Slope stability of Cohesionless Materials: Shaking Frequency Effect

K. Ichii
Senior Researcher, Port and Airport Research Institute, Japan

H. Ohmi
Student intern, Port and Airport Research Institute, Japan

ABSTRACT: It is necessary to clarify the effect of input motion characteristics on slope failure problem in order to appropriately determine input motion time histories. Even for the seismic design based on the pseudo-static approach, difference between real seismic loading and pseudo-static loading should be understood correctly to define the seismic loading level adequately. For this purpose, shaking frequency dependency on slope stability for cohesionless materials was experimentally investigated. Based on a series of model slope failure tests, general tendency for the difference between static inclination, 10 Hz and 20 Hz sinusoidal shakings was clarified. A simple model considering a particle rotation was proposed to explain these test results. This model for frequency dependency enables a quantitative discussion of negligible pulse-like motions for the slope stability.

KEYWORDS: Slope stability, Shaking frequency, Cohesionless material, Sand, Input motion.

1 INTRODUCTION

The pseudo-static approaches have been widely used in seismic design procedure of structures. Although real seismic loadings are dynamic, cyclic, and non-stationary, a constant static loading shall be applied in these approaches for simplicity. Based on the assumption of pseudo-static loading, a number of design techniques have been proposed. Mononobe (1924) and Okabe (1924) proposed dynamic earth pressures for retaining walls. For the design of slopes, horizontal seismic loading is added on the force balance equilibrium in the framework of rigorous limit equilibrium. For example, in the design of high embankment for Japanese airports, a seismic coefficient for horizontal shaking have to be considered in the framework of modified Fellenius method (MOT, 1998).

Differences between real seismic loading and static loading applied in these pseudo-static approach have been investigated in many ways. Noda et al. (1975) summarized the relationships between seismic coefficients in design for case histories of damaged gravity-type quay walls and peak ground accelerations at these sites. A similar approach was also applied for slope and embankment (Matsuo et al., 1984; Matsuo and Itabashi, 1984).

Instead of calibrating the appropriate seismic coefficient for design, the allowable residual deformation concept can be used (e.g., Newmark, 1965; Seed, 1966). In these deformation-based approaches, the sliding mass model should evaluate residual deformation. Applicability of the sliding mass model for sandy slope was experimentally investigated (Goodman and Seed, 1966). Sarma (1975) extended this model to include excess pore pressure generation. Although the procedure of sliding block mass is straightforward, threshold acceleration level and input motion time histories are difficult to be determined appropriately.

Recently, advanced computer technology has enabled to conduct numerical simulation of slope failure by finite element analysis. (e.g., Wakai and Ugai, 2003; Wakai et al., 2001; Iai et al., 1998) However, the determination of appropriate parameters and input motion time histories are still problems to be solved.

It is necessary to clarify the effect of input motion characteristics on slope failure problem in order to appropriately determine input motion time histories. A simple definition of input motion characteristics based on the difference between pseudo-static loading and seismic loading is illustrated
teristics based on the difference between pseudo-static loading and seismic loading is illustrated in Fig. 1. Although the pseudo-static loading is supposed to continue forever, real seismic loading exists only for a limited duration. It should be called a transient effect. Although the pseudo-static loading is supposed to be a constant value, real seismic loading is applied again and again. It should be called a repetitive loading effect. Furthermore, even though within the scheme of endless repetitive loading, some differences might be exist in the differences of loading frequency. It should be called a frequency dependency effect, and the mixture of the repetitive effect and the frequency dependency effect are usually considered the dynamic loading effect.

This research experimentally evaluates the magnitude of the shaking frequency dependency effects on slope stability problem for cohesionless material.

![Figure 1](image)

**Figure 1** A schematic view for real seismic loading and pseudo-static loading

### 2 MODEL TESTS

#### 2.1 Model preparation

The model slope is 100 mm tall with 1:1.5 initial inclination (33.7°), placed on a foundation layer with 100 mm thickness. The model cross section is shown in Fig. 2. A rubber layer is placed on the taller side to reduce the wall shaking. The model is placed in transparent plastic case with 364 mm length, 160 mm width and 300 mm height.

In order to build many models with the same initial condition, a shaking apparatus was used to compact sand. First, sand is compacted for each 20mm thickness by sinusoidal shaking 10Hz, 700Gal. This is used as the foundation layer of slopes. Then, with a slope face-making tool installed on the plastic case as shown in Fig. 3, sand is installed and compacted for each 20mm thickness. Sinusoidal shaking of 10 Hz was also used for compaction; however, three levels of compaction, 300Gal, 500Gal and 700Gal was carried out. In order to stabilize the slope by over-compaction, an extra sand layer of 30mm thickness was added. A vacuum was used to remove extra sand.

Soma silica sand No.5 was used. Particle size distribution curve and the results of CD tests are shown in Figs. 4 and 5, respectively.

Although the authors tried to maintain the test conditions through the test with this procedure, relative density of the slopes are not consistent. The list of cases and initial conditions for these cases are summarized in Table 1.
2.2 Static inclination tests

A hydraulic apparatus was used to gradually incline the model. Since only the surface failure occurred and no deformation at inside the slope was observed, the residual inclinations at top and bottom of the slope were measured. Since a handy inclinometer with 95 mm scale was used, the measured inclination was an average inclination for about 100 mm range. Fig. 6 shows the test results. Instead of inclination, the ratio of acceleration parallel and transverse to the bottom of slope was used in the figure.
Although the angle at rest for granular materials is dependent on many aspects such as the boundary conditions (Sakaguchi et al., 1995), the internal friction angle of sand is identical to the upper limit of slope angle (e.g., Funtai Kogakkai, 1998). Thus, the upper limit of the angle based on the measured internal friction angle by triaxial test is also shown in this figure. The measured slope angles at failure are consistent with the theoretical upper limit.

Once the slope angle exceeds a limit level, slope failure has started. However, it will continue beyond the unstable part of the slope as shown in Fig. 7. Thus, measured slope angle after failure is usually less than the limit slope angle. This mechanism of slope failure cycle also explains why the measured slope angle after failure is far less than the theoretical angle.

![Graphs showing measured slope angles in static inclination test](image)

(a) Measured slope angle at the top  
(b) Measured slope angle at the bottom

Figure. 6 Measured slope angles in static inclination test

![Diagram illustrating mechanism of cyclic slope failure in static inclination test](image)

Figure. 7 Mechanism of cyclic slope failure in static inclination test

### 2.3 Shake tests

An electro-magnetic shaking table was used for shaking table tests. Sinusoidal shakings with 10Hz and 20Hz were used, and peak level of input was set to increase about 50Gal step from 50Gal to about 600Gal. The duration of shaking is long enough to stop the slope failure completely. Thus, the loading duration effect was negligible in this series of shaking test. The measurement is same as static inclination test; however, acceleration at the top of the slope was measured to quantify the amplification of shaking. Although some amplification was observed, no major amplification due to resonance were observed.

The failure slope angles at the top of the slopes are summarized in Fig. 8 for all of the static and shaking tests. Since the static tests can be regarded the shaking with infinite period (zero frequency), the failure angles are in the order of static (0Hz) < 10Hz < 20Hz, except for the slope with 10Hz, 700Gal compaction. Since the relative density of model slopes scattered much as shown in Table 1,
this tendency could be the difference of test conditions. However, this tendency is clearer at the bottom part of the slope where the conditions are unique since these sand are fallen from the top of the slope. Therefore, the authors believe that frequency dependency effect on the slope failure is observed in these tests.

Since the sinusoidal shakings in these tests continued endlessly, this frequency dependency should be explained as the difference of the particle behavior in 1 cycle of motions.

![Graph](image)

(a) Stable slope angle at the top  
(b) Stable slope angle at the bottom  
Figure 8 Stable slope angles at the top of the slope

3 NUMERICAL MODELING

3.1 A simple model for frequency dependency

Since the failure mode of the slope in these static tests and shaking tests are surface failure, a particle on the slope is focused. When the particle with rotational friction of $a/r$ (Sakaguchi et al., 2002) is subjected to horizontal loading, the particle will start to rotate as shown in Fig. 9. Then, if the rotated angle exceeds a limit, the particle turned over and deformation or failure of slope will occur. However, if the horizontal loading stops before the rotation exceeds the limit, the particle turns back to suit the former position, and no deformation or failure will occur. When the force $H$, parallel to the slope, and $V$, transverse to the slope, are acting on a sand particle with rotational friction $a/r$ placed on the slope angle of $\phi$, as shown in Fig. 10, the rotational moment acting on the particle is as follows,

$$ M = H r - V a $$  

where,

$$ H = mg \sin \phi + mh \cos \phi, \quad (2) $$  

$$ V = mg \cos \phi - mh \sin \phi, \quad (3) $$

$m$: mass of the particle,  
g: gravity acceleration;  
$h$: horizontal loading (acceleration).

When the particle have uplifted and rotated for the angle of $\theta$, the rotational moment is a function of the uplifted angle:

$$ M(\theta) = H(r \cos \theta + a \sin \theta) - V(a \cos \theta - r \sin \theta) \quad (4) $$
The rotational moment of inertia $I$ is,

$$I = m(a^2 + r^2) = mr^2 \{1 + (a/r)^2\} \quad (5)$$

The acceleration of uplifting rotation is,

$$\ddot{\theta} = \frac{M(\theta)}{I} \quad (6)$$

Therefore, the criteria for deformation or failure is based on the limit angle $\phi$ as,

$$\int \frac{M(\theta)}{I} dt > \phi: \text{Deformation or failure} \quad (7)$$

$$< \phi: \text{Stable} \quad (8)$$

The limit angle for rotation is related with the rotational angle $a/r$ as,

$$\phi = \tan^{-1}(a/r) \quad (9)$$

For simplicity, the integral in equation (7) should be computed in discrete form. In this case, time integration scheme such as the central difference method can be applied as,

$$\{\theta\}_{n+1} = 2\{\theta\}_n - \{\theta\}_{n-1} + \frac{M(\theta)}{I} \Delta t^2 \quad (10)$$

The duration of integration is for the duration of positive moment $M(\theta)$ in the first cycle of loading, from $t_0$ to $t_1$ in Fig. 11.

Sinusoidal loading can be expressed as,

$$h(t) = K_{\phi} g \sin \omega t \quad (11)$$
where, \( K_h \): seismic coefficient
\( \omega \): angular frequency

Thus, the parameters for this model are only five; \( a/r \): rotational friction angle, \( r \): distance from center of particle to contact plane, \( K_h \): seismic coefficient, \( \omega \): angular frequency of loading and \( \phi \): initial slope angle. The largest value of initial slope angle which maintain stability under sinusoidal shaking is referenced to ‘angle at cyclic loading’ in this paper.

![Graph](image)

Figure 12 Results of shaking frequency effect

### 3.2 Calculated results

Figure 12 shows the comparison between shake test results and calculation. Rotational friction angle of \( a/r = 0.87297 \) (\( \phi = 41.12^\circ \)) and \( r = 0.0065 \) (m), which is identical with \( D_{90} \), were used. Although discrepancy still remains, calculated results shows good agreement in general tendency.

### 4 DISCUSSION

Although the existence of frequency dependency effect on slope stability were not verified enough, the shake test results and proposed model reveals that frequency dependency effect could exist and be important for slopes with large-size particles since the proposed model include not only the friction parameter \( a/r \) but also the size parameter \( r \). It implies that a threshold of frequency to be considered could exist for a slope with a large particle. Thus, it will give useful information in the determination of input motion for design of rock fill dams, if the relationships between the threshold frequency to be considered and particle size were clarified.

Earthquake engineers know that pulses with high amplitude but short duration will not affect the overall stability of structures. However, quantitative discussion for permissible pulse has not been made yet. The proposed model is a simple but the first model to discuss the permissible pulse-like motion for the slope stability.
5 CONCLUSION

A series of model tests for slope failure were conducted to clarify the shaking frequency dependency effect, and a simple model for the evaluation of the effect was proposed. Major findings of the research are as follows.

1) Failure angle of the slope under a peak horizontal acceleration is in the order of static < 10 Hz < 20 Hz in this research. Although, this difference could be the difference of initial conditions, the author believes it is due to the shaking frequency effect since this phenomena is clearer at the bottom of the slope than the top.

2) A simple model for the stability of particle considering particle rotation was proposed. The model calculation explains the general tendency of the test results well. This model for frequency dependency enables a quantitative discussion of the permissible pulse-like motion for the slope stability, for the first time.

REFERENCES


MOT; Ministry of Transport, Japan: Design policy of high embankment for airport, 224p, 1998. (in Japanese)


