ; \quad O(\nabla h/h) = O(1), \tag{2}$$

where a is a typical wave amplitude and ℓ is a representative wavelength.

2.2 New Form of the Boussinesq Equations

A more general form of the Boussinesq equations that is a applicable to a wider range of water depths was derived by Nwogu(1992). The velocity, u_a, at an arbitrary distance $z=z_a(x,y)$ from the still water level is used as the velocity variable, instead of the depth-averaged velocity, $\bar{u}$. This improves the dispersion characteristics of Boussinesq equations in intermediate and deep water. The new form of the equations can be written as:

$$\left.\begin{aligned} \eta_t+\nabla\cdot[(h+\eta)u_a]+\nabla\cdot\left[\left(\frac{z_a^2}{2}-\frac{h^2}{6}\right)h\nabla(\nabla\cdot u_a)+\left(z_a+\frac{h}{2}\right)h\nabla[\nabla\cdot(hu_a)]\right] &= 0 \\ u_{at}+g\nabla\eta+(u_a\cdot\nabla)u_a+\left[\frac{z_a^2}{2}\nabla(\nabla\cdot u_{at})+z_a\nabla[\nabla\cdot(hu_{at})]\right] &= 0 \end{aligned}\right\} \tag{3}$$

The new form of the equations is similar to the standard form of the equations, except for an additional frequency dispersion term in the continuity equation.

2.3 Linear Dispersion Properties

The linear dispersion characteristics of the new set of equations is compared to that of the standard set of equations in this section. The linearized version of the standard equations for one horizontal dimension and constant depth can be expressed as:

$$\eta_t+h\bar{u}_x = 0 \tag{4}$$

$$\bar{u}_t+g\eta_x-\frac{h^2}{3}\bar{u}_{xxt} = 0 \tag{5}$$

The dispersion relation of the foregoing set of equations can be obtained by substituting a small amplitude periodic wave form with frequency ω and wavenumber k into the equations, and letting the discriminant vanish for a non-trivial solution, yielding

$$C^2=\frac{\omega^2}{k^2}=gh\left[\frac{1}{1+\frac{1}{3}(kh)^2}\right] \tag{6}$$

where C is the phase speed. The linearized, one-dimensional, constant depth version of the new Boussinesq equations can be written as:

$$\eta_t + hu_{\alpha x} + (\alpha + 1/3)h^3 u_{\alpha xxx} = 0 \quad (7)$$
$$u_{\alpha t} + g\eta_x + \alpha h^2 u_{\alpha xxt} = 0 \quad (8)$$

where $\alpha = 0.5(z_\alpha/h)^2 + (z_\alpha/h)$. Its dispersion relation can be obtained in a similar manner and is given by

$$C^2 = gh\left[\frac{1 - \left(\alpha + \frac{1}{3}\right)(kh)^2}{1 - \alpha(kh)^2}\right] \quad (9)$$

Although the foregoing relation is similar to the second-order dispersion relation presented by Witting(1984) and later used by Madsen *et al.* (1991), there are significant differences between the new set of Boussinesq equations used in this paper and those of Madsen *et al.* and Witting. Madsen *et al.* (1991) start off by assuming a dispersion relation of the form given by Witting(1984). They then introduce a new term to the momentum equation to produce the desired dispersion relation. The depth-averaged velocity is still used as the velocity variable and the equations are applicable in water of constant dapth only, i.e. their equations cannot be applied to shoaling and refraction problems. The present set of equations were consistently derived from the continuity equation and Euler's equations of motion and are applicable in water of varying depth.

As was noted by Madsen *et al.* (1991), depending on the velocity variable used or the value of α, different dispersion relations are obtained. If the velocity at the seabed ($z_\alpha = -h$) is used, $\alpha = -1/2$. Alternatively, if the velocity at the still water level ($z_\alpha = 0$) is used, $\alpha = 0$. The standard form of the Boussinesq equations which uses the depth-averaged velocity corresponds to $\alpha = -1/3$. The exact linear dispersion relation for Airy waves is given by

$$C^2 = gh\frac{\tanh kh}{kh} \quad (10)$$

The phase speeds for different values of α, normalized with respect to the linear theory phase speed of Eg.(10), are plotted as a function of relative depth is **Fig.1.** The relative depth is defined as the ratio of the water depth, h, to the equivalent deep water wavelength $\ell_0 = 2\pi g/\omega^2$. The "deep water" depth limit corresponds to $h/\ell_0 = 0.5$. The different dispersion equations are all equivalent in relatively shallow water($h/\ell_0 < 0.02$), but gradually depart from the Airy solution with increasing depth. The velocity at the still water level gives the poorest fit. The value $\alpha = -2/5$ was obtained by Witting(1984), using a(1,1) Padé apporoximation of tanh kh. The (1,1) Padé approximant gives a second-order dispersion relation that is accurate up to $O[(kh)^4]$, while the dispersion relation for the standard Boussinesq equations of Eg.(6) is only accurate up to $O[(kh)^2]$.

An optimum value of α for the range $0 < h/\ell_0 < 0.5$ was obtained by minimizing the sum of the relative error in the phase speed over the entire range. This gave a value $\alpha = -0.390$, corresponding to a velocity at an elevation $z_\alpha = -0.53h$, The normalized phase speed for this value of α is also shown is **Fig.1.** It gives a difference of less than 2% for the phase speed over the entire range. By comparison, the standard form of the Boussinesq equations($\alpha = -1/3$) has a phase speed error of 85% at a maximum h/ℓ_0 of 0.48.

The group velocity, C_g, which is associated with the propagation of wave energy(or the wave envelope), is also important in wave shoaling studies. The wave front, as well as the alternate groups of large and small waves that occur in irregular wave trains travel at the group velocity. The group velocity for the new form of Boussinesq equations is given by

$$C_g = \frac{d\omega}{dk} = C\left[1 - \frac{(kh)^2/3}{[1-\alpha(kh)^2][1-(\alpha+1/3)(kh)^2]}\right] \tag{11}$$

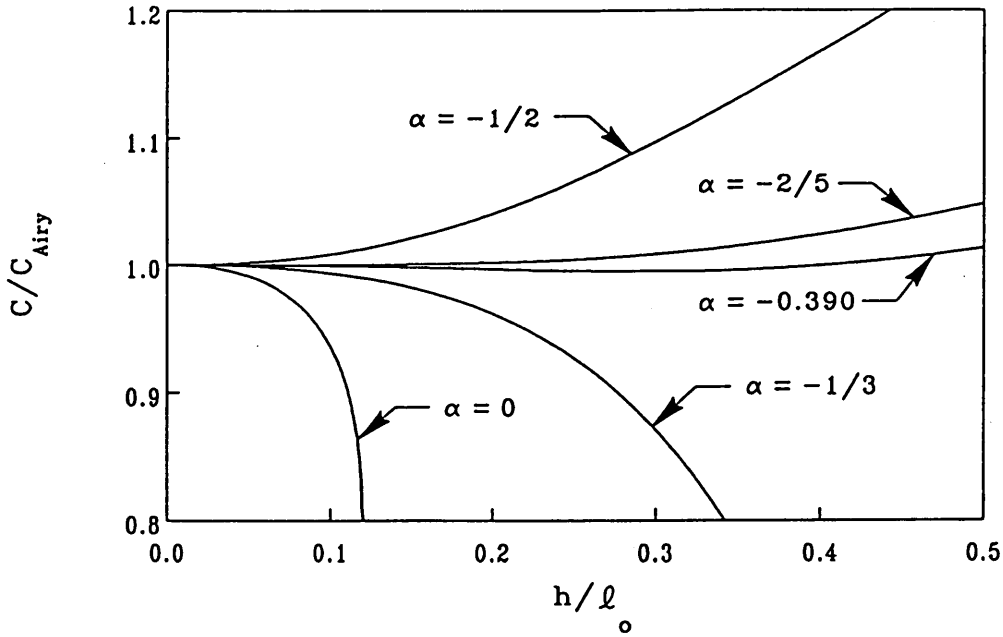

Fig.1 Comparison of the normalized phase speeds for different values of α.

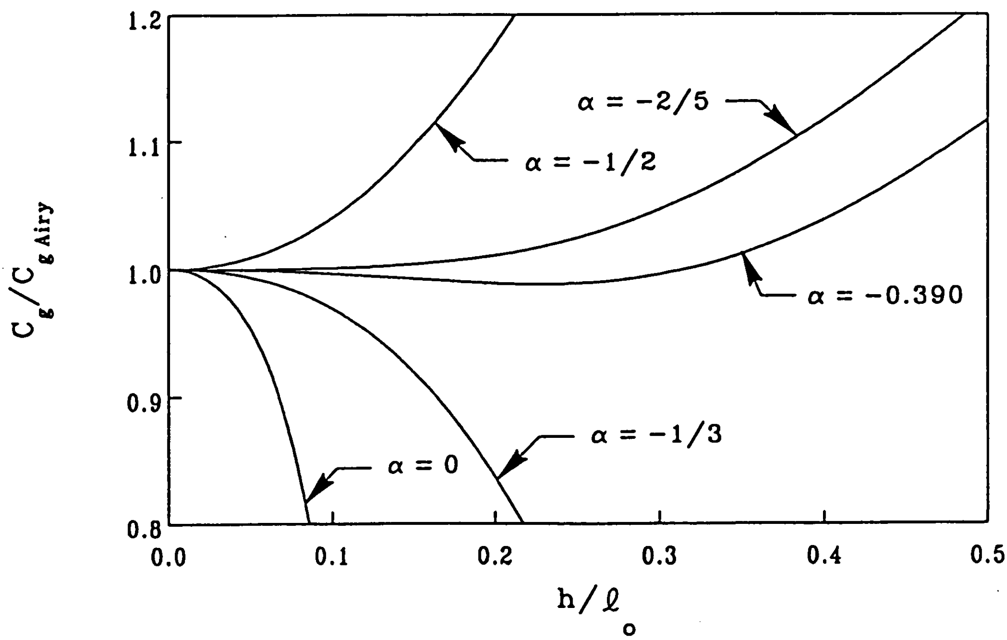

Fig.2 Comparison of the normalized group velocities for different values of α.

The normalized group velocities for different values of α are plotted as a function of relative depth(h/ℓ_0) in **Fig.2.** The group velocities are observed to deviate more rapidly away from the Airy relation than the phase velocities. The new value of α has a maximum group velocity error of 12%, compared to 100% for the standard Boussinesq equations.

In intermediate water depths with $h/\ell_0<0.3$, the differences between the phase and group velocities of the new Boussinesq model and Airy theory become negligible. For example, an α value of -0.393 gives errors of less than 0.2% for the phase speed and 1% for the group velocity over that range. In order to further illustrate the improvement in the dispersion properties of the new set of equations over the standard set of equations, if we apply a 1% maximum error criterion to the phase speed, $\alpha=-1/3$ gives a maximam $h/\ell_0=0.12$ while $\alpha=-0.393$ gives a maximum $h/\ell_0=0.42$. Applying the same error criterion to the group velocity gives $h/\ell_0=0.06$ for $\alpha=-1/3$ and $h/\ell_0=0.30$ for $\alpha=-0.393$. The new set of Boussinesq equations can thus model coastal areas to five times deeper than could be previously modelled with the same level of accuracy in the linear dispersion characteristics.

3 NUMERICAL RESULTS

The one-dimensional version of the governing differential equations (3) have been solved numerically using an iterative Crank-Nicolson finite difference method. Details of the third-order accurate finite difference scheme are given is Nwogu(1992). Appropriate boundary conditions are used to specify the waves propagating into the domain and absorb the waves propagating out. The numerical model has been used to simulate the shoaling of normally incident, long-crested waves on a constant slope beach. The ability of the new equations to model the propagation of both deep and shallow water waves is investigated. Examples considered include solitary, regular and irregular waves.

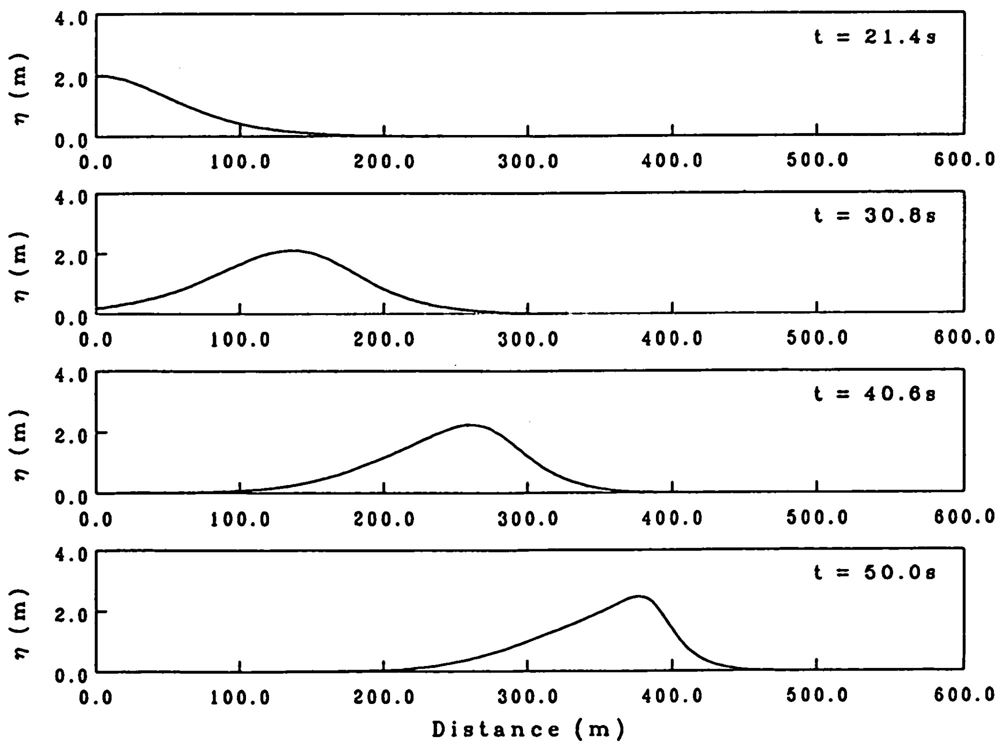

Fig.3 Propagation of a solitary wave on a constant slope beach ($\tan\beta=1/30$).

3.1 Solitary Waves

The numerical model was initially tested with the solitary wave example given by Peregrine(1967). Consider a solitary wave of height, $H=2$m, propagating on a beach with slope $\tan\beta=1/30$. The water depth at the incident wave boundary is 20m. The grid sizes for the computations were $\Delta x=2$m and $\Delta t=\Delta x/\sqrt{gh_0}$. The input surface elevation and velocity time series had the crest located at $t_0=21.4$s. **Figure 3** shows the variation of the water surface elevation with distance at different instants of time. The surface profiles computed by the new equations are very similar to those presented by Peregrine, and show the important features of a shoaling solitary wave train, such as the steepening of the crest front.

3.2 Regular Waves

The ability of the new Boussinesq model to simulate the propagation of a regular wave from deep to shallow water is now investigated. The standard form of the Boussinesq equations cannot simulate the shoaling of waves from deep water since its dispersion relation does not converge for $h/\ell_0>0.48$. Consider a wave train with period, $T=6.7$s, and height, $H=1.0$m, propagating on a beach with a slope of 1:25. The water depth at the incident boundary is 35m, while the depth at the absorbing boundary is 1.8m. The relative depth(h_0/ℓ_0) at the incident boundary is 0.5. Computations have been carried out using $\alpha=-0.390$ and grid resolutions $\Delta x=\ell_0/48$ and $\Delta t=T/48$. **Figure 4** shows the spatial profile of the propagating wave at an instant of time ($t=174.2$s). As expected, the wavelength decreases as the water depth decreases. The surface elevation also changes from a symmetric profile in deep water to an asymmetric one in shallow water.

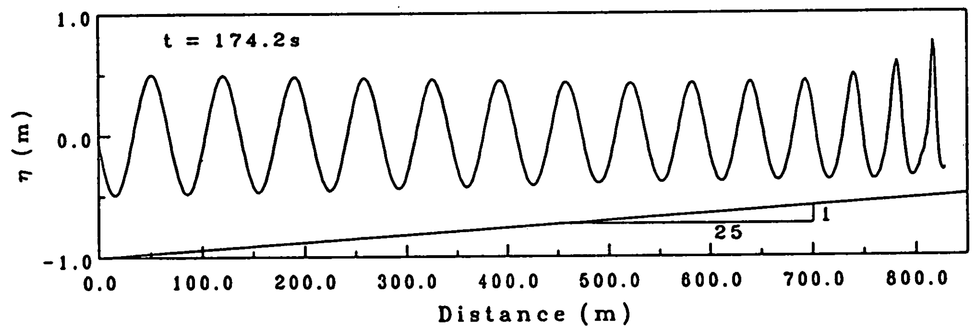

Fig.4 Propagation of a regular wave on a constant slope beach ($T=6.7$s, $\tan\beta=1/25$).

Figure 5 compares the change of wave height with water depth of the Boussinesq model with shoaling relationship predicted by linear wave theory, based on the conservation of energy flux. The Boussinesq model predicts a significant attenuation(~10%) of the wave height in intermediate water depths, before increasing in very shallow water. This is due to the cumulative effect of the neglected higher order terms in the Boussinesq model. Although the numerical model can simulate the propagation of a regular wave from deep to shallow water, the resulting error in wave height may be unacceptable for some engineering applications.

In intermediate water depths, the differences between the dispersion characteristics of the new Boussinesq and Airy waves become practically negligible. Let us now consider the shoaling of a wave train with $T=8.6$s and $H=1.0$m from intermediate to shallow water on the same beach. The relative depth at the incident boundary is now 0.3. Grid resolutions, $\Delta x=\ell_0/48$, and $\Delta t=T/48$, were also used in the simulation. **Figure 6** shows a comparison of the spatial profile of the shoaling wave obtained using $\alpha=-0.393$ and $\alpha=-1/3$ at an instant of time($t=150.5$s). The new Boussinesq model($\alpha=-0.393$) seems to reasonably model the shoaling process without the noticeable decrease in wave height that was observed for the deep water wave. The standard Boussinesq model($\alpha=-1/3$) has errors of 13% and 44% in the phase and group velocities respectively, leading to the observed phase lag and wave height differences.

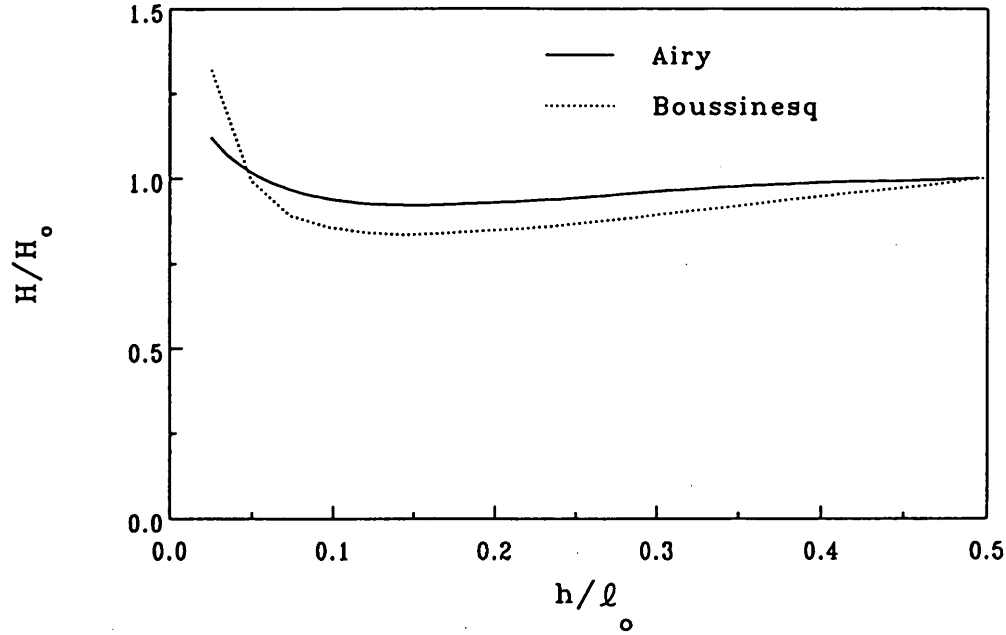

Fig.5 Comparison of the shoaling curves of Boussinesq and Airy models for a deep water incident wave.

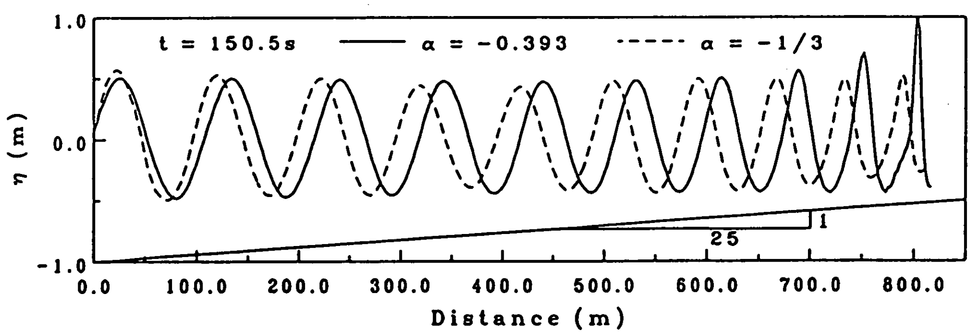

Fig.6 Propagation of a regular wave on a constant slope beach(T=8.6s, $\tan\beta=1/25$).

The shoaling curves of the new Boussinesq and Airy models compared in **Fig.7** for the intermediate water depth wave. The differences between both models is negligible in intermediate water depths. In shallow depths($h/\ell_0<0.06$) where the effect of nonlinearities become more important, the Boussinesq model predicts a larger wave height. The wave ultimately becomes unstable at a water depth of 2.2m when the ratio H/h is 0.75. This is quite close to the breaking or limiting wave height predicted by higher order wave theories in shallow water. However, the Boussinesq model only includes the first order of nonlinearity.

The above results demonstrate that the new Boussinesq model can reasonably simulate the shoaling of periodic waves from intermediate depths, but not from the deep water limit. A 1% maximum error for the group velocity would appear to be a reasonable criterion for regular wave shoaling studies.

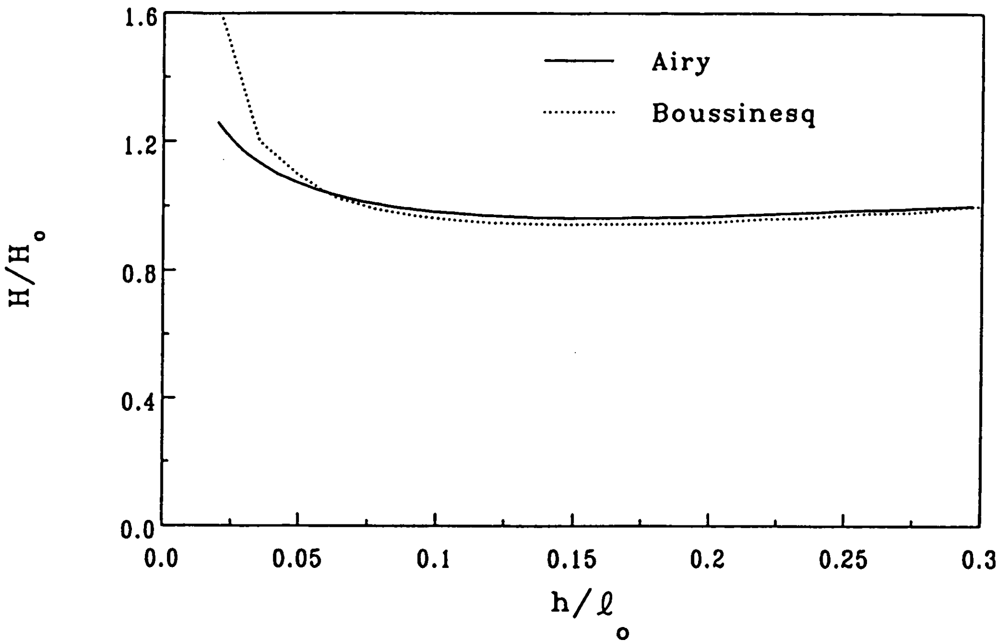

Fig.7 Comparison of the shoaling curves of Boussinesq and Airy models for an intermediate water depth incident wave.

3.3 Irregular Waves

A more important application of the new Boussinesq model is to the shoaling of irregular waves. There is at present no practical theoretical model that can simulate the transformation of irregular waves, from linear waves in deep water, to nonlinear waves in shallow water. The standard Boussinesq model can only be applied to problems where the maximum frequency of interest in the incident wave spectrum corresponds to a relative depth of about 0.2. This restricts its region of applicability to relatively shallow water, where nonlinearities are already significant. However, the new Boussinesq model can be applied to problems where the maximum frequency of interest corresponds to a relative depth of about 0.6 at the incident boundary.

The new Boussinesq model is now used to examine the shoaling of an irregular wave train on a 1:100 beach from a water depth of 40m at the incident boundary to 10m at the absorbing boundary. The input surface elevation and velocity time series were synthesized as a superposition of regular waves with random phases. A storm with a duration of 4096s was simulated at a time interval of 0.5s. The sea state was synthesized from a JONSWAP spectrum with significant height H_s=2m, peak period T_p=13.5s, and γ=3.3. The relative depth at the incident boundary is 0.56 for the maximum wave frequency of interest($\sim 2f_p$). The finite difference computations were carried out with a grid size Δx=3.5m. The generated surface profiles were checked to ensure that the waves did not break before reaching the absorbing boundary.

The spectral densities of the water surface elevation time histories obtained at different water depths are shown in **Fig.8.** The spectral estimates were averaged over frequency bands of width 0.004Hz(62dof). The spectral density at the 25m depth is quite similar to the input spectrum at the 40m depth. However, the spectrum at the 15m and 10m depths exhibit additional low and high frequency energy due to nonlinear wave interactions. Boussinesq models contain nonlinear terms and are thus able to generate and propagate waves at the sum and difference frequencies of the incident waves.

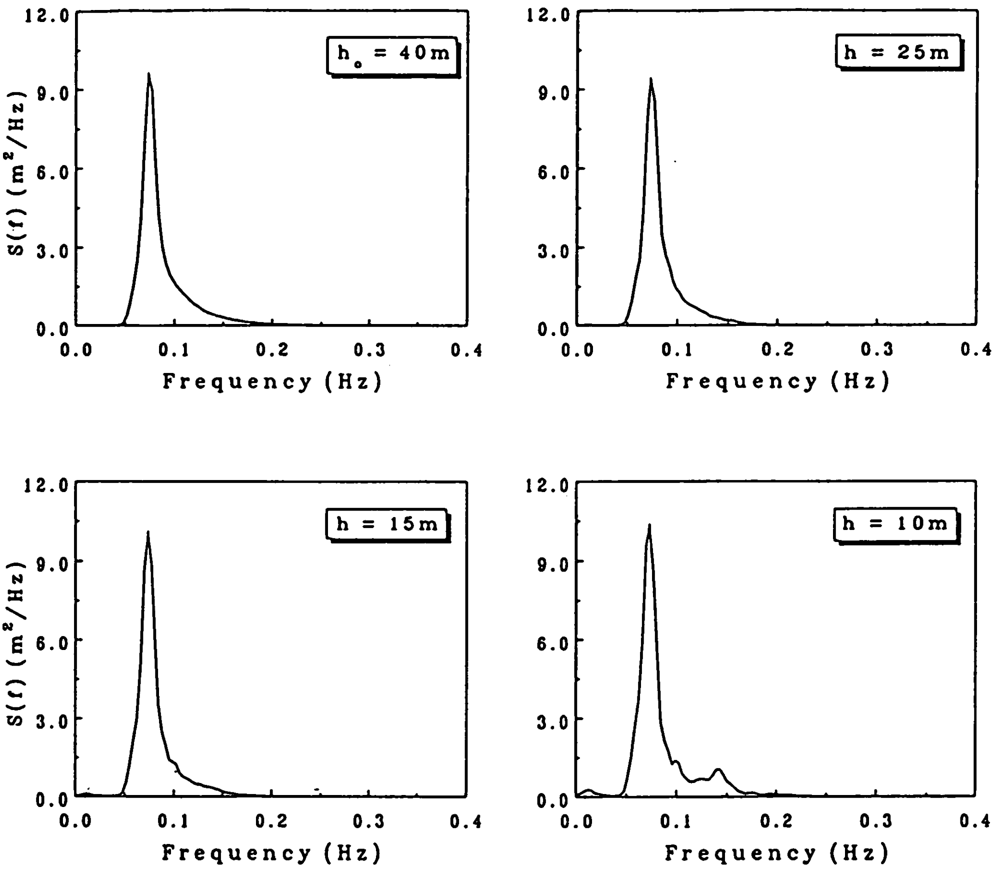

Fig.8 Spectral densities of the surface elevation of an irregular wave train propagating on a constant slope beach ($\tan\beta = 1/100$).

An important consideration in many coastal engineering problems is the long period waves(~1−5min) that are generated from the incident short wind waves. These long waves are also referred to as infragravity waves or surf beats. The Boussinesq model can simulate the generation and propagation of two types of long waves. The first one is the set-down beneath wave groups or bound long waves which increases in magnitude during the shoaling process due to nonlinear interactions. The second type is the free long waves that are generated by a reflection of the bound long waves from a beach. The absorbing boundary condition used in the numerical model absorbs the short waves but reflects the long waves.

4 FIELD EXPERIMENTS

As part of an ongoing study of beach processes in the surf zone, the Port and Harbour Research Institute carries out field measurements at the Hazaki Oceanographical Research Facility(HORF) near Kashima, Japan. The facility consists of a 427m long pier built on a sandy beach facing the Pacific Ocean. During the typhoon season, additional field data is collected in deeper water outside the surf zone. A detailed description of the facility and field experiments is given by Katoh *et al.* (1991). In this paper, we shall only examine part of the data that was collected during the week of 25 February 1989 to 1 March 1989. Three ultrasonic wave gauges were deployed at water depths of 9, 14 and 24m. The wave gauges were located at distances of 1.3, 2.1, and 3.2km, respectively from the shoreline. Biaxial electromagnetic current meters were also installed at the 9m and 14m depths to provide information on the directional distribution of wave energy. Data was continuously sampled at a rate of 2Hz for two hour durations, at six hour time intervals.

In order to assess the validity of applying the one-dimensional Boussinesq model to the field data, a preliminary analysis was initially carried out to estimate the mean angle of propagation and directional spread of the storm waves. The maximum entropy method, as implemented by Nwogu *et al.* (1987), was used to estimate the directional distribution of energy from the measured water surface elevation and horizontal orbital velocity data. The current meter data was transformed so that a direction of 0° represents waves propagating normal to the shoreline. **Figure 9** shows the directional distributions at the peak frequency for the storm that occurred on 28 February, 1989 at 13:30 hrs. The storm had a significant wave height, $H_s=2.2$m, and peak period, $T_p=13.4$s, at a water depth of 24m. The directional distribution at the 14m depth has a mean direction($\bar{\theta}$) of 8°, and standard deviation, $\sigma_\theta \simeq 15°$, while the distribution at the 9m depth has a $\bar{\theta}$ of 4° and $\sigma_\theta \simeq 15°$. These distributions are equivalent to a cosine power function($D(\theta) \sim \cos^{16}\theta$) which is relatively narrowbanded. Hence, the effects of refraction on the wave spectra at the 9m and 14m depths should be minimal.

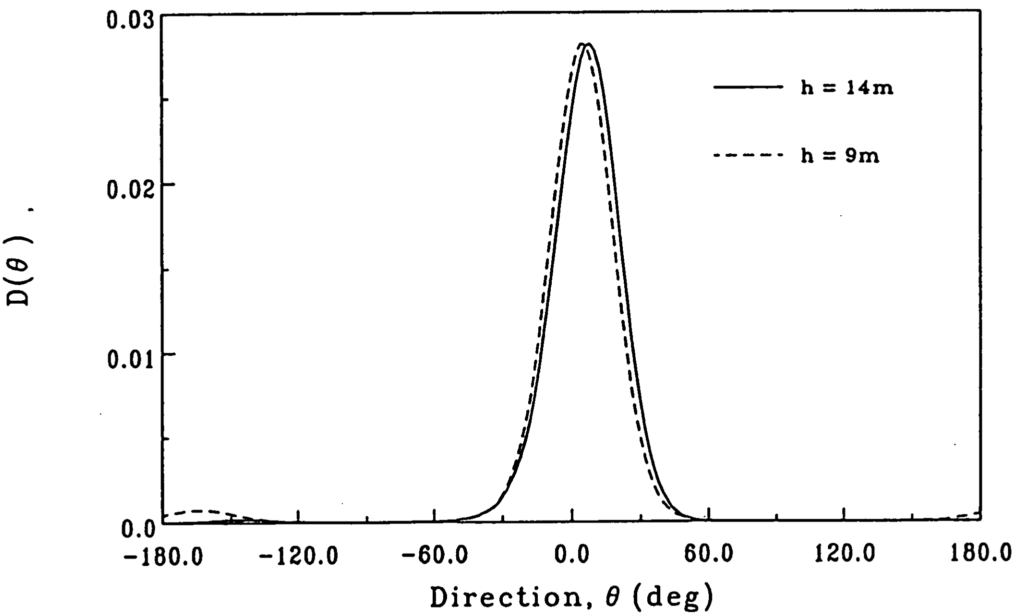

Fig.9 MEM estimated directional distributions at f_p.

The measured water surface elevation at the 24m depth was input into the numerical model as the offshore boundary condition. The numerical model was then used to simulate the water surface elevations at the 9m and 14m depths. The computations were carried out using a spatial resolution of 2m. The spectral densities of the measured surface elevations are compared with the numerical predictions in **Figs.10 and 11**. Good agreement is observed between the field data and the numerical model. The Boussinesq model slightly overestimates the long period wave energy at the 9m depth. This might be due to the reflected long waves being re-reflected at the offshore boundary in the numerical model or the directional spreading of the short waves reducing the amount of induced long wave energy.

In summary, the new Boussinesq model represents the best practical tool available at the present time for simulating the shoaling of irregular waves from "deep" to shallow water. It is a nonlinear model and can thus simulate the generation and propagation of waves at the sum and difference frequencies of the short waves. It can also reproduce the horizontal and vertical asymmetry of the water surface profile in shallow water. Although Boussinesq theory only includes weak nonlinearity, it can model the transformation of linear waves in intermediate water to very steep, near breaking waves in shallow water. The standard Boussinesq model is limited to shallow water because it only incorporates the leading order of frequency dispersion. The new Boussinesq model is, however, applicable to deeper water because of its second order dispersion properties.

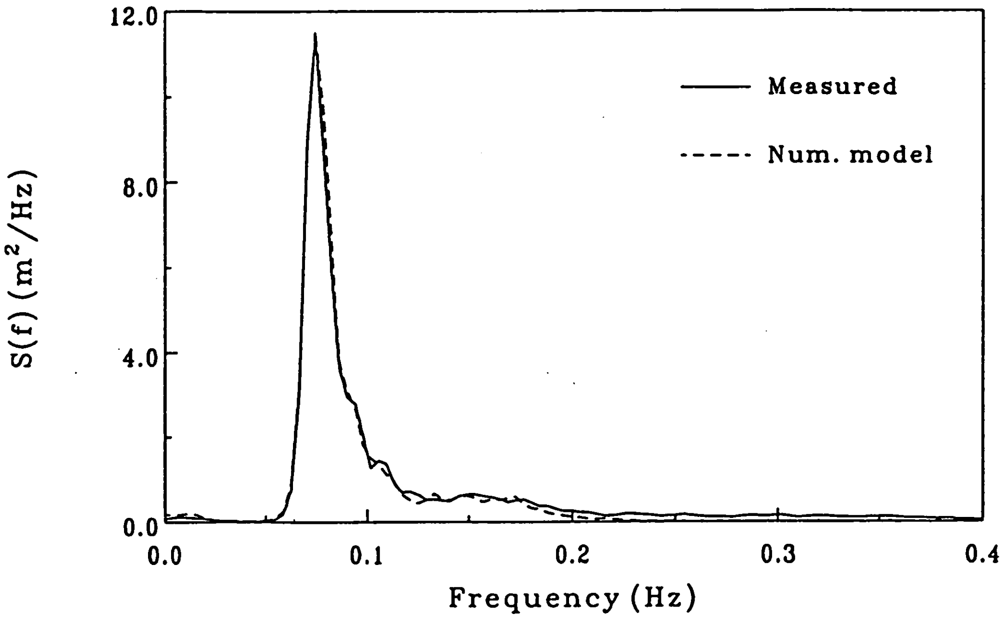

Fig.10 Comparison of measured and predicted surface elevation spectra at 14m depth.

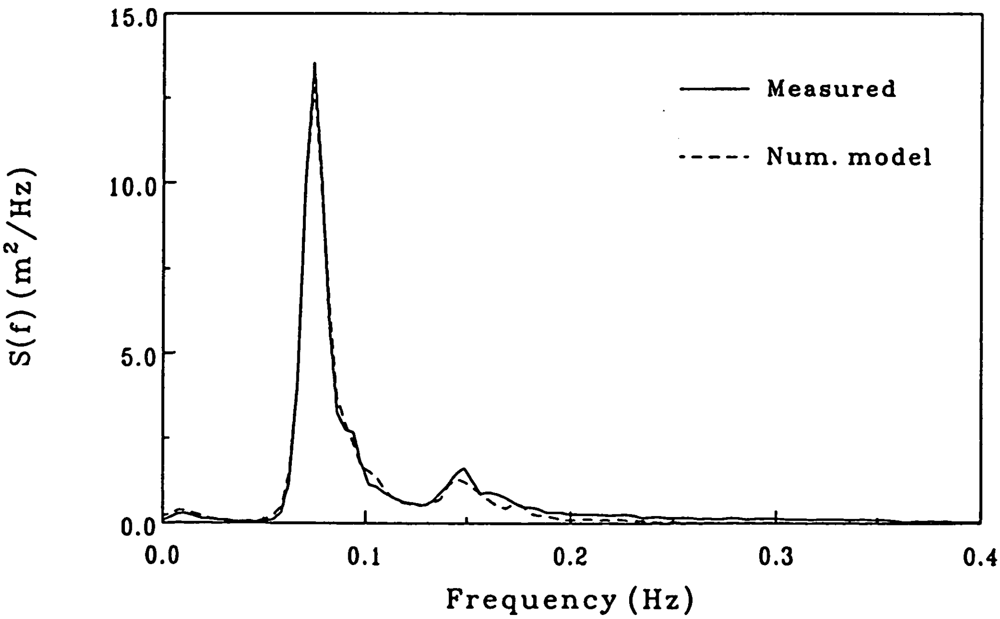

Fig.11 Comparison of measured and predicted surface elevation spectra at 9m depth.

5 CONCLUSIONS

A new set of Boussinesq type equations has been used to simulate the shoaling of regular and irregular waves from deep to shallow water. The new equations are applicable to water depths three to five times deeper than could be previously modelled because of its improved frequency dispersion properties. A finite difference method has been used to solve the equations. Numerical comparisons have shown that the new equations can reasonably model the propagation of regular waves from intermediate to shallow water. The numerical model has also been applied to field data obtained from HORF. The results demonstrate that the new Boussinesq model can reproduce several nonlinear effects that occur in the shoaling of irregular waves from deep to shallow water, including the generation of long period (infragravity) and short period waves. Additional work is being carried out to extend the model to include wave breaking in the surf zone. *(Received on March 31, 1992)*

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LIST OF SYMBOLS

a	: Typical wave amplitude
C	: Phase speed
C_g	: Group velocity
g	: Gravitaional acceleration
$D(\theta)$	: Angular spreading
H	: Wave height
H_0	: Deep water wave height
h	: Water depth
h_0	: Deep water depth
f	: Frequency
f_p	: Spectral peak frequency
k	: Wavenumber
l	: Representative wavelength
l_0	: Deep water wave length
$S(f)$	: Frequency spectrum
$\bar{u}$	: Depth-averaged horizontal velocity
u_α	: Horizontal velocity at z_α
x	: Horizontal axis
z	: Vertical axis possitive upwards
z_α	: Vertical distance from still water level
α	: $\alpha=0.5(z_\alpha/h)^2+(z_\alpha/h)$
β	: Bottom slope angle
η	: Water surface elevation
θ	: Wave direction
ω	: Angular frequency