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REPORT OF THE PORT &
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1. An Analysis of the Hydraulic Transport of Solids in Horizontal Pipelines

Tokuji YAGI*
Tadasu OKUDE**
Shoji MIYAZAKI***
Akio KOREISHI**

Synopsis

A head loss equation in the hydraulic transport of solids in horizontal pipelines has been developed theoretically assuming a flow model with both suspended solids and a sliding bed. Based on an analysis of the experimental results, two factors in the theoretical equation, or an apparent friction factor and the spatial solid concentration, were determined as a function of the flow parameter, $\psi=[V^2/gD(s-1)]\sqrt{C_d}$.

Two head loss parameters, $\phi=(i_m-i_w)/i_wC$ and $\phi^*=(i_m-i_w)/i_wq$, were examined against ψ. While, theoretical and experimental discussions were devoted to the ratio of the solid concentrations, q/C, and revealed that q increased steeply in the range of ψ less than, approximately, 3. Considerations into the critical velocity made it possible to introduce a dimensionless parameter, N_c, which is directly corresponding to a minimum head loss point and therefore available for judging the flow regime.

The experiments were conducted with a 155.2 mm pipe using sand of 0.91 mm in diameter and gravel of 8.75 mm, 27.5 mm and 45 mm, and with a 100.3 mm pipe using sand of 0.25 mm, 0.65 mm and 1.28 mm and gravel of 7.0 mm. Besides these, the experimental data collected in 1957 with a 80.7 mm pipe using sand of 0.75 mm and gravel of 8.0 mm were reexamined.

* Chief of the Hydraulic Transportation Laboratory, Machinery Division
** Member of the Hydraulic Transportation Laboratory, Machinery Division
*** Chief of the Sludge Treatment Laboratory, Marine Hydrodynamics Division

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1. 水平管路における土砂水力輸送の解析

八木得次*・奥出　律**・宮崎昭児***・是石昭夫**

要　旨

水平管路における土砂水力輸送時の圧力損失を，浮遊粒子と摺動粒子が共存する流れモデルを想定し理論的に導いた。理論式中の2つの変数，粒子群の見かけ摩擦係数と管内濃度は，実験により流れ変数 $\psi=[V^2/gD(s-1)]\sqrt{C_d}$ の関数として表わすことができた。

圧力損失係数，$\phi=(i_m-i_w)/i_wC$ および $\phi^*=(i_m-i_w)/i_wq$ と ψ との関係を調べると共に，管内濃度と吐出濃度の比を理論と実験の両面から検討し，$\psi<3$ の領域で管内濃度が著しく増大することがわかった。また限界流速を考察し最小圧力損失点に相当する無次元量 N_c を導き，これが流れ領域の判定に利用できることを示した。

実験は，155.2mm の管路で 0.91mm，8.75mm，27.5mm，45mm の砂と砂利を，また 100.3 mm の管路で 0.25mm，0.65mm，1.28mm，7.0mm の砂と砂利を輸送して行なった。この外，以前に行なった 80.7mm管路による 0.75mm と 8.0mm の砂と砂利のデータも再整理した。

* 機　材　部　流体輸送研究室長
** 機　材　部　流体輸送研究室
*** 海洋水理部　汚泥処理研究室長

Contents

1. Introduction

The recent remarkable enlargement of the scale of port and harbour works in our country has brought a sharp increase in the volume of earth that must be dredged to improve port areas and waterways, or that is required to construct reclaimed lands.(1) It may be instructive to mention that the total amount of earth that must be removed to make a new port in Kashima is estimated to be 120 million cubic meters, and that soil in the quantity of 69 million cubic meters is required for creating an artificial port island, being constructed in the Port of Kobe.(2) It is also interesting to note that in the "Five-Year Plan for Port Improvement" drafted in 1971, the amount of earth necessary for making another 40,000-hectare reclaimed areas for industrial use was predicted to reach into 3,300 million cubic meters.(3)

In order to convey such big amount of soil efficiently and economically within a limited construction period, a very careful consideration has to be given to the selection of an optimum conveyance method at the time of planning. Use of dump-trucks, motor-scrapers, belt-conveyors and hydraulic pipelines is the conventional but the most popular method of conveying soil by land.(2)(6) An application of the dredger-pipeline system and the barge-line system is quite practical for transporting soil by sea.(2)(4)(5) Estimation and comparison of the transportation capability and of the cost of construction of each method will be made from a pure technical viewpoint,(6) and an optimum method will be determined finally after special attention should be given to the social environment.

Whichever method we may adopt, the practical and reliable technical notes are required for evaluating the method. As far as the hydraulic transport of solid materials by pipelines is concerned, the most important design requirement is to transport the specified capacity of solid material continuously without clogging the line. To accomplish this, it is significant to predict head losses in the system with accuracy, or to determine the optimum operation conditions represented by the flow velocity and the concentration of solids, at which we can expect the maximum production at the minimum power consumption.

Interest in the transportation of solid materials by fluids in pipelines has increased in scope and volume as this means of transportation has a wide variety of application in industry......we may well say that one of the major applications of it exists in dredging operations.

Owing to its great potentialities, many investigators have devoted themselves exclusively to a study of the solid-liquid mixture flows in pipes theoretically and experimentally. The emphasis of their research has been on the prediction of the head loss in pipes that is one of the major factors governing the system. Unfortunately, however, no generally accepted theory and equation to describe the head loss accurately under various flow conditions has yet been established, for the flow mechanics of solid-liquid mixtures is so complicated.

The Hydraulic Transportation Laboratory of the Port and Harbour Research Institute, Ministry of Transport, has been engaged in the study of multi-phase flows as part of the hydraulic dredging studies. A series of laboratory experiments have been conducted for the purpose of obtaining the more practical relation that predicts the head loss in pipes in direct relation to dredging operations. In the experiments, we have concentrated our effort on collecting as many data as possible for the wide range of flow regime and on measuring the spatial concentration of solids in pipes.

In this paper, an analysis of the pressure drop for settling mixtures in horizontal pipes has been developed assuming a flow model with both suspended solids and a sliding bed. A coefficient in the theoretical equation, which may be defined as an apparent friction factor of solids in pipes, was determined by analyzing the experimental results. This paper is also devoted to a discussion of the ratio of the solid concentrations (the spatial solid concentration to the delivered one), of the critical velocity and of the soil coefficient that has been proposed for dredging operations. In order to evaluate the present study, the results were compared with those reported in the literature.

All experimental data totaling 2,213 points were compiled in the form of computer cards and written in the computer magnetic tape. By using the computer and the curve plotter on line, the calculated results were plotted directly on the diagram.

2. Theoretical Considerations

2.1 Assumption of a flow model

It is well known that the flow regimes or modes of transportation of solid-liquid mixtures by pipelines are affected by the following variables to be of importance:

(a) Pipeline factors: (i) diameter; (ii) slope; (iii) material, including relative roughness of pipe.

(b) Fluid factors: (i) density; (ii) viscosity; (iii) temperature.

(c) Solid particles factors: (i) density; (ii) particle size and particle size distribution; (iii) shape.

(d) System factors: (i) velocity of flow; (ii) concentration of solids; (iii) acceleration of gravity.

The classification of flow regimes differs slightly with each investigator. Fig. 1 is one of the classifications for a given liquid, solid material and pipe size, described qualitatively only.[7] There are four regimes; homogeneous flow, heterogeneous flow, saltation and stationary bed flow. Heterogeneous flow defined here is a regime in which all solid particles are in suspension, but the vertical solid concentration gradient is not

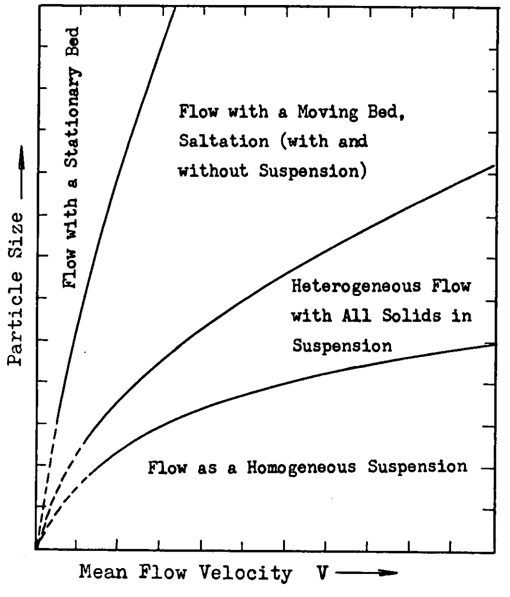

Fig.-1 Flow regimes for given fluid, solid and pipe size, qualitatively only[7] uniform. Flow by saltation and flow with a sliding bed are usually grouped together. This is due to the fact that the solid particles on the surface of the sliding bed may be picked up and conveyed further along the pipe by saltation. The flow behaviour of these particles is unstable and they often cause irregularities (ripples and dunes), which increase the head loss distinctively.

Heterogeneous flow is probably the most important regime of solids transportation through pipelines because the amount of material transported per unit power consumption may become maximum.[7] In the case of dredging, however, dredged materials often contain coarse solid particles like gravel and shells in fine sand since a dredging pump sucks up sea bottom materials excavated by a rotating cutter head and directly delivers them to the discharge line. Then the flow mode which we should encounter in this field is supposed to be saltation rather than heterogeneous flow, more strictly speaking, it may be the flow in a transition zone between above two regimes. This assumption can be justified by the fact that the rate of pipe abrasion is heavy at about one-third of the bottom pipe boundary.

We shall consider a flow model with a sliding bed at the bottom, suspended particles in the middle and a pure liquid layer at the upper portion of the pipe, as shown in Fig. 2. Observing this flow regime through a lucid pipe, we know, in most cases, that the solid particles in suspension cause irregularities and that there exists the difference in local flow velocities of solid particles both in suspension and in sliding. However, we shall assume that the moving velocity of every particle is equal and that they move horizontally only. We do not consider collisions among solid particles here. The particles are uniform in size, shape (to be spheres) and density.

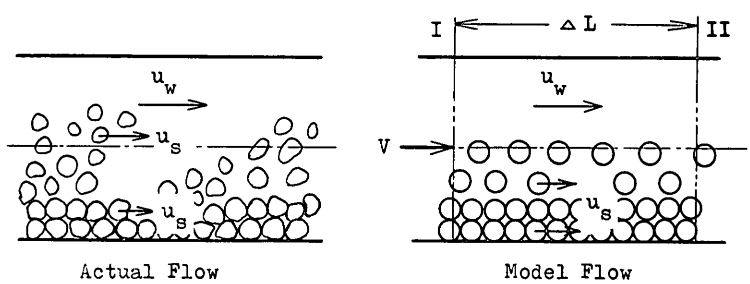

Fig.-2 A flow model with suspended solids and a sliding bed

2.2 An equation of motion of solids in a horizontal pipe

Probable forces that may act on a solid particle flowing in a horizontal pipe are:

(a) Drag force......due to the slip velocity, or the difference of flow velocities between solids and the fluid. It is usually given by

$$C_s a_s \frac{\gamma_w}{2g}(u_w - u_s)^2$$

in which a_s is the projected area of a solid particle in the direction of flow, γ_w is the unit weight of fluid (water), g is the gravitational constant, u_w and u_s are the mean flow velocity of water and of a solid particle, respectively, and C_s is the drag coefficient of a solid particle.

The drag coefficient, C_s, is known as a function of the particle Reynolds number, R_{es}, and expressed by $C_s = k/R_{es}^{\varepsilon}$, in which k and ε are the constants, of which numerical values are given by Stokes' Law, Allen's Law and Newton's Law of settling according to the separate ranges of R_{es}.[8]

(b) Friction resistance along the pipe......which acts on a solid contacting with the pipe boundary. Considering the buoyance on a solid in the fluid, it can be given by

$$\xi_s(\gamma_s-\gamma_w)v_s$$

in which ξ_s is the sliding coefficient of friction between a solid and the pipe wall, γ_s is the unit weight of solid particle and v_s is the volume of a particle.

(c) Gravity force......which acts on a particle in the vertical direction and of which magnitude is equal to the submerged weight of particle.

(d) Lift force......caused by asymmetry of the pressure distribution over the surface of a particle or by circulation around it.

(e) Forces due to collisions among solid particles......caused by collisions among solid particles resulting from the difference of their local flow velocities.

In this paper, however, we take no account of the forces in the vertical direction, then those forces described at (c) to (e), owing to our convenient treatment.

Taking the X-axis in the direction of flow, the mean flow velocity of the mixture is expressed by $dX/dt=V$ and its acceleration by $d^2X/dt^2=dV/dt$. Then, a general equation of motion of Z solid particles presenting in a given space, between a length of the horizontal pipe, ΔL, may be given as follows:

$$Z\frac{(\gamma_s-\gamma_w)v_s}{g}\frac{dV}{dt}=Z\eta_D C_s a_s\frac{\gamma_w}{2g}(u_w-u_s)^2-Z\eta_F\xi_s(\gamma_s-\gamma_w)v_s \tag{2.1}$$

in which η_D and η_F are the coefficients of correction to consider the difference of drag forces and that of friction forces on each particle, respectively. These coefficients of correction must be introduced into the equation of motion because the available drag coefficient, C_s, is not obtained for a particle suffering interference of neighbouring particles, but obtained for a single unbounded particle, and because all particles do not always act on, at least contact with, the pipe boundary as a bed load.

If the flow is steady, the mean flow velocity of the two phases, u_w and u_s, may hold constant, i.e., $dV/dt=0$. Under this state, Eq. (2.1) becomes

$$\eta_D C_s a_s\frac{\gamma_w}{2g}(u_w-u_s)^2=\eta_F\xi_s(\gamma_s-\gamma_w)v_s \tag{2.2}$$

The "steady-state" drag coefficient of a solid particle, C_d, is defined by

$$C_d=\frac{2g(\gamma_s-\gamma_w)}{\gamma_w v_t^2}\frac{v_s}{a_s} \tag{2.3}$$

in which v_t is the terminal settling velocity of a solid particle derived from the relation that the drag force acting on a particle falling at its terminal velocity, without acceleration, in unbounded still water is equal to the submerged weight of particle.

As we assumed that the particle was a sphere with a diameter d, combination of Eqs. (2.2) and (2.3) yields

$$\frac{u_w-u_s}{v_t}=\sqrt{\frac{\eta_F}{\eta_D}\frac{C_d}{C_s}\xi_s} \tag{2.4}$$

This theoretical approach is an application of the equation of motion developed by Uematsu and Kano[9] in the paper on the pneumatic conveyance of granular solids. Ayukawa and Ochi[10] have also developed the similar approach but they assumed C_s

$=C_d$ as the flow of materials used in their experiment was in Newtonian region.

2.3 Pressure drop for settling mixtures

As shown in Fig. 2, we shall consider a length, ΔL, of the horizontal pipe in which a solid-water mixture is flowing steadily at the mean flow velocity, V, at that of solids, u_s, and at that of water, u_w. If the pressure of the flowing mixture decreases from p to $p-\Delta p$ between the two sections, I and II, the pressure differential, Δp, may be greater than the pressure differential, Δp_w, to be caused in the case of water alone flowing in the same section at the same mean flow velocity, V. This increase in the pressure drop, $\Delta p-\Delta p_w$, is directly related to the presence of solid particles in the fluid medium. It can be considered as the excess pressure drop, Δp_s, that was expended to convey solids from the sections I to II.

Δp_s may consist of the two components; the pressure drop, Δp_{ss}, expended to transport the suspended particles and the pressure drop, Δp_{sf}, expended to convey the sliding bed. This conception can be formularized as

$$\Delta p=\Delta p_w+\Delta p_s=\Delta p_w+\Delta p_{ss}+\Delta p_{sf} \tag{2.5}$$

If we can assume that the magnitude of forces necessary for transporting the suspended solids is equal to that of drag forces acting on them, and that forces necessary for conveying the sliding bed are equal to the friction forces between the sliding solids and the pipe wall, the following equilibrium of forces may hold for the mixture in this section:

$$pA=\Delta p_w A+Z_s\eta_D C_s a_s\frac{\gamma_w}{2g}(u_w-u_s)^2+Z_f\xi_s(\gamma_s-\gamma_w)v_s+(p-\Delta p)A$$

or

$$\Delta p=\Delta p_w+\frac{1}{A}\left[Z_s\eta_D C_s a_s\frac{\gamma_w}{2g}(u_w-u_s)^2+Z_f\xi_s(\gamma_s-\gamma_w)v_s\right] \tag{2.6}$$

in which A is the cross-sectional area of pipe, and Z_s and Z_f are the number of particles in suspension and in sliding, respectively. It is impossible to know the momentary numerical values of Z_s and Z_f, but the average ones may be obtained by

$$Z_s=\kappa A\Delta Lq/v_s \quad \text{and} \quad Z_f=(1-\kappa)A\Delta Lq/v_s \tag{2.7}$$

in which q is the spatial solid concentration (by volume) in the pipe, κ is the ratio of the number of particles in suspension to all particles presenting in the given space.

η_D is the similar coefficient of correction as we considered in the previous section. Eq. (2.6) does not include another coefficient of correction, η_F, because in this case all sliding solids act on the bottom of the pipe directly as a bed load.

Substitution of Eqs. (2.2) and (2.7) into Eq. (2.6) yields

$$\Delta p=\Delta p_w+[1+\kappa(\eta_F-1)]\xi_s(\gamma_s-\gamma_w)q\Delta L \tag{2.8}$$

Introducing a new parameter, μ, which is defined by

$$\mu=[1+\kappa(\eta_F-1)]\xi_s \tag{2.9}$$

Eq. (2.8) becomes

$$\Delta p=\Delta p_w+\mu(\gamma_s-\gamma_w)q\Delta L \tag{2.10}$$

If all particles would flow as a bed load motion, the value of κ is equal to zero and μ to ξ_s. This is the critical flow model. In actual flows, however, we seldom observe such uniform sliding motion since there exists the difference in local flow velocities of solid particles which usually have a wide particle size distribution. This means that μ should be treated not as a constant but as a function of pipeline, fluid, solid particles and system parameters.

Eq. (2.9) shows that μ is dimensionless and equivalent to the coefficient of friction between the moving solids in the flowing fluid and the pipe boundary. Then, it may be called the "apparent friction factor" of solids in pipes.

Δp_w represents the pipe friction loss for water alone, and then is given by the Darcy-Weisbach formula as

$$\Delta p_w=\lambda_w\frac{\Delta L}{D}\frac{\gamma_w}{2g}V^2 \tag{2.11}$$

in which λ_w is the friction factor of the pipeline for water and D is the diameter of pipe.

The second term on the right-side of Eq. (2.10) can also be expressed with the form of the Darcy-Weisbach formula. If we define the modified Froude number, F_D, by

$$F_D=\frac{V^2}{gD(s-1)} \tag{2.12}$$

in which s is the specific gravity of solid particles, or γ_s/γ_w, Eq. (2.10) can be rewritten as

$$\Delta p=\left(1+\frac{\mu}{\lambda_w}\frac{2q}{F_D}\right)\Delta p_w \tag{2.13}$$

The hydraulic gradient for mixture, i_m, will be derived from Eq. (2.13) as

$$i_m=\left(1+\frac{\mu}{\lambda_w}\frac{2q}{F_D}\right)i_w \tag{2.14}$$

in which i_w is the hydraulic gradient for water, being derived from Eq. (2.11) as

$$i_w=\lambda_w\frac{V^2}{2gD} \tag{2.15}$$

3. Experimental System and Procedure

3.1 Experimental system

The experimental systems used in the present study are shown in Figs. 3 and 4.

As seen in Fig. 3, by opening the feeder gate at the bottom of the sand feeder, the solid material stocked in the hopper slides down on the chute into the sand well continuously, and is sucked up with water from an intake of the suction pipe set in the well. The discharge pipe is flexible at the outlet and therefore the mixture can be led either to the cyclone or to the measuring tank by changing its position. The mixture discharged into the cyclone returns to the hopper directly, while the mixture led to the measuring tank is discharged to the solids recovery bucket after measuring its volume and weight. The capacity of the hopper is sufficiently large to store enough

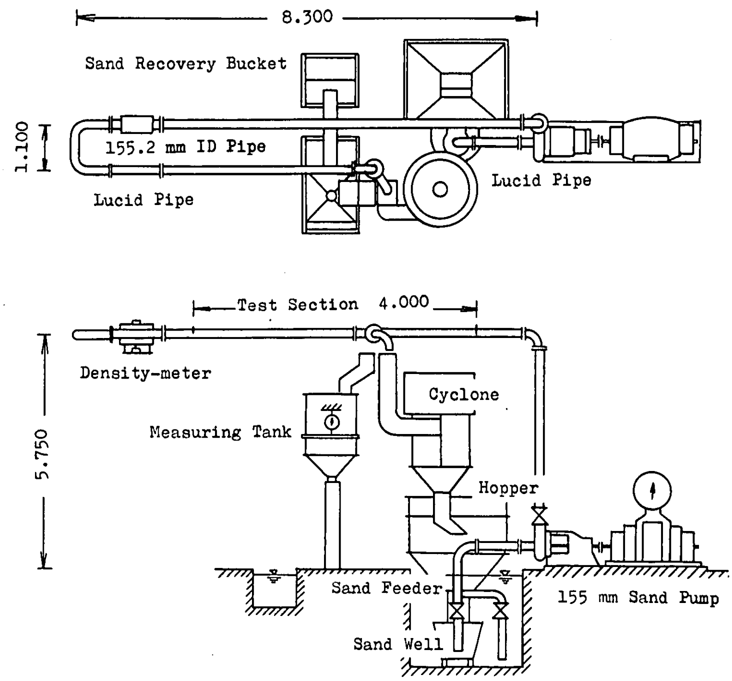

Fig.-3 Schematic description of experimental system with a 155mm pipe

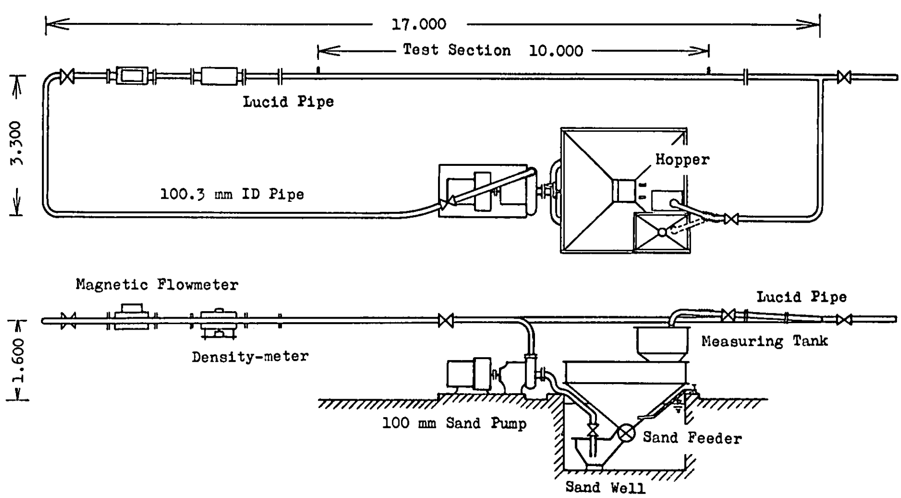

Fig.-4 Schematic description of experimental system with a 100mm pipe

volume of solids as we can adopt the experimental procedure described below.

The revolution of the pump is controllable. The pressure was picked up out of the side wall and the distance between the two taps was 4.0 meters. The pressure differential was measured by means of the Hg-manometer.

The layout of the system shown in Fig. 4 is fundamentally the same as that of Fig. 3, except that the length of the horizontal section was extended to 17 meters, and that the pipe of the test section can easily be changed with pipes with different sizes, and that the weighing system was installed above the hopper to save much trouble of recovering the solids delivered.

The revolution of the pump is controlable. The pressure taps are fixed on the side wall spacing 10.0 meters. A set of magnetic flow-meter, density-meter and differential pressure transducer were equipped with the system. All of their signals were led to the amplifier and recorded by the oscillograph continuously.

The density-meter used in the experiments is shown in Fig. 5, of which original model had been used by Hasegawa et al.[11] It has a floating pipe connected with fixed ones with a pair of thin rubber sleeves. Stiffness of the rubber sleeve may sometimes cause hysteresis, but if we are very carefull in chosing its quality this disadvantage can be minimized. As one countermeasure to reduce its thickness, the static pressure of the fluid in and out-side of the sleeve is balanced.

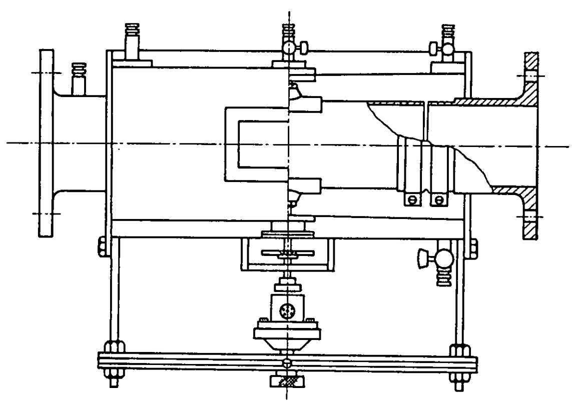

Fig.-5 Density-meter

3.2 Experimental procedure

Firstly, the flow rate and the pressure drop were measured under transporting water alone. Then, the feeder gate was opened and its opening area was adjusted observing the flow behaviour through the lucid pipe and checking the values of the spatial concentration of solids detected by the density-meter. Upon confirming that the flow becomes steady, the flow rate, the sptial and delivered solid concentrations, and the pressure drop were measured at the same moment. The measurement was repeated two times at a certain interval. Then, the feeder gate was closed and the measurement for the flow of water alone was taken again. This procedure was repeated many times changing the flow rate and the concentration.

The flow rate was mainly adjusted by controlling the revolutions of pump. In the case of low revolutions, however, the pump was unable to suck up the materials in the well. In this case, the opening area of a sluice valve adjacent to the pump was controlled. Solid concentration could be changed by adjusting the opening area of the feeder gate.

3.3 Materials transported

Materials transported are listed in Table 1. The terminal settling velocity of a solid particle was measured in a 155.2mm lucid pipe in which still water was filled. To consider a wall-effect on the falling velocity, correction was made by the following equation.[8]

$$v_t = v_{tm}/[1-(d/155.2)^{1.5}]$$

in which v_t is the free-settling velocity, or terminal settling velocity, of a single particle and v_{tm} is the observed value. Correction for a concentration-effect on the falling velocity was not considered this time.

The number of the experimental data reached into 914 points for the 155-mm pipe experiment and into 614 points for the 100-mm pipe experiment. Besides these, additional 685 data points collected by Hasegawa et al.(11) with a 80mm pipe were reexamined. All data were compiled in the form of computer cards and written in the computer magnetic tape. Eliminating suspicious data, we had 1,656 available points for the mixture flow.

Table-1 Solid particles and pipes used in the experiment

Solids Transported	Range of Particle Size in mm	d in mm	s	v_t in m/s	D(1) in mm	Solid Conc. up to C	Solid Conc. up to q	No. of Data after elimination
sand	0.5–1.5	0.91	2.63	0.123	155.2	0.24	0.29	275
gravel	7.5–10	8.75	2.70	0.397		0.28	0.38	212
gravel	25–30	27.5	2.65	0.801		0.22	0.24	133
pebble	40–50	45	2.65	1.038		0.15	0.16	173
sand	0.18–0.42	0.25	2.67	0.034	100.3	0.11	0.12	41
sand	0.5–1.0	0.65	2.63	0.095		0.16	0.17	116
sand	1.0–2.0	1.28	2.63	0.150		0.22	0.27	148
gravel	5.0–10	7.0	2.70	0.352		0.20	0.32	61
sand(2)	0.5–1.0	0.75	2.63	0.105	80.7	0.34	0.40	215
gravel	6.0–10	8.0	2.61	0.379		0.30	0.31	282

(1) Galvanized steel pipe
(2) Data collected by Hasegawa et al.(11)

4. Analysis of Experimental Results

4.1 Head loss parameters, ϕ and ϕ^*

The head loss parameter, ϕ, is a dimensionless variable usually defined by the following expression:

$$\phi = \frac{i_m - i_w}{i_w C} \tag{4.1}$$

in which i_m and i_w are the hydraulic gradient for mixture and for water at the mean flow velocity, V, respectively, and C is the concentration of solids delivered (by volume).

Properly speaking, the spatial solid concentration, q, should be used instead of the delivered one because we are discussing the flow of mixture in pipes. However, the delivered concentration has often been used for convenience. It is because the accurate measurement of the spatial solid concentration is comparatively difficult on one hand, but because the delivered one is not only measured easily but also reliable in accuracy on the other. If we use the spatial solid concentration in Eq. (4.1) and express the head loss parameter with ϕ^*, we shall obtain

$$\phi^* = \frac{i_m - i_w}{i_w q} \tag{4.2}$$

The theoretical head loss parameters, ϕ_0 and $\phi_0{}^*$, can be obtained from Eq. (2.14) as

$$\phi_0=\frac{\mu}{\lambda_w}\frac{q}{C}\frac{2}{F_D} \tag{4.3}$$

and

$$\phi_0{}^*=\frac{\mu}{\lambda_w}\frac{2}{F_D} \tag{4.4}$$

The relationship which represents the flow properties of mixture must clearly include effects of those parameters described in the previous section. Durand[12] used a term $(V^2/gD)\sqrt{C_d}$ against ϕ. Worster[13] took account of the specific gravity of solids and adopted a term $V^2/[gD(s-1)]$, which is the similar parameter to F_D in Eq. (2.12). By including an effect of the specific gravity of solids, the Durand's term can be modified as

$$\psi=\frac{V^2}{gD(s-1)}\sqrt{C_d}=F_D\sqrt{C_d} \tag{4.5}$$

We have tried to use the both parameters, ψ and F_D, against ϕ, but found that the former was more proper and universal in expressing the flow properties of mixture. Validity of the parameter, ψ, is also confirmed by Terada et al.[14]

In order to examine the correlation between ϕ and ψ, all experimental data were plotted on the logarithmic graph paper separately as shown in Figs. **6.1** to **6.10**. Although all data points seem to lay almost on a straight line, a distinguishable difference can be seen in the gradient between the case for d less than 1.28mm and that for d greater than 7.0mm, in other words, between the case for sand and that for gravel. This fact agrees well with the results of other investigators who proved the difference was closely related to the particle size and that the criterion for distinguishing between above two discussed cases might exist in d of, approximately, 2.0mm.

Dividing the range of particle size into two separate groups, or sand and gravel, we obtained the summarized figures of ϕ versus ψ relation as shown in Figs. **7.1** and **7.2**. Although the comparative extent of the scatter of the data is observed, it can be generally expressed with the equation of

$$\phi=K\psi^m \tag{4.6}$$

The values of K and m of the present study were obtained by the least squares method as Table 2. What is obvious from the comparison between Figs. **7.1** and **7.2** is that in the range of ψ less than, approximately, 3, the values of ϕ for sand are very close to those for gravel. This may mean that once a stationary bed has been formed the head loss in pipes will be affected only by the effevtive cross-sectional area of pipe rather than by particle sizes.

Plotting the values of ϕ^* and ψ in the same manner, we obtain Figs. **8.1** and **8.2**. The tendency of the correlation between the two parameters is quite similar to that of ϕ versus ψ, except that the absolute values of ϕ^* are smaller than those of ϕ over all range of ψ, especially in the range less than 3, where the depth of deposit seems to increase and therefore the spatial solid concentration becomes much larger than the delivered one. This point will be discussed in detail below.

It is possible to express the correlation with a single formula as we did for the ϕ versus ψ relation. However, as indicated in Figs. 8.1 and 8.2, the best-fit equation had

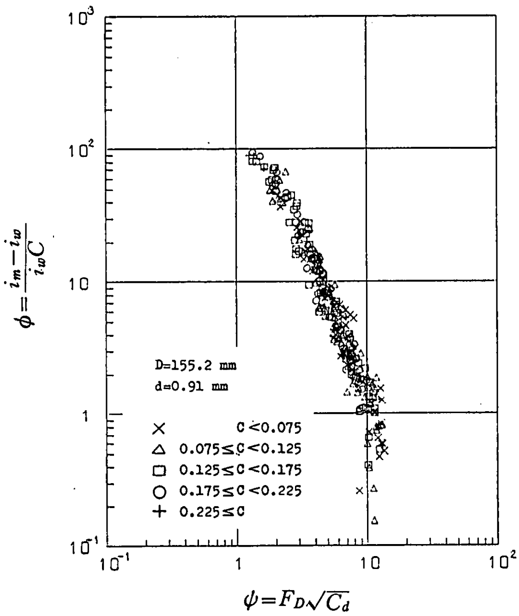

Fig.-6.1 ϕ vs. ψ for $D=155.2$mm and $d=0.91$mm

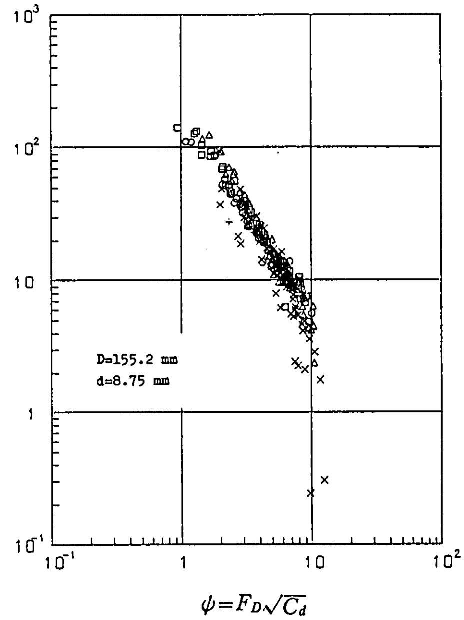

Fig.-6.2 ϕ vs. ψ for $D=155.2$mm and $d=8.75$mm

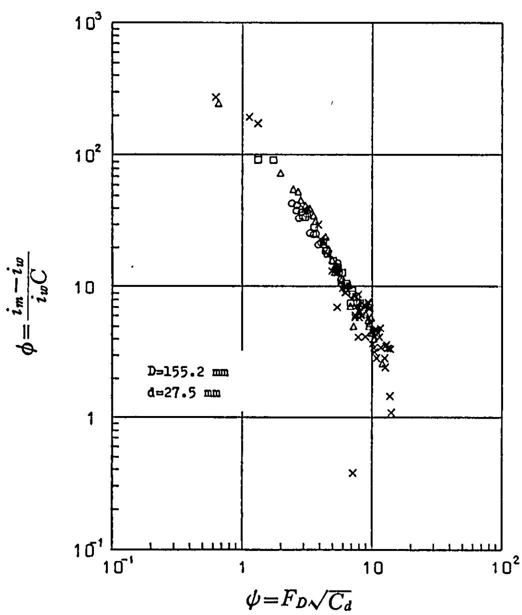

Fig.-6.3 ϕ vs. ψ for $D=155.2$mm and $d=27.5$mm

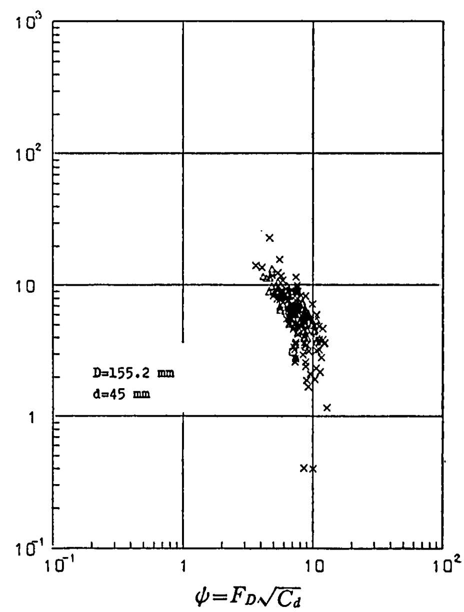

Fig.-6.4 ϕ vs. ψ for $D=155.2$mm and $d=45$mm

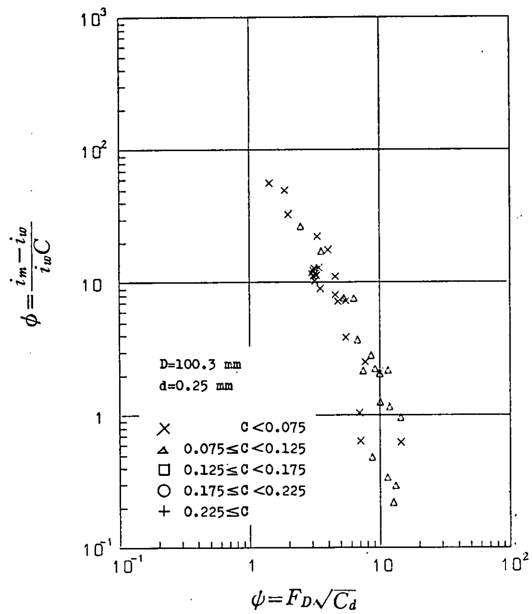

Fig.-6.5 ϕ vs. ψ for $D=100.3$mm and $d=0.25$mm

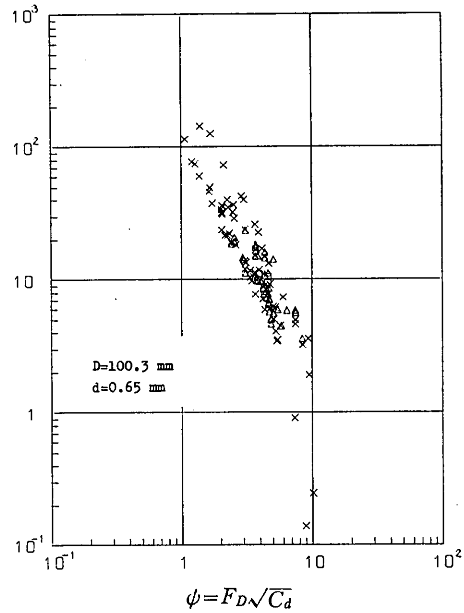

Fig.-6.6 ϕ vs. ψ for $D=100.3$mm and $d=0.65$mm

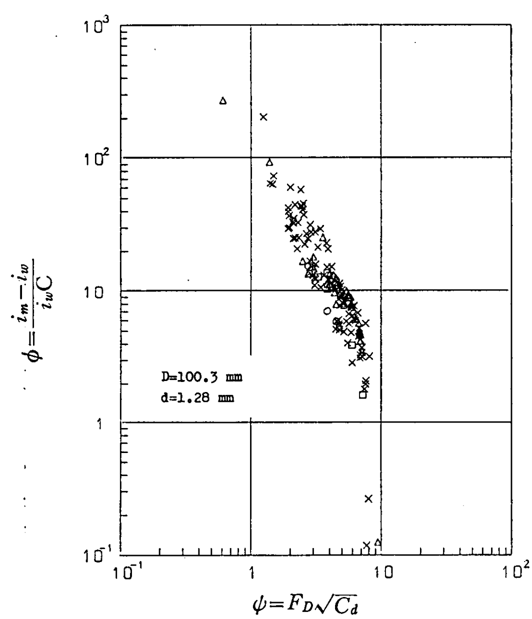

Fig.-6.7 ϕ vs. ψ for $D=100.3$mm and $d=1.28$mm

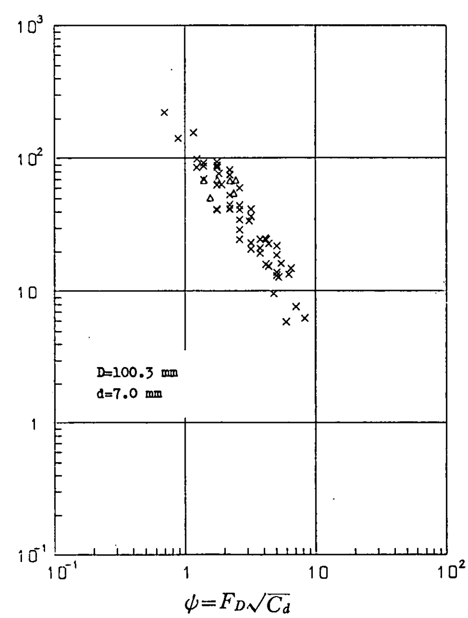

Fig.-6.8 ϕ vs. ψ for $D=100.3$mm and $d=7.0$mm

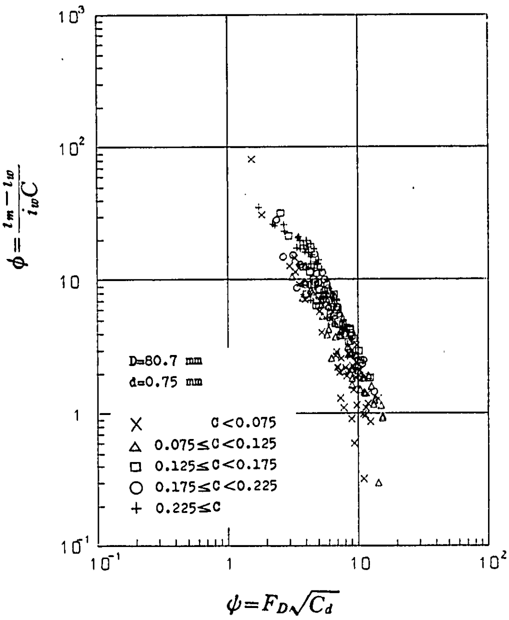

Fig.-6.9 ϕ vs. ψ for D=80.7mm and d=0.75mm

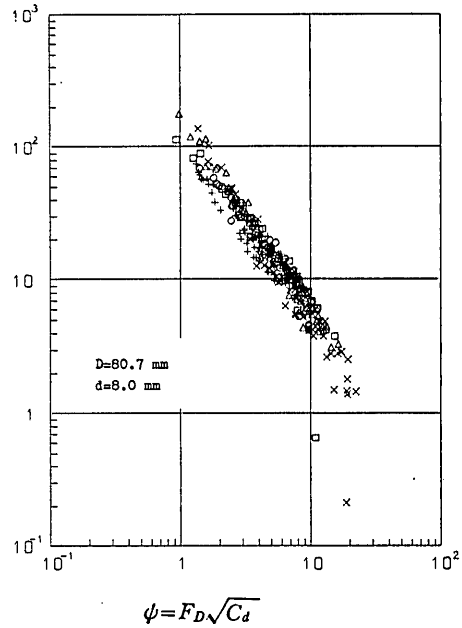

Fig.-6.10 ϕ vs. ψ for D=80.7mm and d=8.0mm

better be given for the separate ranges of ψ, or ψ<and>3. If the general form of this correlation is given by

$$\phi^*=K'\psi^{m'} \tag{4.7}$$

the values of K' and m' were observed as Table 3.

Table-2 The values of K and m in $\phi=K\psi^m$

Kind of Material	K	m
sand	200	−2.10
gravel	180	−1.55

Table-3 The values of K' and m' in $\phi^*=K'\psi^{m'}$

Kind of Material	Range of ψ	K'	m'
sand	ψ<3	100	−1.55
	ψ>3	180	−2.09
gravel	ψ<3	98	−1.16
	ψ>3	138	−1.46

4.2 Apparent friction factor, μ

The parameter, μ, which was defined by Eq. (2.9) and introduced in Eq. (2.8), has a physical significance as a kind of coefficient of friction between solids and the pipe wall when the solids are transported by fluids in pipes.

As seen in Eq. (2.9), it involves three variables; κ, η_F and ξ_s. As for the sliding coefficient of friction, ξ_s, Kuzuhara[15] and Terada[16] measured its values for gravel

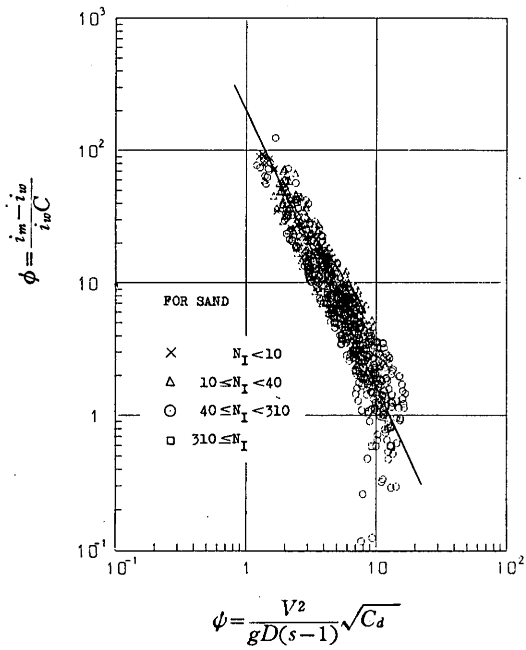

Fig.-7.1 Relationship between ϕ and ψ for sand

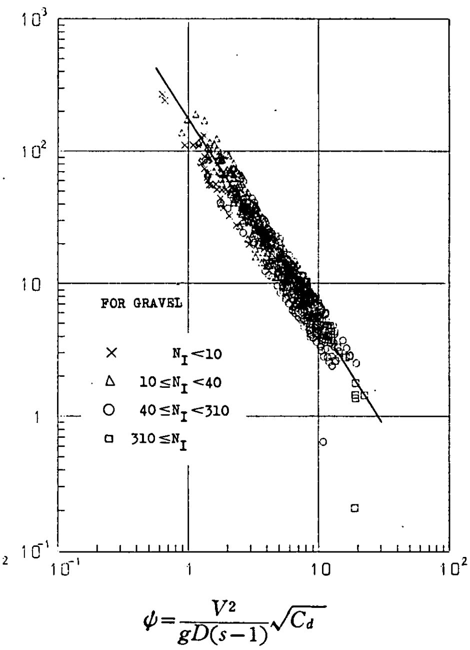

Fig.-7.2 Relationship between ϕ and ψ for gravel

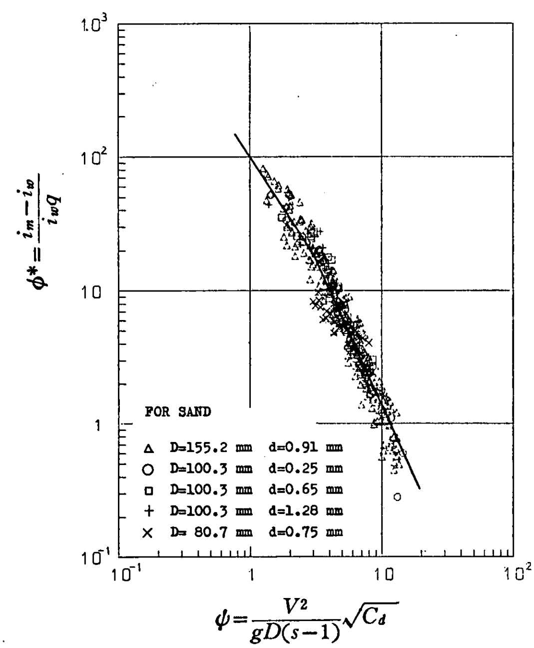

Fig.-8.1 Relationship between ϕ^* and ψ for sand

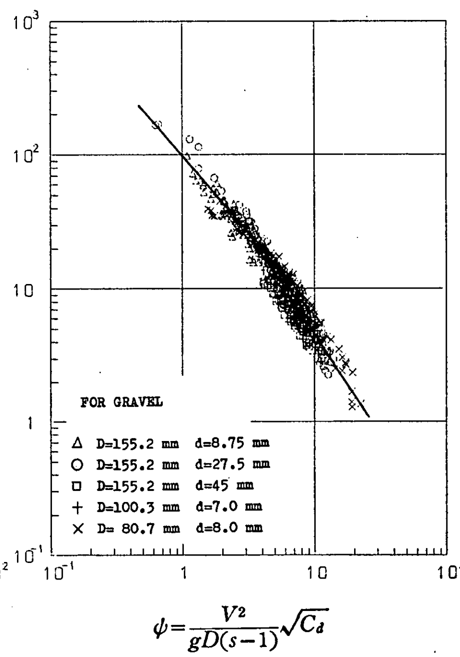

Fig.-8.2 Relationship between ϕ^* and ψ for gravel in different methods. To know the values of κ and η_F is very difficult, probably impossible, because they are directly related to the flow condition and vary particularly with the flow velocity and the concentration of solids just as the flow regime changes with those parameters. This will mean that the resultant parameter, μ, must be treated as a function of the flow parameters.

The parameter, μ, can be derived from either Eq. (4.3) or Eq. (4.4) as

$$\frac{\mu}{\lambda_w}=\phi_0\frac{C}{q}\frac{F_D}{2}=\phi_0{}^*\frac{F_D}{2} \tag{4.8}$$

In order to use the parameter, ψ, we multiply both members by $\sqrt{C_d}$ and substitute Eqs. (4.5) and (4.7) into the above equation:

$$\frac{\mu}{\lambda_w}\sqrt{C_d}=\frac{K'}{2}\psi^{m'+1} \tag{4.9}$$

The values of K' and m' in Table 3 give us the relationship between $(\mu/\lambda_w)\sqrt{C_d}$ and ψ as Fig. 9.

It is reasonable to think that κ and η_F have values within the following ranges; $0\leq\kappa\leq1$ and $0<\eta_F\leq1$. When κ is zero or η_F is unit, in other words, all solid particles move as a sliding bed, the parameter, μ, is equal to ξ_s, which is a maximum value of μ as far as Eq. (2.9) is concerned. In actual flows, however, there exists the difference in local flow velocities of solid particles which causes friction forces among particles.

With a decrease of the mean flow velocity, the differential becomes conspicuous and friction forces, or shear stresses, among particles increase, consequently, the values of μ become much greater than the ordinary sliding coefficient of friction. In this study, the parameter is obtained from Eq. (4.9) and therefore it will be able to include both effects of friction between solids and the pipe wall and of shear stresses among particles.

While, with an increase of the mean flow velocity, the values of κ increase gradually, resulting in an decrease of μ. In the heterogeneous flow regime, solid particles may contact with the pipe wall instantaneously, but as the flow is fully steady μ may approach to a constant value. In the present study, no such tendency can be seen due to insufficient number of data in the heterogeneous flow regime.

Assuming that friction forces between solids and the pipe wall are equal to the product of the coefficient of friction and the weight of solids in the pipe, and that the same magnitude of forces as friction forces might act on the pipe as reaction forces, Noda et al.(17) measured this reaction forces and calculated back the coefficient of friction, μ_s, in their term.

The meaning of μ and μ_s are almost the same and therefore we can compare both values on the same level. The available data were collected with a 52.5mm stainless steel pipe transporting two kinds of crushed rock (andesite) with d_e, sphere equivalent diameter, of 3.5mm and of 6.9mm, with the specific gravity of 2.65. If we read the values of λ_w from Moody diagram for smooth pipes and assume that C_d is equal to 0.54 for the small material and to 0.50 for the large one, their data can be reproduced as Fig. 9.

Although the values of μ_s for the material with d_e of 3.5mm roughly agree with the present study, those for the material with d_e of 6.9mm deviate from our results. This deviation may depend on the assumption of the values of C_d.

4.3 Ratio of the solid concentrations, q/C

The slip which occurs between solid particles and the liquid results in the spatial

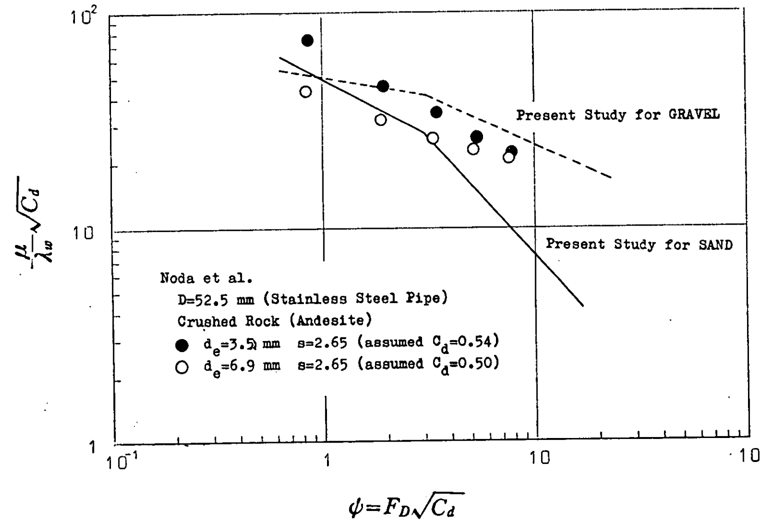

Fig.-9 Apparent friction factor, μ

solid concentration in the pipe, q, being greater than the concentration of solids delivered, C. As far as a solid-liquid mixture flows steadily without forming a stationary bed, the difference between the above two concentrations is not so great. But to know the ratio of q/C is sometimes very helpful for design purpose as well as for operation of the system.

When a solid-liquid mixture flows steadily without deposit, the following relationship should hold:

$$\frac{u_s}{V}=\frac{C}{q} \quad \text{and} \quad \frac{u_w}{V}=\frac{1-C}{1-q} \tag{4.10}$$

Introducing a new function φ_0, which is defined by

$$\varphi_0=\sqrt{\frac{\eta_F}{\eta_D}\frac{C_d}{C_s}\xi_s\frac{v_t}{V}} \tag{4.11}$$

we obtain the following equation after substituting Eqs. (4.10) and (4.11) into Eq. (2.4):

$$\varphi_0=\frac{1-C}{1-q}-\frac{C}{q} \tag{4.12}$$

The above equation can be solved for q as

$$q=\frac{1}{2}\left[\left(1-\frac{1}{\varphi_0}\right)+\sqrt{\left(1-\frac{1}{\varphi_0}\right)^2+\frac{4C}{\varphi_0}}\right] \tag{4.13}$$

Then,

$$\frac{q}{C}=\frac{1}{2C}\left[\left(1-\frac{1}{\varphi_0}\right)+\sqrt{\left(1-\frac{1}{\varphi_0}\right)^2+\frac{4C}{\varphi_0}}\right] \tag{4.14}$$

The function φ_0 cannot be obtained theoretically because it involves three unknown variables; η_F, η_D and C_s. But from Eq. (4.12) and the experimental data of q and C, we can suppose its form. In this study, it was considered with respect to the parameter, ψ, of which results are shown in Figs. 10.1 and 10.2.

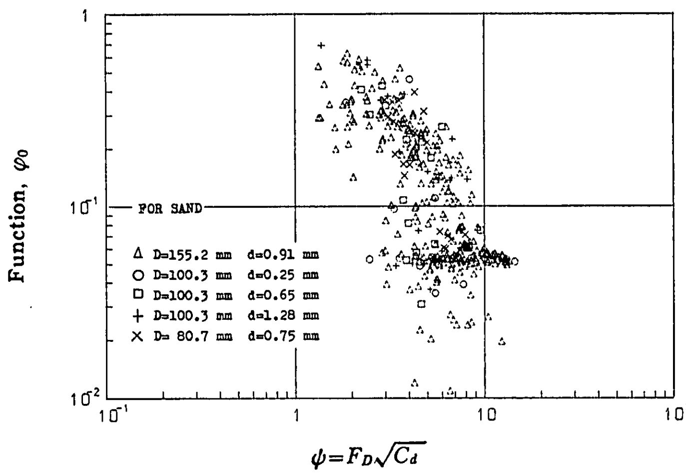

Fig.-10. 1 Function, φ_0, versus flow parameter, ψ, under transporting sand

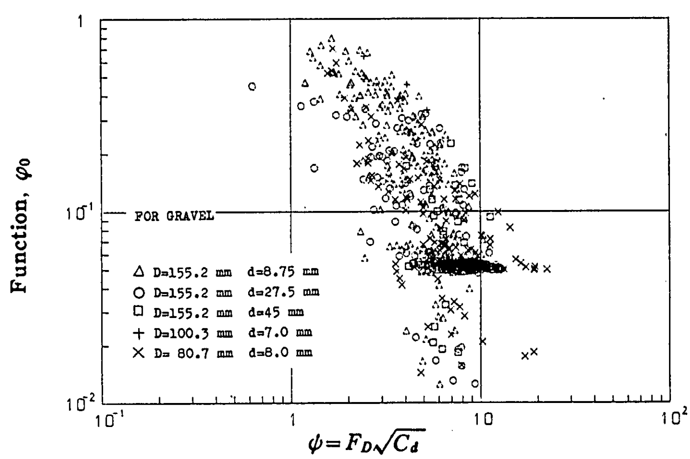

Fig.-10. 2 Function, φ_0, versus flow parameter, ψ, under transporting gravel

The experimental values are very scattered. It may be partly because the flow of settling mixtures is unstable and partly because the data of q lack sufficient accuracy compared with those of C. The largest cause for the error in the values of q is effects of stiffness of the rubber sleeves.

The correlation between φ_0 and ψ is quite similar for both cases; for sand and for gravel. If we can express it with an equation in the form of

$$\varphi_0=1.3\,\psi^{-1.4} \tag{4.15}$$

substitution of Eq. (4.15) into Eq. (4.14) yields the ratio of the solid concentrations, q/C, against ψ. In this paper, however, we only show one of the results for gravel as the correlation between q/C and a term $(\lambda_w/2)[V^2/gD(s-1)]$, which is shown in Fig. **11**, in order to compare our results with those of Worster and Denny, and of Ayukawa.[10]

The marked points in the figure are obtained from the direct fraction of measured values of q and C. Although some problems may be involved in Eq. (4.15), we may well conclude that Eqs. (4.14) and (4.15) are available for acquiring information concerning q/C.

Ayukawa and Ochi[10] did not measured the concentration of solids in the pipe, but they derived an equation for predicting the ratio, q/C, which is given by

$$\frac{q}{C}=2/[(1-\varphi_A)+\sqrt{(1-\varphi_A)^2+4C\varphi_A}] \tag{4.16-a}$$

in which φ_A is a function obtained experimentally as

$$\varphi_A=[0.90\ \xi_s(d/D)^{-0.707}F_d^{-2.72}]^{1/2} \tag{4.16-b}$$

F_d is the particle Froude number defined by $V/\sqrt{gd(s-1)}$. (We transformed their formula using our terms.)

The calculated values of q/C by the above Ayukawa's equation are plotted together with the result of Worster and Denny for a given condition, i.e., $\xi_s=0.697$ for gravel,[10] $\lambda_w=0.022$ for galvanized steel pipe and $C=20\%$, as shown in Fig. 11.

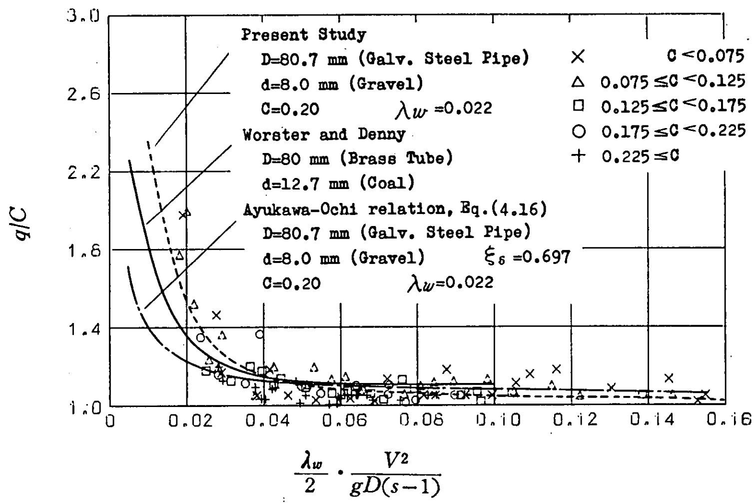

Fig. 11 Ratio of the solid concentrations, q/C

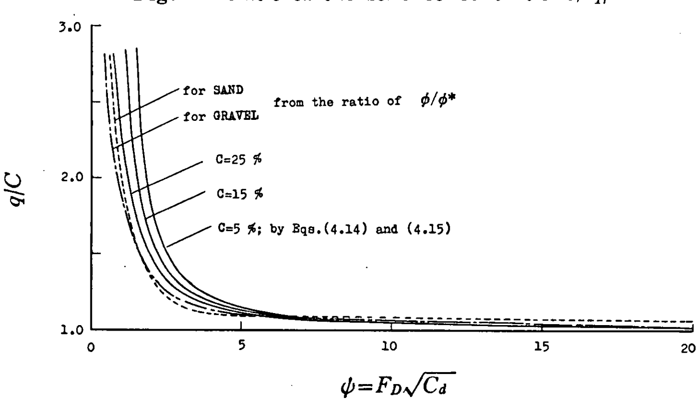

Fig.-12 Comparison of q/C

At low velocities, the result of the present study is greatest in the three. Although only an average line is plotted, the result of Worster and Denny falls between that of Ayukawa's equation on one hand and that of the present study on the other. At high velocities, the values of q/C calculated by Eq. (4.14) approach gradually to unit, which means that, at much more high velocities, there is no slip between solid particles and the fluid carrier. But such a range of flow is beyond the scope of this research. Every curve has a similar tendency in the range of $(\lambda_w/2)[V^2/gD(s-1)]$ less than, approximately, 0.03, where they increase steeply. This may mean that with the formation of deposit the difference of the two solid concentrations becomes conspicuous. The value of $(\lambda_w/2)[V^2/gD(s-1)]=0.03$ is nearly corresponding to $\psi=3$.

We can also obtain the ratio of the solid concentrations, q/C, as a fraction of ϕ and ϕ^*, or from Eqs. (4.6) and (4.7). Fig. 12 gives the comparison between the calculated values of q/C by Eq. (4.14) and those obtained from the fraction, ϕ/ϕ^*. Considering the extent of the scatter of the data collected, we may tolerate the difference.

4.4 Evaluation of the head loss equations

The results of the present study should be evaluated as compared with those of other investigators, for our experimental conditions are limited.

Most representative equations to describe the head loss in horizontal pipes for heterogeneous flow are, at the present, the Durand-Condolios relation(18) and the Newitt et al. relation(19). The Durand-Condolios relation is given by

$$\phi_D=\frac{i_m-i_w}{i_wC}=K_D\left[\frac{V^2}{gD(s-1)}\sqrt{C_d}\right]^{-3/2} \tag{4.17}$$

Durand and Condolios did not give the numerical values of the constant, K_D, but many investigators have examined their data with Eq. (4.17) and reported the different values of K_D, being equal to 81, or 121, or 150.(20)(21)(22)

Newitt et al. gave Eq. (4.18) for suspension flow, or

$$\phi_N=\frac{i_m-i_w}{i_wC}=1,100(s-1)\frac{v_t}{V}\frac{gD}{V^2} \tag{4.18}$$

If it can be rewritten as

$$\phi_N=K_N\left[\frac{V^2}{gD(s-1)}\sqrt{C_d}\right]^{-3/2} \tag{4.19}$$

the coefficient, K_N, is a function of a particle diameter, d, pipe size, D, and particle drag coefficient, C_d. The calculated values of K_N for sandy materials used in the present experiments changed from 93 to 150, having an average value of 121.

Zandi and Govatos(20) collected over 2,500 data points from many different sources and investigated the validity of the head loss equations proposed by Durand and Condolios, Newitt et al. and Worster. Although they found that the Durand-Condolios relation, with $K_D=81$ in Eq. (4.17), was capable of predicting the head loss for the heterogeneous flow most accurately, they proposed the more reasonable relations by dividing the values of ψ into the two separate ranges, or

$$\text{for}\quad \psi<10:\quad \phi_Z=\frac{i_m-i_w}{i_wC}=280.0\ \psi^{-1.93} \tag{4.20-a}$$

$$\text{for}\quad \psi>10:\quad \phi_Z=6.3\ \psi^{-0.354} \tag{4.20-b}$$

To compare the present results with above discussed relations, all of them are plotted together in Figs. 13.1 and 13.2, which give the relationship between ϕ and ψ, and ϕ^* and ψ, respectively.

It is obvious from the figures that our results fall between the Durand-Condnlios relation on one hand and the Zandi-Govatos relation on the other, and therefore they

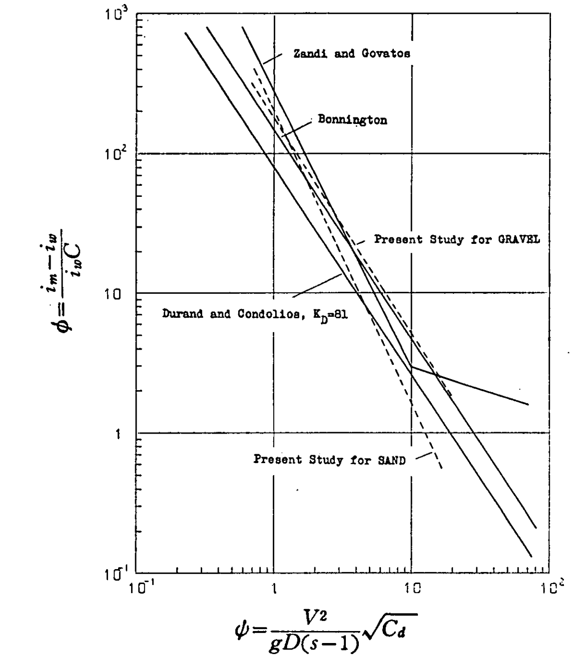

Fig.-13.1 Relationship between ϕ and ψ for sand and gravel

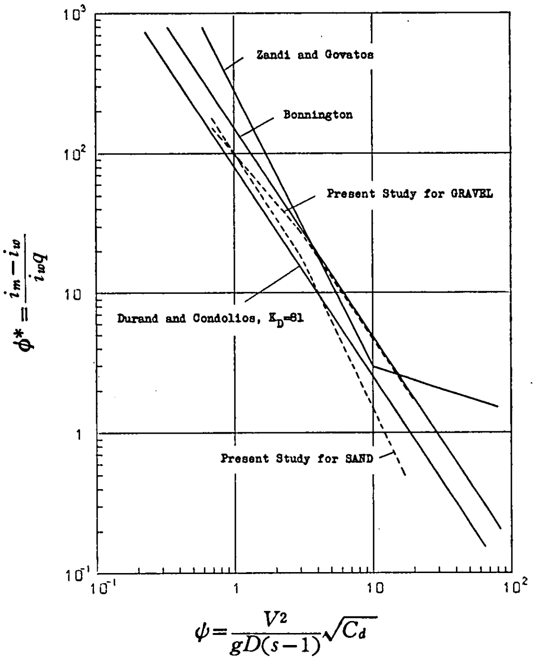

Fig.-13.2 Relationship between ϕ^* and ψ for sand and gravel

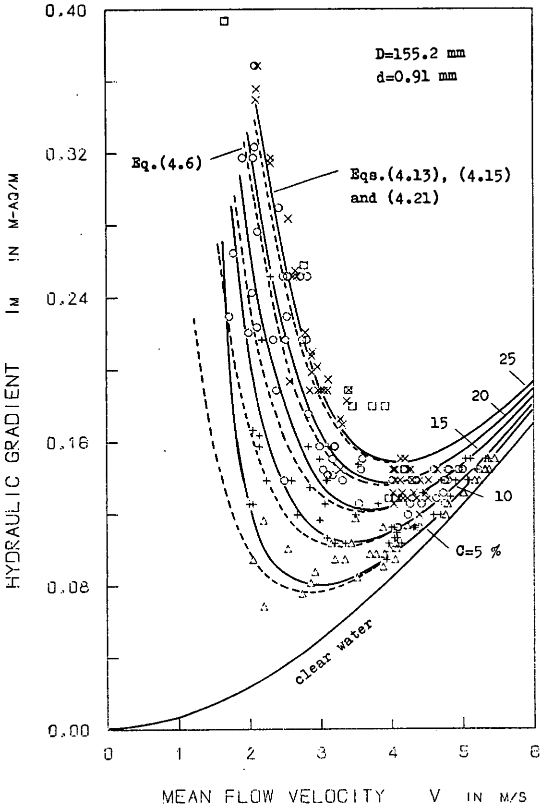

Fig.-14.1 Flow velocity versus head loss with concentration, C, as parameter, for sand in a 155.2mm pipe

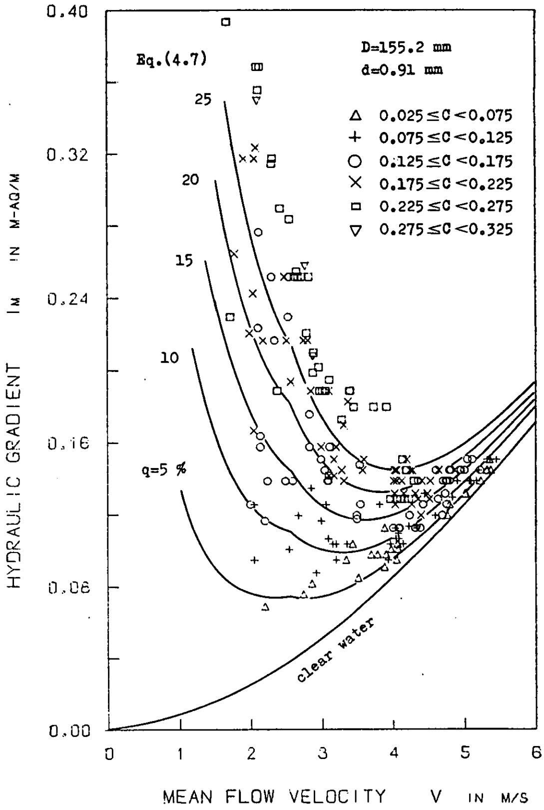

Fig.-14.2 Flow velocity versus head loss, with concentration, q, as parameter, for sand in a 155.2mm pipe

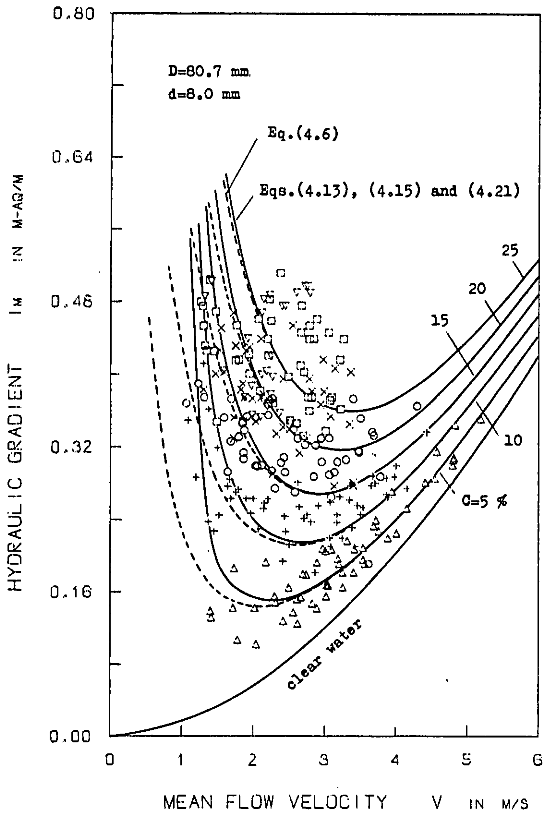

Fig.-14.3 Flow velocity versus head loss, with concentration, C, as parameter, for gravel in a 80.7mm pipe

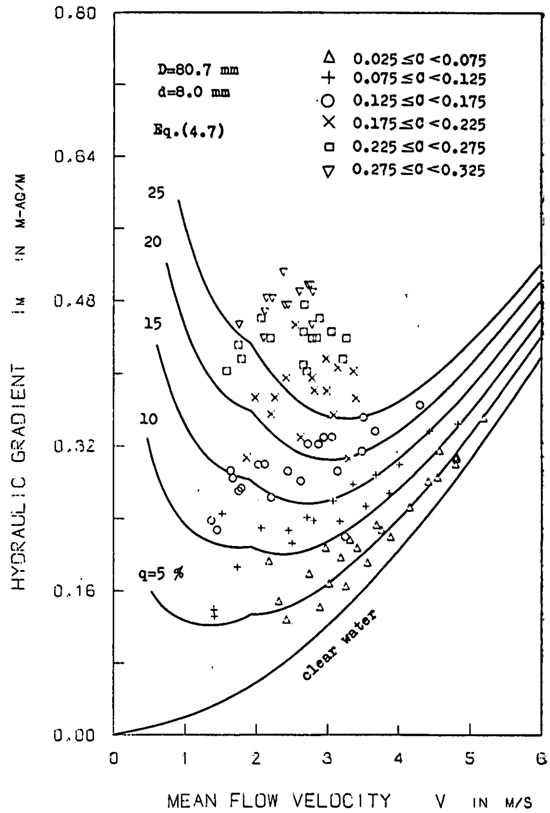

Fig.-14.4 Flow velocity versus head loss, with concentration, q, as parameter, for gravel in a 80.7mm pipe

are very close to the Bonnington's result.[22]

From Eqs. (2.14) and (4.9), the theoretical head loss can be expressed by Eq. (4.21), or

$$i_m = (1 + qK'\psi^{m'})i_w \tag{4.21}$$

If the concentration of solids delivered, C, is given, we can compute the value of q by Eqs. (4.13) and (4.15), and then predict the head loss, i_m, by Eq. (4.21). While, we can also predict the head loss by Eq. (4.6). Figs. **14.1** and **14.3** show the comparison between the two predicted values. The difference of both values are remarkable at low velocities, but at high velocities there is no problem in using either equation for predicting the head loss. It is obvious from the figures that Eq. (4.21) with q of Eq. (4.13) is capable of predicting the head loss more accurately than Eq. (4.6). The relationship between V and i_m with concentration, q, as parameter are given as Figs. **14.2** and **14.4**, in which the solid curves are predicted by Eq. (4.7).

4.5 Critical Velocities

The term "critical velocity" is sometimes ambiguous as it is used without definition. Some investigators use it as the velocity which distinguishes heterogeneous flow from flow with a moving bed, i.e., the velocity at which some of the suspended particles begin to settle and move along the bottom pipe boundary. If the flow velocity decreases below the above criterion, the thichening of the sliding bed grows and the differential of local flow velocities of solid particles becomes conspicuous resulting in the formation of a deposit on the bottom of the pipe. Some investigators take the stand that the critical velocity is this velocity corresponding to the onset of the formation of a stationary bed.

It is very difficult, however, to identify a velocity at which the solid particles closest to the bottom pipe boundary transit from suspension to sliding, or sliding to deposit. Therefore, as far as the critical velocity is concerned, it seems to be advantageous to adopt the velocity corresponding to a minimum point on a head loss versus velocity curve of constant concentration of solids. This definition is quite reasonable not only from a viewpoint of economic operation of the system but also from a viewpoint of simplification in computing the values.

From Eq. (4.21).

$$i_m=\frac{\lambda_w}{2}\frac{s-1}{\sqrt{C_d}}(\psi+qK'\psi^{m'+1}) \tag{4.22}$$

Differentiating i_m with respect to V and equating di_m/dV to zero, the critical value, ψ_c, or the critical velocity, V_c, corresponding to a minimum head loss point is obtained as follows:

$$\psi_c=\frac{V_c^2}{gD(s-1)}\sqrt{C_d}=[-qK'(m'+1)]^{-1/m'} \tag{4.23}$$

The relationship between ψ_c and q is given as Fig. 15.

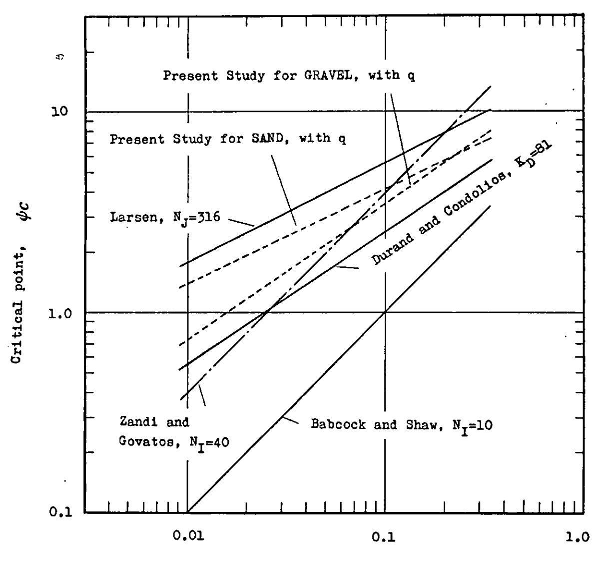

Fig.-15 Critical point, ψ_c, versus concentration of solids, q or C

Although there is no problem for practical use, we must note, according to the present analysis, that the head loss versus flow velocity curve of constant concentration may have two minimum points, as shown in Figs. 14.2 and 14.4. One minimum point exists in the range of $\psi<3$ and the other in $\psi>3$. The latter may, generally speaking, correspond to the critical velocity, but how should we explain the former? If only we express the $\phi^*-\psi$ relation with a single equation as we did for the $\phi-\psi$ relation, there occurs no such problem and we might miss it. Temporally, as we can find no substantial reason to justify or deny this result, we dare show it here.

By substituting the values of K' and m' in Table 3 into Eq. (4.23), the critical velocity and the resultant minimum head loss can be predicted for a given solid material, pipe size and concentration of solids.

As for critical velocities, Durand[12] proposed a dimensionless parameter, F_L, defined by $V_L/\sqrt{2gD(s-1)}$, in which V_L is the "limit deposit velocity" in his term. This V_L is said to correspond fairly accurately to the minimum head loss point for each velocity-head loss curve. Graf et al.[23] also obtained the similar parameter based on the dimensionless analysis.

Expressing the left-side of Eq. (4.23) with the Durand's parameter, F_L, we obtain Fig. 16 as the relationship between F_L and particle diameter, d, with concentration, q, as parameter. At low concentrations, the results of the present study agree well with those of Graf. et. al., who have analyzed the data collected by Gibert, but at high concentrations, the present results agree with those of Durand for uniform material. The figure also shows that there appears a peak on each line of low concentrations but that with an increase of the concentration a peak disappears and the critical velocities are almost independent of the particle size.

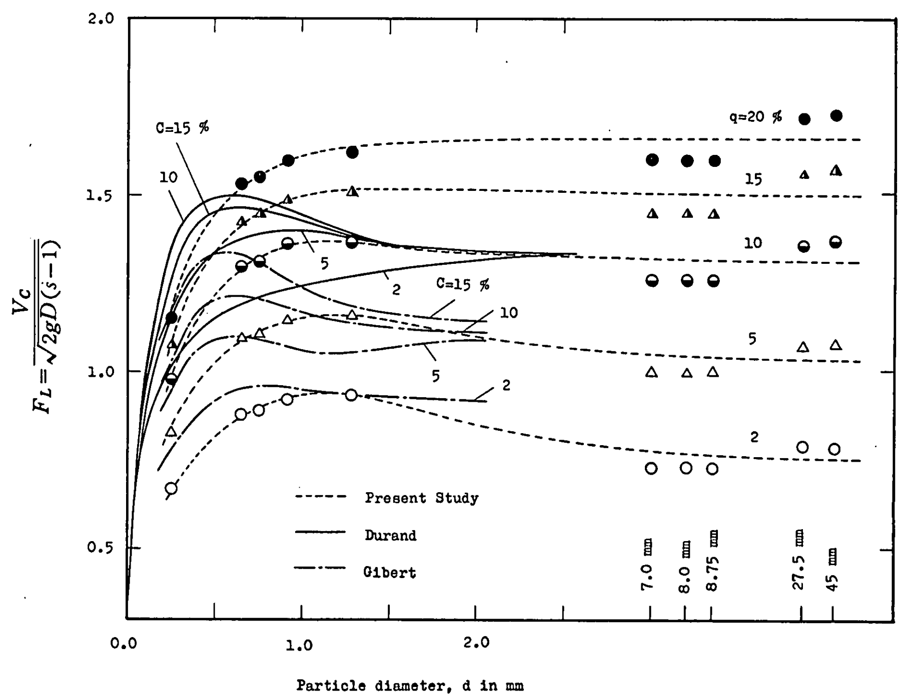

Fig.-16 Critical velocity versus particle diameter

If Eq. (4.23) can be rewritten as

$$\frac{\psi_c^{-m'}}{q}=-K'(m'+1)=N_c \tag{4.24}$$

N_c is the characteristic number capable of indicating a critical point on a velocity versus head loss curve of constant concentration. The present study gives $N_c=196$ and $m'=-2.09$ for sand, and $N_c=63$ and $m'=-1.46$ for gravel.

Zandi and Govatos[20] proposed a dimensionless I-number for classification of the regime of flow, which has a form of

$$N_I=\frac{V^2\sqrt{C_d}}{CgD(s-1)}=\frac{\psi}{C} \tag{4.25}$$

and concluded that the value of $N_I=40$ might be taken as the upper limit of the transition between saltation to heterogeneous flow.

Babcock and Shaw[24] suggested that the value of $N_I=10$ might be more reasonable for the division of the heterogeneous flow from flow with moving bed.

Larsen[25] also discussed the Zandi and Govats proposal suggesting a minor simplification of the values of constant and exponent in Eq. (4.21), i.e., $\phi_Z=316\,\psi^{-2}$ for $\psi<10$, and $\phi_Z=6.8\,\psi^{-1/3}$ for $\psi>10$. In his discussion, he proposed the introduction of a modified J-number, $N_J=\psi^2/C$, for describing the flow condition, and gave $N_J=316$ as a criterion.

Fig. 15 also shows the comparison between ψ_c and C of above discussed relations. It reveals that within the range of concentration in practical operations the Zandi-Govatos criterion is greater than that of the present study but the difference is not so great. A criterion suggested by Babcock and Shaw may have smaller values compared with other results discussed here. Zandi's N_I has a simpler form than N_c given by Eq. (4.24), but the latter is derived mathematically.

4.6 Correlation with the soil coefficient, β

In dredging operations, evaluation of the performance of dredging pump is very important. Upon determing its principal dimensions, special attention wil be given to the total head, the flow rate and the power required. The total head is closely related to the length of discharge pipeline, in other words, to the total head loss in the system, and the flow rate to the critical velocity.

Usually, dredging materials to be transported in pipes have a wide distribution of particle size. But, at present, little is known about the transportation of nonuniform materials, and it is no exaggeration to say that the flow velocity of mixture sometimes determined by experience of engineers.

For the purpose of predicting the head loss for soil-water mixture in pipes, we have practically used the following relation:

$$i_m=[1+\beta(\gamma_m/\gamma_w-1)]\,i_w \tag{4.26}$$

in which γ_m is the unit weight of mixture and β is the parameter to consider an effect of the kind of dredging materials on the head loss, then it is called the soil coefficient.[26]

Hasegawa et al.[11] conducted laboratory experiments to investigate the variation of the values of β and found that they became almost constant in the heterogeneous flow regime but they have increasing values for the flow with a moving bed. The practical values of β were obtained by their extensive field tests for cutter suction dredgers and given as Table 4.

The parameter, β, can be obtained mathematically from previous equations as

$$\beta=\frac{i_m-i_w}{i_w(\gamma_m/\gamma_w-1)}=\frac{K'}{s-1}\psi^{m'} \tag{4.27}$$

Table-4 The soil coefficient, β, after Hasegawa et al.[26]

Flow Velocity of Water required, V_{pw} in m/s	β	Kind of Dredging Material
2.5	2	mud
3.5	3	fine sand
4.5	4	coarse sand; sand containing small gravel
5.5	5	gravel

At present, the values of β corresponding to ψ_c are important. Then, substituting Eq. (4.23) into Eq. (4.27), we obtain

$$\beta_c=\frac{1}{s-1}\frac{-1}{m'+1}\frac{1}{q} \tag{4.28}$$

in which β_c is the critical value of β corresponding to ψ_c.

We learn from their paper[26] that an average pipe size and concentration of solids, at that time, were 500mm in diameter and 10%, respectively. Then, if we take $q=0.10$, β_c will be computed as 5.6 for sand and as 13.2 for gravel. These values are greater than those given in Table 4, and its difference is remarkable for gravel. It is, we may suppose, dependent on the fact that soils described in Table 4 were classified from a viewpoint of dredging operations and therefore they are not so uniform as the materials used in the laboratory experiments. This may be illustrated more clearly by the following examples:

In the case of coarse sand with the specific gravity of 2.65, the required flow velocity of water, V_{pw}, is read as 4.5 m/s and the soil coefficient, β, as 4. If it is transported in a 500 mm pipe with concentration of 10%, the flow velocity of the mixture, V, results in 3.75 m/s according to Eq. (10) in their paper, which is corresponding to $F_L=0.93$. This value agrees with the result of limit deposit velocity for nonuniform material given by Durand and Condolios.[7]

While, in the case of gravel with the specific gravity of 2.65, V_{pw} is read as 5.5m/s and β as 5. If it is transported at the similar flow condition, V will be computed as 4.43 m/s, which is corresponding to $F_L=1.10$. This value is also close to the previous Durand-Condolios result for nonuniform material.

5. Conclusions

The conclusions that may be drawn from this study are summarized as follows;

1 By introducing an apparent friction factor, a theoretical analysis of the pressure drop for settling mixtures has been developed.

2 The head loss parameter, ϕ, was examined against the flow parameter, ψ, and two independent relationships were obtained for sand and for gravel. The difference was observed between the constant and the exponent in the head loss equation of the present study and those of Durand and Condolios, or those of Zandi and Govatos. But the result for gravel was very close to that of Bonnington.

3 Another form of the head loss parameter, ϕ^*, which includes the spatial solid concentration, q, instead of the delivered one, C, was discussed with respect to the parameter, ψ. Although the relationship has a similar tendency to the $\phi-\psi$ relation,

the best-fit equation should be given for the two separate ranges of ψ, or ψ <and> 3. It is because this regime is closely related to the formation of deposit in pipes.

5 At low velocities, values of the apparent friction factor are greater than an ordinary sliding coefficient of friction between solid and the pipe wall because of shear stresses among solid particles in moving bed. With an increase of the flow velocity, they decrease sharply.

6 The ratio of the solid concentrations, q/C, has been derived semi-theoretically. It is almost constant at high velocities, but increases steeply for values of ψ less than, especially, 3, which is corresponding to the formation of the stationary bed.

7 The critical velocity, which is defined as the velocity corresponding to a minimum head loss point, agrees with the result of Graf et al. at low concentrations, but at high concentrations it is close to the Durand's limit deposit velocity for uniform material.

8 A characteristic number, N_c, available for judging the flow regime is proposed by Eq. (4.24). A minor disadvantage in using it is that the form is a little complicated compared with the I-number proposed by Zandi and Govatos. But the former was derived mathematically and directly corresponds to a minimum head loss point.

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Appendix I.——Notation

The following symbols have been used in this paper:

a_s =projected area of a solid particle in the direction of flow;
A =cross-sectional area of pipe;
C =concentration of solids delivered (by volume);
C_d =steady-state drag coefficient of a solid particle, defined by Eq. (2.3);
C_s =drag coefficient of a solid particle;
d =particle diameter at d_{50};
D =pipe diameter;
F_d =particle Found number defined by $V/\sqrt{gd(s-1)}$;
F_D =modified Froude number defined by $V^2/gD(s-1)$, or Eq. (2.12);
F_L =parameter defined by Durand as $V_L/\sqrt{2gD(s-1)}$;
g =gravitational constant;
i_m =hydraulic gradient for mixture;
i_w =hydraulic gradient for water;
k =constant;
K =constant in Eq. (4.6);
K_D =constant in the Durand-Condolios relation, or Eq. (4.17);
K_N =constant in the Newitt et al. relation, or Eq. (4.19);
K' =constant in Eq. (4.7);
ΔL =a given length of the horizontal pipe;
m =exponent in Eq. (4.6);
m' =exponent in Eq. (4.7);
N_c =characteristic number capable of indicating a critical point on a flow velocity

versus head loss curve of constant concentration, defined by Eq. (4.24);
N_I =I-number proposed by Zandi and Govatos, or Eq. (4.25);
p =pressure in the pipe;
Δp =pressure drop in the pipe;
Δp_s =excess pressure drop due to the presence of solid particles;
Δp_{ss} =excess pressure drop expended to transport the suspended solids;
Δp_{sf} =excess pressure drop expended to convey the sliding bed;
Δp_w =pressure drop for water alone;
q =spatial solid concentration (by volume);
R_{es} =particle Reynolds number;
s =specific gravity of solid particles;
t =time;
u_s =mean flow velocity of solid phase;
u_w =mean flow velocity of liquid phase;
v_s =volume of a solid particle;
v_t =terminal settling velocity of a solid particle in still water;
V =mean flow velocity of mixture;
V_c =critical velocity corresponding to a minimum head loss point;
V_L =limit deposit velocity defined by Durand;
Z =number of solid particles in a given space;
Z_f =number of solid particles in sliding in a given space;
Z_s =number of solid particles in suspension in a given space;
β =soil coefficient;
β_c =critical soil coefficient corresponding to V_c;
γ_m =unit weight of mixture;
γ_s =unit weight of solid particles;
γ_w =unit weight of water;
ε =exponent;
κ =ratio of the number of suspended solid particles to all particles in a given space;
λ_w =Darcy-Weisbach friction factor for water;
ξ_s =sliding coefficient of friction between a solid particle and the pipe wall;
η_D, η_F =coefficient of correction introduced in Eq. (2.1);
μ =apparent friction factor defined by Eq. (2.9);
φ_A =Ayukawa's function given by Eq. (4. 16-b);
φ_0 =function defined by Eq. (4.11);
ϕ =head loss parameter defined by $(i_m-i_w)/i_wC$;
ϕ^* =head loss parameter defined by $(i_m-i_w)/i_wq$;
ϕ_0 =theoretical head loss parameter defined by Eq. (4.3);
ϕ_0^* =theoretical head loss parameter defined by Eq. (4.4);
ϕ_D =head loss parameter in the Durand-Condolios relation, or Eq. (4.17);
ϕ_N =head loss parameter in the Newitt et al. relation, or Eq. (4.18);
ϕ_Z =head loss parameter in the Zandi-Govatos relation, or Eq. (4.20);
ψ =flow parameter defined by Eq. (4.5); and
ψ_c =critical value of the flow parameter corresponding to a minimum head loss point.

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