運輸省港湾技術研究所

港湾技術研究所報告

REPORT OF THE PORT AND HARBOUR RESEARCH INSTITUTE

MINISTRY OF TRANSPORT

VOL. 18 NO. 1 MAR. 1979

NAGASE, YOKOSUKA, JAPAN

港湾技術研究所報告 (REPORT OF P.H.R.I.)

第18巻 第1号 (Vol. 18, No. 1), 1979年3月 (Mar. 1979)

目 次 (CONTENTS)

REPORT OF THE PORT AND
HARBOUR RESEARCH INSTITUTE
Vol. 18, No. 1 (March 1979)

1. A Review on Statistical Interpretation of Wave Data

Yoshimi GODA*

Synopsis

Various aspects of wave statistics are reviewed on the basis of field observation reports mostly published in last several years. Wave statistics are classified into three categories of short-term, wave climate, and long-term statistics, according to their time scales. In the short-term wave statistics, a small but consistent deviation of wave heights from the Rayleigh distribution has been observed. The joint distribution of wave heights and periods is now paid attention from the viewpoints of theoretical as well as field investigations. In the wave climate statistics, the persistency of sea state has become a subject of discussion. As to the long-term wave statistics, attention is called for the long-term wave variability and the reliability of data base.

A survey of some twenty reports on observed wave data is made to examine the relationships between various characteristic wave periods. It indicates that $T_{H_{1/3}}$ is the most reliable period parameter, while $\bar{T}_z$ is the least reliable one being affected by the performance of wave recording systems and other factors. A generalized form of the JONSWAP spectrum, which covers the Bretschneider-Mitsuyasu spectrum and the ISSC spectrum, is proposed with the parameters of $H_{1/3}$, T_p, and shape factor with suggestions for conversion of characteristic wave periods into T_p.

---

* Director of Marine Hydrodynamics Division.

港湾技術研究所報告
第18巻 第1号 (1979. 3)

1. 波浪資料の統計的取扱に関する考察

合田良実*

要旨

波浪統計は対象とする時間スケールによって短期統計，波候統計，および長期統計の3種に大別される。こうした波浪統計については，現地観測成果に基づく報告が最近に至って数多く発表されているので，これらについて概括的な考察を行なった。

短期統計，または波群中の個別の波の統計的性質に関しては，波高分布が僅かながらレーリー分布からずれることが認められる。また，波高と周期の結合分布に関する二つの理論が最近発表されたことにより，これを現地データで検討する事例が増えている。さらに，有義波高や平均周期などの代表値は不規則な波形に対する一つの統計量であるから，その統計的変動幅についても注意する必要がある。

波候統計のうち波高などの出現確率については，対数正規分布およびワイブル分布のあてはめが行なわれるが，いずれも観測結果の近似的表示の一法であり，波浪データに対する適合性の理論的根拠を示すことはできない。また，最近は高波の持続性に関する統計解析が試みられている。

長期統計は全波計と極値統計に分けられる。前者は長期間における全ての波の波高出現度数を求めるもので，構造物の疲労解析などに利用される。後者は所定の再現期間に対応する最高波高，有義波高の値を推定するための統計解析である。いずれも解析手法が幾つか提案されているが，作業にあたっては波浪状況の長期変動および使用データの信頼性に十分注意しなければならない。

また本報告では，不規則波の周期に関する代表値を6種選び，それらの相互関係を現地観測報告約20編について調査した。この結果，有義波周期が最も統計的変動性が少なくてかつスペクトルのピーク周期に最も近いこと，平均周期は波高計の機種，記録・解析方法などによって影響されて変動が著しいことなどが明らかになった。また，目視観測による波周期は有義波周期にほぼ等しいようである。なお，波浪の標準スペクトルとしては有義波高とピーク周期をパラメーターとして表示し，使用する波浪データに応じて周期の変換を行なうのが適当であり，これによって Bretschneider-光易型スペクトルと ISSC スペクトルとを調和させることができる。さらに，JONSWAP スペクトルを波高，周期，および形状パラメータを用いて表示する式を提示した。

---

* 海洋水理部長

CONTENTS

1. Introduction

Sea waves had eluded man's intensive effort to quantify their characteristics for long time, but the development in ocean instrumentations in a recent few decades has been providing us with ever increasing volume of wave data from various parts of the sea. Many reports have been published, especially in last several years, which discuss various aspects of observed sea waves. There are also available several statistical theories which provide the basis for interpretation of sea wave data. Time is opportune for presenting a review of these studies from the engineer's viewpoint. Since a previous review work by *Notle* [1973] concentrated on the statistics of extreme sea state and another by *Battjes* [1977] put some emphasis on theoretical aspects, the present review will make effort to provide a concise but overall view of wave statistics on the basis of field observation reports. A detailed discussion will also be given on the relationships between characteristic wave periods for the purpose of selecting a most appropriate period parameter for the nominal wave spectrum.

It should be mentioned here that the present review was originally prepared as a draft contribution to the report of the Committee I.1 on Environmental Conditions of the Seventh International Ship Structures Congress to be held in Paris, 1979. With the encouragement of the Committee Chairman, Dr. N. Hogben, an enlarged version of the draft contribution is presented hereupon.

Notation—The symbols employed in this report are listed with their definitions in the appendix. Most of them are in accordance with the PIANC recommendation [1973].

2. Classification of Wave Statistics

Interpretation of wave data is made in many forms, depending upon the purpose of analysis and application. A classification of wave statistics into the short-term and the long-term statistics is quite common, especially in the field of naval architecture. When dealing with stationary offshore and coastal structures, however, another category of wave statistics with the time scale of a year or so becomes indispensable, which is called the wave climate statistics. In the two-way classification, the wave climate statistics is treated as a part of the long-term statistics. But the former aims at the general description of seasonal and annual wave conditions at specific locality, whereas the latter is oriented for design applications for structures with the lifetime of several tens of years. The methodology of analysis is different, and therefore the distinction between the two is better made for clarifying the interpretation of wave data. Thus, various items of wave statistics, although they are not exhaustive, are listed in the following under the headings of the short-term, the wave climate, and the long-term statistics:

1) Short-Term Wave Statistics
    a) distributions of individual wave heights and surface elevations
    b) distributions of individual wave periods
    c) joint distribution of individual wave heights and periods

- d) distribution of individual wave steepness
- e) statistics of wave groups

2) Wave Climate Statistics
    - a) marginal distributions of sea state parameters
    - b) joint distributions of sea state parameters
    - c) statistics of sea state persistency
3) Long-Term Wave Statistics
    - a) long-term distribution of individual wave heights and wave-induced loads
    - b) estimate of extreme, individual waves in terms of return periods
    - c) estimate of extreme sea state in terms of return periods

The term of sea state is used here to denote the wave condition described by means of height, period, and direction parameters. The wave height parameter commonly employed is H_v (visually observed height) or $H_s=H_{1/3}$ (significant wave height or the mean of the heights of highest one-third zero up-crossing waves). The wave period parameter commonly employed is T_v (visually observed period), $\bar{T}_z$ (mean of zero up-cossing wave periods), $T_{m_{0,1}}$ (mean period defined with the zetoth and first moment of wave spectrum), or $T_{H_{1/3}}$ (significant wave period or the mean of the periods of highest one-third zero up-crossing waves). The diversity of wave period parameters often causes inconvenience in data utilization and sometimes misinterpretation, too. To clarify the question of appropriate wave period parameter, discussion is made in Chapter 4 on the relationships between characteristic wave periods.

3. Short-Term Wave Statistics

3.1 Wave Height Distribution

The marginal distribution of individual wave heights is well approximated by the Rayleighan of the following form:

$$P[x]=1-\exp[-a^2x^2], \tag{1}$$

where

$P[x]$: distribution function or the probability that x does not exceed a given value
$x=H/H_*$: nondimensional wave height
H_* : arbitrary reference height

$$a=\frac{H_*}{(8m_0)^{1/2}}=\begin{cases} 1/\sqrt{8} : & H_*=\sqrt{m_0}=\eta_{\rm rms} \\ \sqrt{\pi}/2 : & H_*=\bar{H} \\ 1 \quad : & H_*=H_{\rm rms} \end{cases} \tag{2}$$

m_0 : zeroth moment of wave frequency spectrum.

The applicability of the Rayleigh distribution to sea waves was proved by *Longuet-Higgins* [1952] for the case of narrow-band spectrum. A number of wave observation data (*Bretschneider* 1959 and others) have provided further evidence of its applicability to waves with broad-band spectra under the condition of individual waves being defined by the zero crossing method. A numerical simulation study by *Goda* [1970] also indicates that the quasi-Rayleigh distributions of zero up-crossing wave heights are observed irrespective of spectral forms.

A small but consistent deviation of wave heights from the Rayleigh distribution has been found with regard to the ratio of H_s to $\sqrt{m_0}=\eta_{\mathrm{rms}}$; most of observations suggests the ratio of 3.8 on the average (*Wilson* and *Baird* 1972, *Liu* and *Robbins* 1974, *Soejima et al.* 1975, and others) instead of the theoretical value of 4.004. The ratio of 3.8 is also realized in numerical simulation of sea wave profiles (*Goda* 1970). As to the height of highest wave in a group of waves relative to $\sqrt{m_0}$, *Haring et al.* [1976] as well as *Forristall* [1978] show the observed ones lower than the theoretical ones on the average by about 10% for extreme wave conditions, especially at the Gulf of Mexico. *Forristall* [1978] further proposes an empirical distribution of the following form for zero up-crossing wave heights:

$$P[\xi]=1-\exp\left[-\xi^{\lambda}/\mu\right], \tag{3}$$

where

$$\xi=H/\sqrt{m_0}, \quad \lambda=2.126, \quad \text{and} \quad \mu=8.42.$$

3.2 Wave Period Distribution

The marginal distribution of individual wave periods is closely related with the joint distribution of wave heights and periods. A semi-empirical proposal of the Rayleigh distribution for T^2 by *Bretschneider* [1959] provides a fair approximation to wind waves, though he could not formulate the joint distribution in a closed form. *Longuet-Higgins* [1975] has derived the theory of the joint distribution for waves with a very narrow band spectrum, which can be rewritten in the following form:

$$p(x,\tau)=\frac{2a^3x^2}{\sqrt{\pi}\,\nu}\exp\left\{-a^2x^2\left[1+\frac{(\tau-1)^2}{\nu^2}\right]\right\}, \tag{4}$$

where,

$p(x,\tau)$: joint probability density function of nondimensional wave heights and periods
$\tau=T/\bar{T}_z$: nondimensional wave period

$$\nu=\left[\frac{m_0m_2}{m_1^2}-1\right]^{1/2}: \text{ spectral width parameter} \tag{5}$$

$$m_n=\int_0^{\infty}f^nS(f)df: \ n\text{-th moment of frequency spectrum} \tag{6}$$

The marginal distribution of wave periods is derived from Eq. (4) as

$$p(\tau)=\frac{\nu^2}{2[\nu^2+(\tau-1)^2]^{3/2}}\,. \tag{7}$$

The joint distribution of Eq. (4) has its axis of symmetry at $\tau=1$, or $T=\bar{T}_z$, and yields no correlation between wave heights and periods. On the other hand, ocean waves exhibit a distinctly positive correlation, especially at the portion of low waves [*e.g.*, *Chakrabarti* and *Cooley* 1977, and *Goda* 1978]. This tendency is formulated in the theory by the group of CNEXO [*Arhan et al.* 1976, *Cavanié et al.* 1976, and *Ezraty et al.* 1977], which is basically for the joint distribution of the amplitudes and periods of positive maxima. The time interval between successive positive maxima is estimated by extending the theory of *Cartwright* and *Longuet-Higgins* [1956], and is employed as the substitute of zero up-crossing wave period. The proposed joint and marginal distributions are as follows:

$$p(\xi,\zeta)=\frac{\alpha^3\xi^2}{4\sqrt{2\pi}\,\varepsilon(1-\varepsilon^2)\zeta^5}\exp\left\{-\frac{\xi^2}{8\varepsilon^2\zeta^4}[(\zeta^2-\alpha^2)^2+\alpha^4\beta^2]\right\}, \tag{8}$$

where,

$\xi=H/\sqrt{m_0}$: nondimensional wave height
$\zeta=\bar{\zeta}\tau=\bar{\zeta}T/\bar{T}_z$: nondimensional wave period

$$\alpha=\frac{1}{2}(1+\sqrt{1-\varepsilon^2}) \tag{9}$$

$$\beta=\varepsilon/\sqrt{1-\varepsilon^2} \tag{10}$$

$$\varepsilon=\left[1-\frac{m_2{}^2}{m_0m_4}\right]^{1/2}\equiv\varepsilon_S:\ \text{spectral width parameter.} \tag{11}$$

The mean value of nondimensional wave period, $\bar{\zeta}$, is numerically obtained from the marginal distribution of wave period of the following:

$$p(\zeta)=\frac{\alpha^3\beta^2\zeta}{[(\zeta^2-\alpha^2)^2+\alpha^4\beta^2]^{3/2}}\,. \tag{12}$$

When this theory was applied for ocean waves by the group of CNEXO, the spectral width parameter was estimated by the formula of

$$\varepsilon=[1-(N_0/N_c)^2]^{1/2}\equiv\varepsilon_T\,, \tag{13}$$

where N_0 and N_c denote the numbers of zero up-crossings and maxima of surface elevation in a wave record. Thus, ε was treated as an empirical parameter derived by the wave-by-wave analysis.

Though the theory by CNEXO is theoretically inconsistent especially in its approximation of the mean zero up-crossing wave period with the mean interval of positive maxima as pointed out by *Battjes* [1977], it provides a good approximation to the joint distribution of heights and periods of ocean waves. A shortcoming of the theory is that the asymmetry of the joint distribution with respect to the wave period becomes too pronounced with the increase of the spectral width parameter and the theory predicts the probability of long periods much higher than the observation. It is observed that the portion of high waves in the joint distribution retains the symmetry around their mean value of periods irrespective of the value of spectral width parameter, even though the mean period of high waves increases relative to $\bar{T}_z$ as the spectral width parameter increases [*Goda* 1978].

3.3 Statistical Variability of Sea State Parameters

The characteristic heights and periods of ocean waves such as H_s, $T_{H_{1/3}}$, etc.

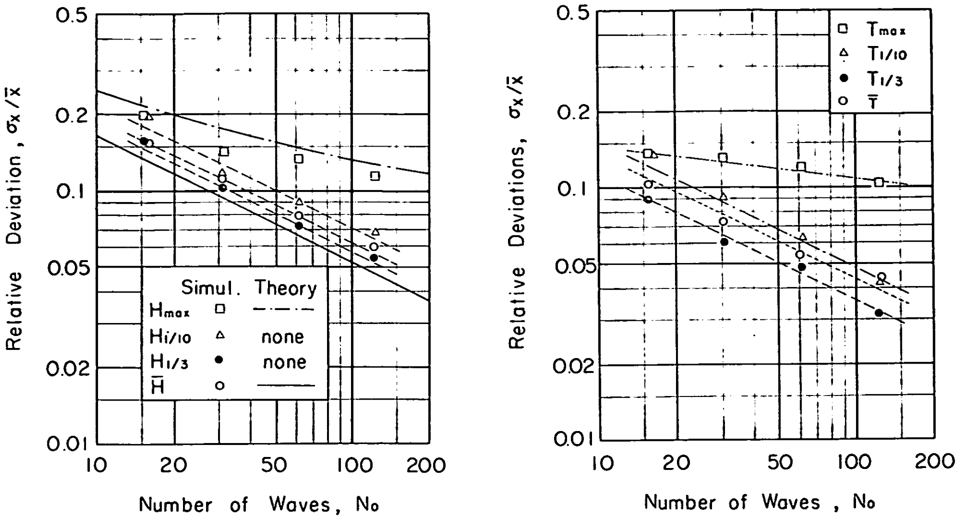

Fig. 1 Ratios of Standard Deviations to the Means of Characteristic Heights and Periods of Zero Up-crossing Waves (*Goda* 1977)

are subject to statistical variability due to irregularity of wave profiles. A numerical simulation study by *Goda* [1977a] with a directional spectrum has yielded the variation of standard deviations of various characteristic heights and periods in terms of the number of zero up-crossing waves as shown in **Fig. 1**. The significant height and period are found to be the stablest parameters. Nevertheless they have the standard deviations of about 6% and 4%, respectively, when estimated from a single sample of one hundred waves long. Interpretation of wave data should be made with due consideration for such statistical variability.

4. Wave Climate Statistics

4.1 Distributions of Sea State Parameters

The simplest wave climate statistic may be the monthly, seasonal and annual means of sea state parameters. They are quite useful to describe the wave conditions at respective locality. Another wave climate statistic often analyzed is the marginal distribution of wave height parameter in the form of non-exceedance or cumulative probability. The log-normal distribution of the following has been fitted to many data since *Jasper* [1955] with a varying degree of success:

$$p(y)=\frac{1}{\sqrt{2\pi}\,\sigma_y}\exp\left\{-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right\}, \qquad (14)$$

where,

$y=\log H$
$p(y)$: probability density function of y
μ_y : mean of y
σ_y : standard deviation of y

Lawson and *Abernethy* [1975] advocate the use of the log-normal distribution because only two parameters are sufficient to describe the wave height distribution.

Where the log-normal distribution does not fit to the data, the following Weibull distribution is employed to approximate the observed distribution [*Battjes* 1970 and 1972]:

$$P[H \leq X] = \begin{cases} 1-\exp\left[-\left(\dfrac{X-H_c}{H_0}\right)^k\right] : & X \geq H_c \\ 0 \qquad\qquad\qquad\qquad : & X < H_c , \end{cases} \tag{15}$$

where $P[H \leq X]$ denotes the probability that the wave height parameter H does not exceed a given level X. The parameter H_c roughly corresponds to the height of swell persistent at the locality; it is to be selected by inspection of observed data or by several trials with different values for H_c for search of best fitting. The parameters H_0 and k are characteristic ones of the Weibull distribution, and they are determined by the best fitting to the observed wave data.

The choice of the distribution functions for H_s is sometimes debated. Some support the Weibull distribution, while the others are in favour of the log-normal distribution. The arguments are based on the degree of goodness-of-fitness without a theoretical reasoning. In fact, no theoretical justification will be possible for the distributions of sea state parameters. *Nordenstrφm* [1973] commented that the Weibull distribution, used to describe wave heights or response amplitudes, cannot be true or false but only good or bad. The fitting of a distribution function is anyhow an abbreviated form of data presentation when the tabulated form of original ones is cubumsome. One should be content himself with one type of distribution function so long as it provides a good fitting to his observed wave data.

The joint distribution of height and period parameters is also prepared often from the observed wave data. It is mostly in the form of correlation tables with frequencies of observations in respective ranks of height and period, either in absolute numbers or in relative figures per mil [*e.g.*, *Draper* 1966a]. *Lawson* and *Abernethy* [1975] applied the multi-variable log-normal distribution to describe the wave climate off Botany Bay, Australia, but its applicability to other locations will be of a limited extent.

4.2 Sea State Persistency

Apart from the concept of wave group, which is a sequence of high waves among a wave train [*e.g.*, *Goda* 1976], discussion is needed on the persistency of sea state. A measure of sea state persistency is the length of the duration of calm sea or rough sea. As the information on wave duration is useful for planning and execution of maritime works, time series data of sea state are analyzed for the duration of sea state above or below a certain level. *Lawson* and *Abernethy* [1975] found that the average duration (in days) of rough sea could be approximated by $7/H_s^2$ (in meters) off Botany Bay, Australia. *Lawson* and *Youll* [1977] further showed that both the log-normal and Weibull distributions can be employed to describe the duration of storm waves. Theoretical approach has been tried by *Houmb* [1971], *Vik* and *Houmb* [1976], and *Houmb* and *Vik* [1977], who demonstrate the use of the Weibull distribution and have prepared the data for estimation of the wave duration at Norwegian coasts.

Another measure of sea state persistency is the autocorrelation of time series data of wave height parameter. *Goda* [1967] obtained the correlation coefficient of about 0.30 with the lag of one day along the coasts of Japan, while *Lawson* and *Abernethy* [1975] found that the correlation coefficient remained above 0.5 among the data one day apart. The existence of such correlation indicates that the data of wave parameters taken at several hours apart are not statistically independent. The analysis of wave data for extreme wave conditions thus requires the use of annual maxima or a peak value of each storm sea in order to insure statistically independent data. Such a procedure is called the grouping correction by *Nolte* [1973].

5. Long-Term Wave Statistics

5.1 Individual Wave Statistics

The distribution of individual wave heights and wave-induced loads during the lifetime of a structure is needed to make fatigue analysis and others. The Prodeedings of the International Ship Structures Congress almost always discuss the long-term distribution of individual wave heights and wave-induced loads. The present practice of the prediction is due to *Battjes* [1970 and 1972], who combined the observed joint distribution of H_s and $\bar{T}_z$ with the Rayleigh distribution of individual wave heights within a given sea state. The resultant long-term distribution may be fitted with the Weibull distribution similar to Eq. (15), but the exponential distribution may serve as the first approximation. When the joint distribution of H_s and $\bar{T}_z$ is not readily known, the marginal distribution of H_s may be employed for the prediction of long-term distribution of individual wave heights. The technique was initially applied by *Japsper* [1956] and later advocated by *Nordenstrφm* [1969]. A good estimate of the overall mean of $\bar{T}_z$ is crucial in the latter technique for yielding a reliable long-term distribution.

A caution must be taken on the yearly variation of sea state when making prediction of long-term distributions. **Figure** 2 shows the number of individual waves per year exceeding a certain height level off Kashima Port, Japan. It was estimated by the present writer from the data published by *Takahashi et al.* [1975 to 1978] by the method similar to that proposed by *Battjes* [1970 and 1972]. The central curve is based on the data of four years from 1973 to 1976, while the upper and lower curves are based on the one-year data of 1975 and 1976, respectively. Clearly, a single year is too short to yield reliable long-term statistics. *Nolte* [1973] has demonstrated another example that H_{max} for the return period of 100 years in the North Sea is estimated as 11.5 m from the one-year data of 1967, while it becomes nearly threefold or $H_{max}=$ 32 m from the one-year data of 1968. Yearly variations of strong winds

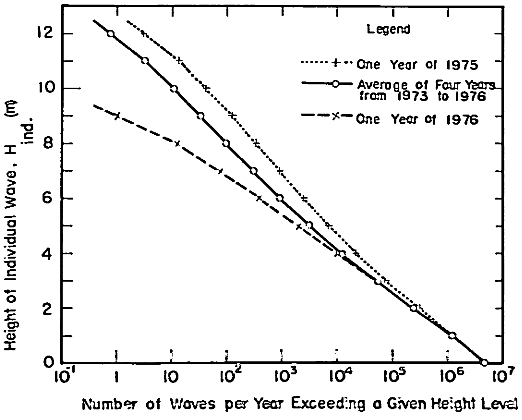

Fig. 2 Cumulative Distribution of Individual Wave Heights off Kashima Port, Japan

and wave conditions were also demonstrated at the Third, Fourth, and Sixth International Ship Structures Congresses [*ISSC* 1967, 1970, 1976]. The wind and wave conditions seem to have the periods of fluctuations of some 12 years and several tens of years. *Yamaguchi et al.* [1978] have made a hindcasting of wind waves at the Lake Biwa for the period of 80 years and demonstrated a long-term fluctuation of annual maxima of significant wave heights. *Ward et al.* [1977] also report by a wave hindcasting study that the period of 1950 to 1959 was the severest decade along the Atlantic coast of the United States during the period of 1900 to 1975.

In consideration of the yearly variation of sea state, efforts should be made for the collection of reliable wave data covering a sufficiently long period. The record of visual observation is one source of data, but its consistency of accuracy over years needs to be examined as discussed by *Houmb et al.* [1978]. A promissing source of data will be a systematic wave hindcasting project for all storms which produced waves exceeding a certain level. The joint distribution of wave height and period parameters can be constructed from the hindcasted sea state records, and the long-term distribution of individual wave heights above a certain level can be estimated.

5.2 Estimation of Extreme Wave Conditions

The concept of extreme waves is associated with that of desing waves for stationary offshore and coastal structures. The extreme individual wave for a given return period is sought for when a structure is designed against a single highest wave. The extreme sea state, usually expressed in terms of significant wave parameters, needs to be predicted when a structure is designed with the technique of spectral response analysis. The latter can also yield the information for the former by converting H_s into $H_{\max}$ with the knowledge of short-term wave height distribution and the duration of extreme H_s.

The methods for the prediction of extreme wave conditions are basically:

1) extraporation of long-term distribution of individual or characteristic wave heights,
2) extreme value analysis with annual maxima, and
3) extreme value analysis with peak wave heights of major storms above a certain level.

Among these methods, the long-term distribution model has the merit of easy application and has been employed at various locations [*e.g.*, *Draper* 1966b]. However, caution should be taken for the statistical dependency of observed data and the long-term variability of wave conditions as there exists a temptation of extraporating a few years data to a period of several tens of years. As cited earlier, *Nolte* [1973] also pointed out the necessity of the grouping correction for the long-term distribution. He presented an addition of two modifications for change of sea state between successive observations.

The techniques of extreme value analysis of wave data have been discussed by many people. *Petruaskas* and *Aagaard* [1970] have proposed a combination of the Weibull and Gumbel distributions of the following for peak storm waves:

$$P[H \leq X] = 1 - \exp\left[-\left(\frac{X-B}{A}\right)^k\right] \quad : \text{ Weibull distribution,} \qquad (16)$$

$$P[H \leq X] = \exp\left[-\exp\left\{-\left(\frac{X-B}{A}\right)\right\}\right]: \text{ Gumbel distribution.} \tag{17}$$

The shape parameter k in Eq. (16) is assigned one of seven values from $k=0.75$ to $k=2.0$. The parameters A and B for both distributions are determined from observed data by the least-square method. The selection of the distribution function is made by the principle of best fitting to the wave data. A distribution function of the Frechet type of the following has been employed by *Thom* [1973] for annual maxima of extreme wave heights:

$$P[H \leq X] = \exp\left[-\left(\frac{X}{\beta_2}\right)^{-\epsilon}\right]. \tag{18}$$

Another function of the Gamma distribution has also been tried by *Yamaguchi et al.* [1978] with the result of better fitting to the wave data than other distributions.

The choice of these distribution functions is governed by the principle of best fitting to the data. Though no theoretical justification for the selection is available at present, comparative studies on the fitting of these functions with reliable hindcasted wave data for long periods may lead to a differentiation of these functions in their applicability. The work of *Yamaguchi et al.* [1978] as well as that of *Copeiro* [1978] can be cited in this respect.

The plotting position or the non-exceedance probability to be assigned to each wave data is another question to be discussed. For the data of N annual maxima or peak values, non-exceedance probability is generally assigned by the following formula:

$$P[H < X_{m,N}] = 1 - \frac{m-\alpha_1}{N+\beta_1}, \tag{19}$$

where $X_{m,N}$ is the m-th largest wave height among N data ($m=1, 2, \cdots, N$). The parameters α_1 and β_1 are given the following values according to the plotting rules:

$$\left.\begin{array}{lll} \text{Hazen plot} & : \alpha_1=1/2, & \beta_1=0 \\ \text{Gumbel plot} & : \alpha_1=0, & \beta_1=1 \\ \text{Gringorten plot} & : \alpha_1=0.44, & \beta_1=0.12 \end{array}\right\} \tag{20}$$

(for the Gumbel distribution)

While Hazen's and Gumbel's plotting rules are intuitive propositions, *Gringorten*'s rule [1963] is based on statistical calculation to minimize the deviation of a sample from the population of Gumbel distribution when fitted to the distribution. *Petruaskas* and *Aagaard* [1970] have numerically determined the values of the α_1 and β_1 for the Weibull distribution with seven different shape parameters.

The factors more important than the distribution function and the plotting position in the analysis of extreme waves are the reliability of the data base and the long-term variability of wave conditions. Though the reliability of data base is sometimes overlooked, *Resio* [1975] demonstrated that the estimate of extreme wave height for the return period of 20 years could differ by the factor of two at Cleveland along the Lake Erie, depending on the selection of data base. He states that the goodness-of-fit of wave data when plotted as a straight line on a semi-logarithmic (or any other probability) paper does not necessarily insure the accuracy of the data as an estimate of extreme.

The long-term variability of wave conditions has been discussed with regards to the long-term distribution of individual wave heights. In consideration of the long lifetime of maritime structures, an estimate of extreme wave conditions needs to be based on the wave data in the order of 50 years [*Nolte* 1973].

6. Discussion on Characteristic Wave Periods

6.1 Survey of Field Data

While the characteristic wave heights are convertible each other through the formulae by the Rayleigh distribution, the relationships between characteristic wave periods have not been established. This fact is partially responsible for the controversy on the standard form of wave spectrum, because any standardization of spectral form requires its modal frequency to be related to some characteristic wave period. A survey of various reports of ocean and lake wave measurements was made in an effort to give light on the problem of characteristic wave periods. The result of survey is compiled in **Table 1**. Additional information is given below.

Data A [*Mitsuyasu*, *Nakayama*, and *Komori* 1971]: The data are those chosen for study of wave generation by winds. The wave period ratio of $T_{H_{1/3}}/\bar{T}_z$ was calculated by the present writer from a table in the report.

Data B [*Goda* and *Nagai* 1974]: Waves are locally generated wind waves but contaminated by low swell to some extent. About one third of the data exhibited multiple spectral peaks, for which the peak frequency was determined by visually fitting a smooth, single peaked spectrum.

Data C [*Yamaguchi* and *Tsuchiya* 1979]: All the data have single peaked spectra. Wave profiles were registered on analogue tape recorder and were digitized later by a A–D converter.

Data D and E [*Bretschneider* 1959]: Waves were read from recording charts by the method which was called the "crest-to-rough" method by the author but seems to be the method similar to the zero up-crossing technique with disregard of minute waves. The wave period ratios were calculated by the writer.

Data F [*Goda* and *Nagai* 1974]: The data are characterized with the slope of spectrum in high frequency range being milder than f^{-5}, probably because of shallow water effect. About one-third of the data exhibited multiple spectral peaks.

Data G [*Takahashi*, *Hirose*, and *Sasaki* 1979]: The data were recorded in digitized form on perforated paper tapes at the time interval of 0.5 seconds. Frequency spectra were calculated by the maximum entropy method (MEM), and the spectral peak frequency was estimated by fitting a smooth spectral form with a Gamma function. Wave period ratios were calculated by the writer from the original data supplied by *Hirose*.

Data H [*Honda* and *Mitsuyasu* 1978]: Wave profiles were recovered from the

Fourier components of vertical acceleration of a clover-leaf buoy with cutoff of very low frequency components. Some of wave period ratios were estimated by the writer as the inverses of the data presented by the authors.

Data I [*Pitt*, *Driver*, and *Ewing* 1978]: The study was carried out principally to investigate intercomparisons between wave-rider buoy, Baylor wave staff of resistance wires type, and laser altimeter. Some differences were observed in their instrumental performances, though they produced broadly comparable results. Wave period ratios for the whole data were provided by *Ewing* [1978] through private communication.

Data J [*Ezraty*, *Laurent*, and *Arhan* 1977]: The data were originally presented as a rank-wise chart against the spectral width parameter, from which the means and standard deviations were recalculated by the writer.

Data K [*Wilson* and *Baird* 1972]: Wave period ratios were estimated from the original scatter diagrams by fitting straight lines through the origin.

Data L [*Lawson* and *Abernethy* 1975]: Wave period ratios were calculated by the authors for waves with H_{rms} greater than about 2 meters.

Data M [*Lawson* 1978]: The data originally in the tabulated form of rank-wise information with respect to H_s were provided to the writer by the author through private communications. There exist some trends of the variation of wave period ratios with respect to H_s. The data with H_s below 1 meter were discarded in the present calculation by the writer as they show quite different values from the rest of data probably because of the strong influence of co-existing swell. Wave observation was made in the offshore and inshore of Newcastle, Australia, but the both data are combined in **Table 1** because the differences in wave period ratios between them are rather small.

Data N [*Arhan*, *Cavanié*, and *Ezraty* 1976]: same as Data J.

Data O [*Haring*, *Osborn*, and *Spencer* 1976]: Tabulated information of 22 groups of wave records was provided to the writer by *Haring* [1978] through private communication. Additional information was read from the diagrams in the paper.

Data P and Q [*Houmb* and *Overik* 1977]: Wave height ratios were calculated by the writer from the data sheet listed in the report.

Data R [*Chakrabarti* and *Cooley* 1977]: The data of $T_{m_{0,2}}$, $\bar{T}_z$, and ε_T were read from the diagrams in the paper. Some wave period ratios were estimated from the original scatter diagrams by fitting straight lines through the origin, and the rest were estimated from other mean values.

Data S [*Ferdinande*, *De Lembre*, and *Aertssen* 1975]: Wave period ratios were calculated by the writer from tables in the report.

Data T [*Earle* 1975]: same as Data S.

Data U [*Haring*, *Osborne*, and *Spencer* 1976]: same as Data O.

6.2 Theoretical Relationships between Characteristic Wave Periods

Before discussing the results of **Table 1**, some theoretical consideration is given on the relationships between characteristic wave periods. According to

Table 1 A Compilation of Relationships between Characteristic Wave Periods

Ident.	Source of Data [Ref.]	Location	Water Depth (m)	Fetch	Wave Classif.	Wave Sensor	Range of $\bar{T}_z$ (s)
A	Mitsuyasu et al. [1971]	Hakata Bay	5	short	deep water	capacitance gage	2.3- 3.3
B	Goda & Nagai [1974]	Nagoya Port	9	short	deep water	capacitance gage	1.8- 3.9
C	Yamaguchi & Tsuchiya [1979]	Lake Biwa	4	short	quasi-deep	capacitance gage	1.7- 3.2
D	Bretschneider [1959]	Fort Peck Resv. & Lake Texoma	not specif.	short	deep water	step-resis. gage	1.2- 4.4
E	Bretschneider [1959]	Lake Okeechobee & Gulf of Mexico	1.5 9	short long	shallow water	step-resis. gage	4.1- 5.3 7.3- 8.2
F	Goda & Nagai [1974]	Coasts around Japan	10-20	long	shallow water	step-resis. gage & invt. echo sounder	4.5-12.0
G	Takahashi et al. [1979]	ditto	20 & 50	long	quasi-deep	invt. echo sounder	5.1-10.3
H	Honda & Mitsuyasu [1978]	Sea around Japan	deep	long	deep water	cloverleaf buoy	2.8- 7.1
I	Pitt et al. [1978]	Southern North Sea	20	long	deep water	various	3 - 7
J	Ezraty et al. [1976]	Bay of Biscay	25	long	deep to shallow	waverider buoy	3 -11
K	Wilson & Baird [1972]	Nova Scotia Coast	36	long	deep to shallow	waverider buoy	3 -10
L	Lawson & Abernethy [1975]	off Botany Bay	75	long	deep water	waverider buoy	4.5- 8.5
M	Lawson [1978]	Newcastle Coast	21 & 80	long	deep to shallow	waverider buoy	2 -11
N	Arhan et al. [1976]	North Sea	deep	long	deep water	waverider buoy	5.5- 9
O	Haring et al. [1976]	various	110-180	long	deep water	waverider buoy	9.0-10.4
P	Houmb & Overik [1977]	Norwegian Coast	140-250	long	deep water	waverider buoy	5.8-10.7
Q	Houmb & Overik [1977]	Famita (North Sea)	not specif.	long	deep water	shipborn wave rec.	9.4-11.7
R	Chakrabarti & Cooley [1977]	Julliet (Atlantic)	very deep	long	deep water	shipborn wave rec.	8 -13
S	Ferdinande et al. [1976]	North Atlantic	very deep	long	deep water	shipborn wave rec.	6.5-13.1
T	Earle [1975]	Gulf of Mexico	100	long	deep water	wavestaff	7.2-11.0
U	Haring et al. [1976]	various	10-100	long	deep to shallow	wavestaff	6.2- 7.8

Table 1 (continued)

Ident.	Nos. of Data	$T_{H_{1/3}}/T_p$	$\bar{T}_z/T_p$	$T_{m_{0,1}}/T_p$	$T_{m_{0,2}}/T_p$	$T_{H_{\max}}/T_{H_{1/3}}$	$T_{H_{1/3}}/\bar{T}_z$	$T_{m_{0,2}}/\bar{T}_z$	ε_T
A	49	0.95[a]	—	—	—	—	1.056 (0.070)	—	—
B	92	0.960 (0.069)	0.906 (0.100)	—	0.751 (0.083)	0.953 (0.139)	1.066 (0.080)	0.832 (0.067)	0.628 (0.087)
C	173	0.889 (0.034)	0.728 (0.041)	—	0.715 (0.036)	0.998 (0.075)	1.222 (0.039)	0.983 (0.025)	0.808 (0.045)[b]
D	38	—	—	—	—	0.976 (0.170)	1.133 (0.065)	—	—
E	7	—	—	—	—	1.034 (0.210)	1.220 (0.063)	—	—
F	79	0.910 (0.047)	0.747 (0.068)	—	0.612 (0.086)	1.033 (0.107)	1.225 (0.078)	0.832 (0.077)	0.750 (0.084)
G	135	0.911 (0.079)	0.784 (0.069)	0.819 (0.078)	0.751 (0.079)	1.023 (0.207)	1.164 (0.054)	0.957 (0.032)	0.619 (0.071)
H	56	0.90 (0.011)	0.75 (0.083)	0.80 (0.079)	0.74[c]	—	1.20 (0.059)	0.99 (0.017)	0.66 (0.055)
I	55	—	—	0.77 (0.09)	0.72 (0.08)	—	—	0.98 (0.05)	—
J	2057	—	—	—	—	—	1.448 (0.136)	—	0.833 (0.069)
K	171	0.91[d]	0.71[d]	—	—	—	1.28[d]	—	—
L	263	—	0.713 (0.063)	—	0.70[c]	—	—	0.976 (0.032)	—
M	280	0.924 (0.072)	0.705 (0.086)	—	0.671 (0.082)	1.045 (0.174)	1.321 (0.102)	0.952 (0.037)	0.831 (0.048)
N	182	—	—	—	—	—	1.260 (0.050)	—	0.865 (0.031)
O	972	0.913 (0.066)	0.789 (0.070)	0.81[c]	0.77[c]	0.99[c]	1.16[c]	0.974 (0.035)	0.624
P	102	0.982 (0.073)	0.688 (0.071)	0.76[c]	0.667 (0.066)	0.939 (0.092)	1.433 (0.066)	0.971 (0.038)	0.800 (0.035)
Q	15	1.079 (0.058)	0.798 (0.050)	0.83[c]	0.710 (0.052)	0.811 (0.069)	1.355 (0.058)	0.890 (0.042)	0.711 (0.042)
R	25	1.00[d]	0.91[d]	0.77[c]	0.68[c]	—	1.10[c]	0.750 (0.056)	0.649 (0.075)
S	45	—	—	0.763 (0.102)	0.712 (0.103)	—	—	—	0.646 (0.078)[b]
T	20	—	—	—	—	0.980 (0.134)	1.240 (0.039)	—	—
U	215	0.779 (0.104)	0.574 (0.108)	0.62[c]	0.53[c]	1.05[c]	1.36[c]	0.918 (0.049)	0.854

Note : The numerals outside and inside the parentheses represent the arithmetic means and standard deviations of respective data.

Remarks: a) mean value assumed by the author, b) values of ε_S, c) estimate from other mean values, d) estimate from original scatter diagrams.

the statistical theory of *Rice* [1944], the mean zero crossing wave period can be estimated from the frequency spectrum as

$$\bar{T}_z=(m_0/m_2)^{1/2}\equiv T_{m_{0,2}}\,, \tag{21}$$

where m_n is the spectrum moment defined by Eq. (6). As the mean wave period actually observed in a wave record usually shows some deviation from the value of Eq. (21) owing to wave nonlinearily and other reasons [*e.g.*, *Collins* 1967, *Goda* 1974], the symbol of $T_{m_{0,2}}$ is employed to denote the value calculated by Eq. (21). Another mean wave period defined with the wave spectrum is

$$T_{m_{0,1}}=m_0/m_1\,. \tag{22}$$

This period has been employed as the characteristic wave period in the formulation of the ISSC spectrum [*ISSC* 1967], which is expressed as

$$S(f)=0.11H_{1/3}{}^2T_{m_{0,1}}{}^{-4}f^{-5}\exp\left[-0.44(T_{m_{0,1}}f)^{-4}\right]. \tag{23}$$

Equation (23) is a special case of the general spectral function of the following form with $m=5$ and $n=4$:

$$S(f)=Af^{-m}\exp\left[-Bf^{-n}\right]. \tag{24}$$

The above spectral function was first employed by *Neumann* [1953] with the values of $m=6$ and $n=2$. Then, *Phillips* [1958] suggested by theoretical arguments that m should be 5, which had been often observed in spectral measurement of wind waves. With this suggestion, *Bretschneider* [1959, 1963] as well as *Pierson* and *Moskowitz* [1964] proposed the spectral function of Eq. (24) with the values of $m=5$ and $n=4$, which provided the basis of Eq. (23). The spectrum proposed by Bretschneider was later modified by *Mitsuyasu* [1970] so as to be in accord with the relationship between wave statistics and spectra. The latter is sometimes called the Bretschneider-Mitsuyasu spectrum.

The frequency at the spectral peak is calculated for the function of Eq. (24) as

$$f_p=\left(\frac{Bn}{m}\right)^{1/n}. \tag{25}$$

The spectral moment defined by Eq. (6) can be calculated for the spectral function of Eq. (24) by using the following formula [*Mitsuyasu* 1970 and *Ferdinande et al.* 1975]:

$$\int_0^\infty Af^{-m}\exp\left[-Bf^{-n}\right]df=\frac{A}{n}\,\frac{\Gamma[(m-1)/n]}{B^{(m-1)/n}}\,, \tag{26}$$

where $\Gamma[\;]$ denotes the Gamma function.

The mean wave periods $T_{m_{0,1}}$ and $T_{m_{0,2}}$ are thus calculated with the above formulae for the case of $m=5$ and $n=4$ in terms of the wave period corresponding to the spectral peak, *i.e.*, $T_p=1/f_p$, as in the following:

$$\left.\begin{aligned} T_{m_{0,1}}&=0.7718T_p\,,\\ T_{m_{0,2}}&=0.7104T_p\,. \end{aligned}\right\} \tag{27}$$

Ferdinande et al. [1975] and *Ferdinande* [1976] have further proposed a use of the following mean wave periods though they have no theoretical ground:

$$\left.\begin{aligned} T_{m_{-1,0}} &= m_{-1}/m_0 , \\ T_{m_{-2,0}} &= (m_{-2}/m_0)^{1/2} . \end{aligned}\right\} \tag{28}$$

These mean wave periods are calculated as

$$\left.\begin{aligned} T_{m_{-1,0}} &= 0.8572\, T_p , \\ T_{m_{-2,0}} &= 0.8903\, T_p . \end{aligned}\right\} \tag{29}$$

Actual wave measurements and spectral analyses have a limitation in the range of frequency resolution up to the Nyquist frequency. Computation of spectral moments of Eq. (6), therefore, becomes less than the theoretical values computed up to the frequency of $f=\infty$. The effect of cutoff frequency on theoretical mean wave periods was noticed by *Collins* [1967] in his comparison with observed wave data. It was further examined by *Rye* and *Svee* [1976]. For example, $T_{m_{0,1}}$ and $T_{m_{0,2}}$ in terms of T_p vary as shown in **Table 2** for the spectrum of Eq. (24) with the values of $m=5$ and $n=4$, when the cutoff frequency is varied.

Table 2 Effect of Cutoff Frequency on Mean Wave Periods
(Bretschneider-Mitsuyasu or Pierson-Moskowitz spectrum)

Wave period	cutoff frequency/peak frequency				Remarks
	2.0	4.0	6.0	∞	
$T_{m_{0,1}}/T_p$	0.846 (0.892)	0.784 (0.844)	0.776 (0.837)	0.772	The numerals inside parentheses are for the JONSWAP spectrum of Eq. (30) with $\gamma=3.3$.
$T_{m_{0,2}}/T_p$	0.821 (0.870)	0.738 (0.802)	0.723 (0.788)	0.710	

The effect of cutoff frequency becomes less pronounced for sharply peaked spectra such as the JONSWAP spectrum to be discussed in the next chapter, but it is still observable. The results of **Table 2** also imply that $T_{m_{0,2}}$ will be much affected by the presence of noises at the high frequency range in spectral measurements.

6.3 Discussion of the Results of the Survey

A general impression of **Table 1** is a quite large dispersion of each wave period ratio. As a quick reference, the simple arithmetic mean and standard deviation of individual mean values of each wave period ratio were calculated with rejection of the highest and lowest values and with disregard of individual data size. The result becomes as in **Table 3**.

Table 3 Overall Means of Wave Period Ratios

Item	$\dfrac{T_{H_{1/3}}}{T_p}$	$\dfrac{\bar{T}_z}{T_p}$	$\dfrac{T_{m_{0,1}}}{T_p}$	$\dfrac{T_{m_{0,2}}}{T_p}$	$\dfrac{T_{H_{max}}}{T_{H_{1/3}}}$	$\dfrac{T_{H_{1/3}}}{\bar{T}_z}$	$\dfrac{T_{m_{0,2}}}{\bar{T}_z}$	ε_T
Mean	0.93	0.76	0.78	0.70	1.00	1.23	0.93	0.74
Standard Deviation	0.03	0.06	0.02	0.04	0.03	0.10	0.06	0.09

For respective wave period ratios, the following observations can be made on the basis of **Table 1** and the findings stated in the original reports:

1) The mean values of $T_{m_{0,1}}/T_p$ and $T_{m_{0,2}}/T_p$ do not deviate much from the theoretical values of the Bretschneider-Mitsuyasu or Pierson-Moskowitz spectrum listed in **Table 2**. This fact suggests the theoretical spectrum of Eq. (24) with the values of $m=5$ and $n=4$ being a fair approximation to ocean wave spectra.
2) The wave period ratios related to $\bar{T}_z$ all exhibit large variations, which imply the unsuitability of $\bar{T}_z$ as the characteristic wave period to be related to wave spectrum.
3) The significant wave period $T_{H_{1/3}}$ is the closest to T_p among various characteristic wave periods with relatively small statistical dispersion; most of data are in the range of $T_{H_{1/3}}/T_p=0.9\sim1.0$. This suggests that $T_{H_{1/3}}$ is a better parameter than $\bar{T}_z$ as the wave period characterizing wave conditions.
4) The mean value of $T_{H_{\max}}/T_{H_{1/3}}$ remains close to 1.0 except for the data set Q. This supports the applicability of the theoretical joint distribution of wave heights and periods by *Longuet-Higgins* [1975] for high waves in wave trains. It also suggests the importance of $T_{H_{1/3}}$ for design of maritime structures.
5) The mean wave period estimated from a spectrum, $T_{m_{0,2}}$, is slightly smaller than the directly counted mean period of $\bar{T}_z$. Some data taken by shipborn wave recorders and surface piercing type sensors indicate that $T_{m_{0,2}}$ may become less than 80% of $\bar{T}_z$. The difference between $T_{m_{0,2}}$ and $\bar{T}_z$ seem to be affected by the frequency response characteristics of wave sensor, recorder, and data processing technique as well as by the degree of wave nonlinearity registered by a wave recording system.
6) The increase in ε tends to increase $T_{H_{1/3}}/\bar{T}_z$ and decrease $\bar{T}_z/T_p$, as noticed by *Haring et al.* [1976] and the group of CNEXO [*Arhan et al.* 1976, *Cavanié et al.* 1976, and *Ezraty et al.* 1977]. Both of them treat ε as an empirical parameter by employing ε_T defined by Eq. (13) instead of ε_S to be estimated by Eq. (11).

The variability of $\bar{T}_z$ is partly due to different performances of wave recording systems employed, especially at high frequency range. A cutoff of high frequency tail of wave spectrum results in the increase of $\bar{T}_z$ and the decrease of ε. *Honda* and *Mitsuyasu* [1977] have proposed an intentional cutoff of spectral components above $2f_p$ for reliable statistical analysis of wave parameter. Characteristic wave periods also seems to be affected by the spectral shape and the peakedness of spectral peak as discussed by *Rye* and *Svee* [1976], though no quantitative analysis with ocean wave data on this problem has been published to the knowledge of the writer.

7. Discussion on Spectral Form of Extreme Waves

There are at least three questions regarding the standard forms of wave spectrum; that is,

1) whether the spectral peak is singular or multiple,
2) how sharp the spectral peak is, and

3) which characteristic wave period should be incorporated into the wave spectrum and how.

For extreme waves, there seems to exist the consensus that the wave spectrum has a single peak [*e.g.*, *Houmb* and *Due* 1978]. Even with six-parameter spectra proposed by *Ochi* and *Hubble* [1976], the duality of spectral peaks diminish as the significant wave height increases.

The question of the peakedness of extreme wave spectra became apparent by the proposal of the JONSWAP spectrum for fetch-limited waves by *Hasselmann et al.* [1973]. Its original functional form is

$$S(f)=\alpha_0 g^2(2\pi)^{-4}f^{-5}\exp\left[-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right]\gamma^{\exp[-(f/f_p-1)^2/2\sigma^2]}, \tag{30}$$

where,

$$\sigma=\begin{cases}\sigma_a & \text{for } f\leq f_p\\ \sigma_b & \text{for } f>f_p,\end{cases}$$

and the parameters α_0 and f_p are related to nondimensional fetch. The shape parameter γ varied from 1 to 7 with the average value of 3.3, while σ_a and σ_b were assigned the values of 0.07 and 0.09, respectively to define a "mean JONSWAP" spectrum. The spectral moments of Eq. (30) were calculated by *Ewing* [1976] for the purpose of rewriting the JONSWAP spectrum in terms of H_v, T_v and γ. The effort was further undertaken by *Goda* [1977b], who arrived at the following approximate expression:

$$S(f)=\alpha_* H_{1/3}{}^2 T_p{}^{-4}f^{-5}\exp\left[-\frac{5}{4}(T_pf)^{-4}\right]\gamma^{\exp[-(T_pf-1)^2/2\gamma^2]}, \tag{31}$$

where,

$$\alpha_*=\frac{0.0624}{0.230+0.0336\gamma-0.185(1.9+\gamma)^{-1}}, \tag{32}$$

$$\sigma=\begin{cases}0.07 & \text{for } f\leq f_p\\ 0.09 & \text{for } f>f_p,\end{cases} \tag{33}$$

and the shape parameter γ can be any value between $\gamma=1\sim10$. The parameter α_* in Eq. (32) has been empirically formulated from the numerically obtained values of m_0 while satisfying the theoretical relation of $H_{1/3}=4.004\sqrt{m_0}$; if the empirical relation of $H_{1/3}=3.8\sqrt{m_0}$ is to be employed, the constant of 0.0624 in the dividend should be replaced by 0.0562.

The dependency of the shape parameter on wave generating conditions is being studied by *Houmb* and *Overik* [1976] and others. Waves generated by strong winds over relatively short fetches seem to have spectral peaks much sharper than the Bretschneider-Mitsuyasu or Pierson-Moskowitz type spectrum. The latter spectrum is a special case of Eq. (31) with $\gamma=1$, which becomes as below:

$$S(f)=0.312H_{1/3}{}^2T_p{}^{-4}f^{-5}\exp\left[-\frac{5}{4}(T_pf)^{-4}\right]. \tag{34}$$

The answer to the third question on characteristic wave period depends on the nature of wave data. As discussed in the preceeding chapter, $T_{H_{1/3}}$ appears to be the most reliable parameter with the relation of $T_{H_{1/3}}=(0.9\sim1.0)T_p$. When the information of $T_{H_{1/3}}$ is available through the records of wave data analysis as being done in Japan or through the wave hindcasting studies by the significant wave method, it is better to incorporate $T_{H_{1/3}}$ into the spectral information. The relation of $T_{H_{1/3}}=T_p/1.05$ originally proposed by *Mitsuyasu* [1970] may be employed for this purpose. If $T_{m_{0,1}}$ is known either by spectral analysis of wave data or by spectral wave hindcasting, it is recommended to be employed into spectral formulation because the relation of $T_{m_{0,1}}$ to T_p is quite stable as seen from the survey of **Table 1**. The theoretical relation of $T_p=1.296T_{m_{0,1}}$ may be employed in conjunction with the Pierson-Moskowitz type spectrum. When $T_{m_{0,2}}$ is given through the data of spectral observation, an estimate of $T_p=1.408T_{m_{0,2}}$ may be employed to yield the spectral peak frequency for the Pierson-Moskowitz type spectrum. The use of the mean wave period $\bar{T}_z$ is not recommended because of its variability unless its relationship with T_p is established for the particular set of wave data.

The nominal frequency spectrum of extreme waves can be thus obtained by replacing the peak period T_p in Eq. (31) or (34) with some other characteristic wave period. **Table 4** provides a conversion of constants for the Bretschneider-Mitsuyasu or Pierson-Moskowitz spectrum with the expression of

$$S(f)=Af^{-5}\exp\left[-Bf^{-4}\right], \tag{35}$$

where A and B are assigned the values specified in **Table 4**.

Table 4 Constants for Nominal Wave Spectrum

Characteristic wave period	A	B	Remarks
T_p	$0.312H_{1/3}^2T_p^{-4}$	$1.25T_p^{-4}$	
$T_{H_{1/3}}=T_p/1.05$	$0.257H_{1/3}^2T_{H_{1/3}}^{-4}$	$1.03T_{H_{1/3}}^{-4}$	Proposal by *Mitsuyasu* [1970]
$T_{m_{0,1}}=0.772T_p$	$0.111H_{1/3}^2T_{m_{0,1}}^{-4}$	$0.44T_{m_{0,1}}^{-4}$	ISSC spectrum
$T_{m_{0,2}}=0.710T_p$	$0.079H_{1/3}^2T_{m_{0,2}}^{-4}$	$0.32T_{m_{0,2}}^{-4}$	

A problem remains with regards to the visually observed data of T_v. *Ferdinande* [1976] advocated that T_v is better represented with $T_{m_{-1,0}}$ or rather $T_{m_{-2,0}}$ instead of $T_{m_{0,1}}$ originally proposed by *ISSC* [1970], on the basis of the normalization of his observed spectra. Reanalysis of his data [*Ferdinande et al.* 1975] by the writer has yielded the average relation of $T_v=0.92T_p$, which is close to the relation of $T_{H_{1/3}}$ to T_p in **Table 1**. *Nordenstrøm* [1973] has once formulated the relation of $\bar{T}_z=0.74T_v$, which suggests $T_v=0.96T_p$ if $\bar{T}_z$ is presumed equal to $T_{m_{0,2}}$. An early compilation of ocean wave data by *Hogben* and *Lumb* [1967] also indicates the mean relation of $T_v=0.89T_p$ though the scatter of data was large. Considering these relations and recalling the fact that *Sverdrup* and *Munk* [1947] introduced the concept of significant wave being roughly equivalent of visually observed wave, T_v seems to be better represented with $T_{H_{1/3}}$.

Ferdinande's proposal for a use of $T_{m_{-2,0}}$ may serve as a mean to estimate the wave periods of $T_{H_{\max}}=T_{H_{1/3}}$ from spectral data, because their relationships with T_p are quite close and the estimate of T_p sometimes becomes erroneous owing to the presence of multiple peaks and other reasons. The possibility of estimating $T_{H_{1/3}}$ from $T_{m_{-2,0}}$ should be explored in statistical analysis of wave data in future.

8. Summary

Topics discussed in the present review are recapitulated in the following to highlight the subjects for future discussion:

1. The wave climate statistics should be separated from the long-term wave statistics for a clearer definition of wave statistics.
2. A small but consistent deviation of wave heights from the Rayleigh distribution has been observed.
3. Statistical theories for the joint distribution of wave heights and periods are available, even though their applicability to ocean wave data is of a limited extent.
4. Numerical data on the statistical variability of characteristic wave heights and periods are presented.
5. The log-normal and Weibull distributions are discussed for fitting to the wave climate statistic of H_s, but the theoretical arguments on the superiority of either one are discouraged.
6. A discussion is given to the persistency of sea state.
7. Attention is called for the yearly variation of sea state and the collection of reliable wave data covering a sufficiently long period for preparation of long-term wave statistics.
8. Several distribution functions for extreme value analysis of wave data as well as plotting rules are discussed, but no recommendation is given for their selection because the long-term wave variability and the reliability of data base are more influential for the estimation of extreme wave condition.
9. A survey of wave measurement reports has been made to examine the relationships between various characteristic wave periods. The survey indicates that $T_{H_{1/3}}$ is the most reliable period parameter while $\bar{T}_z$ is the least reliable one being affected by the performance of wave recording system ond others.
10. The spectral form of extreme waves is discussed with the conclusion that it may be presumed as singly peaked and better described with the parameters of $H_{1/3}$, T_p, and the shape parameter. A generalized form of the JONSWAP spectrum is proposed with the above parameters.
11. Suggestions are made for the wave period parameter to be employed in the formulation of nominal wave spectrum. The visually observed wave period is better considered to be equal to $T_{H_{1/3}}$.

Acknowledgements

The present review was initiated by the encouragements and stimulating suggestions of Dr. N. Hogben, the Chairman of the Committee I.1 on Environmental Conditions for the Seventh International Ship Structures Congress. Compilation of **Table 1** was greatly assisted by the generous provisions of unpublished wave data and information by Messrs. J. A. Ewing, R. E. Haring, S. Hirose, N. V. Lawson, H. Mitsuyasu, and M. Yamaguchi. The writer was also aided by the enlightening comments and discussions on the present work from Messrs. J. A. Battjes, J. A. Ewing, N. Hogben, and N. V. Lawson. The writer wishes to express his sincerest thanks to the persons listed in the above for their invaluable guidance and assistance given to him.

References

1) ARHAN, M., CAVANIÉ, A., and EZRATY, R. [1976]: Etude théorique et expérimentale de la relation hauteur-période des vages de tempête, *Internal Rept., A.R.A.E.*, IFP 24 191.

2) BATTJES, J. A. [1970]: Long-term wave height distribution at seven stations around the British Isles, *Internal Rept., N.I.O.*, No. A. 44.

3) BATTJES, J. A. [1972]: Long-term wave height distribution at seven stations around the British Isles, *Deutchen Hydr. Zeit*, Band 25, Heft 4, pp. 179-189.

4) BATTJES, J. A. [1977]: Probabilistic aspects of ocean waves, *Communications on Hydraulics, Dept. Civil Engg, Delft Univ. Tech.*, Rept. No. 77-2.

5) BRETSCHNEIDER, C. L. [1959]: Wave variability and wave spectra for wind-generated gravity waves, *U.S. Army Corps of Engrs., BEB Tech. Memo.*, No. 113.

6) BRETSCHNEIDER, C. L. [1963]: A one-dimensional gravity wave spectra, *Ocean Wave Spectra*, Prentice-Hall, Inc., pp. 41-56.

7) CARTWRIGHT, D. E., and LONGUET-HIGGINS [1956]: The statistical distribution of the maxima of a random function, *Proc. Royal Soc., A.*, Vol. 237, pp. 212-232.

8) CAVANIÉ, A., ARHAN, A., and EZRATY, R. [1976]: A statistical relationship between individual heights and periods of storm waves, *Proc. BOSS '76*, Vol. II, Trondheim, pp. 354-360.

9) CHAKRABARTI, S. K., and COOLEY, R. P. [1977]: Statistical distribution of periods and heights of ocean waves, *J. Geophys. Res.*, Vol. 82, No. 9, March, pp. 1363-1368.

10) COLLINS, J. I. [1967]: Wave statistics from Hurricane Dora, *Proc. ASCE*, Vol. 93, No. WW2, pp. 59-77.

11) COPEIRO, E. [1978]: Extremal prediction of significant wave height, *Proc. 16th Int. Conf. Coastal Engg.*, Hamburg.

12) DRAPER, L. [1966a]: The analysis and presentation of wave data—a plea for uniformity—, *Proc. 10th Int. Conf. Coastal Engg.*, Tokyo, pp. 1-11.

13) DRAPER, L. [1966b]: Waves at Sekondi, Ghana, *Proc. 10th Int. Conf. Coastal Engg.*, Tokyo, pp. 12-17.

14) EARLE, M. D. [1975]: Extreme wave conditions during Hurricane Camille, *J. Geophys. Res.*, Vol. 80, No. 3, Jan., pp. 377-379.

15) EWING, J. A. [1976]: Contribution to the Rept. Committee I.1 on Environmental Conditions, *Proc. 6th ISSC*, Boston, pp. I.1-16 to I.1-25.

16) EWING, J. A. [1978]: Private communication to Y. Goda, Aug. 8.

17) FERDINANDE, V., DE LEMBRE, R., and AERTSSEN, G. [1975]: Spectres de vagues de l'Atlantique Nord, *Association Technique Maritime et Aéronautique.*

18) FERDINANDE, V. [1976]: Discussion to the Rept. Committee I.1 on Environmental Conditions, *Proc. 6th ISSC*, Boston.

19) FORRISTALL, G. Z. [1978]: On the statistical distribution of wave heights in a storm, *J. Geophys. Res.*, Vol. 83, No. C5, May, pp. 2353-2358.

20) GODA, Y. [1967]: Note on the presentation and utilization of wave observation data, *Tech. Note, Port and Harbour Res. Inst.*, No. 39, pp. 237-255 (*in Japanese*).

21) GODA, Y. [1970]: Numerical experiments on wave statistics with spectral simulation, *Rept. Port and Harbour Res. Inst.*, Vol. 9, No. 3, pp. 3-57.

22) GODA, Y. [1974]: Estimation of wave statistics from spectral information, *Proc. Int. Symp. on Ocean Wave Measurement and Analysis*, ASCE, New Orleans, pp. 320-337.

23) GODA, Y. [1976]: On wave groups, *Proc. BOSS '76*, Vol. I, Trondheim, pp. 115-128.

24) GODA, Y. [1977a]: Numerical experiments on statistical variability of ocean waves, *Rept. Port and Harbour Res. Inst.*, Vol. 16, No. 2, pp. 3-26.

25) GODA, Y. [1977b]: *Design of Harbor Structures against Random Seas—Introduction to Ocean Wave Engineering—*, Kajima Pub. Soc., Tokyo, p. 20 (*in Japanese*).

26) GODA, Y. [1978]: The observed joint distribution of the periods and heights of sea waves, *Proc. 16th Int. Conf. Coastal Engg.*, Hamburg.

27) GODA, Y. and NAGAI, K. [1974]: Investigation of the statistical properties of sea waves with field and simulation data, *Rept. Port and Harbour Res. Inst.*, Vol. 13, No. 1, pp. 3-37 (*in Japanese*).

28) GRINGORTEN, I. I. [1963]: A plotting rule for extreme probability paper, *J. Geophys. Res.*, Vol. 68, No. 3, pp. 813-814.

29) HARING, R. E., OSBORNE, A. R., and SPENCER, L. P. [1976]: Extreme wave parameters based on continental shelf storm wave recorders, *Proc. 15th Int. Conf. Coastal Engg.*, Hawaii, pp. 151-170.

30) HARING, R. E. [1978]: Private communication to Y. Goda, June 8.

31) HASSELMANN, K. et al. [1973]: Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP), *Erg. Deuthschen Hydr. Zeit.*, Reihe A(8°), Nr. 12.

32) HOGBEN, N. and LUMB, F. E. [1967]: *Ocean Wave Statistics*, National Physical Laboratory, Her Majesty's Stationary Office.

33) HONDA, T. and MITSUYASU, H. [1977]: On the joint distribution of the heights and periods of wind waves, *Proc. 24th Coastal Engg. Conf. in Japan*, pp. 83-87 (*in Japanese*).

34) HONDA, T. and MITSUYASU, H. [1978]: On the joint distribution of the heights and periods of ocean waves, *Proc. 25th Coastal Engg. Conf. in Japan*, pp. 75-79 (*in Japanese*).

35) HOUMB, O. G. [1971]: On the duration of storms in the North Sea, *Proc. 1st Conf. on Port and Ocean Engg. under Arctic Conditions*, Trondheim, Vol. 1, pp. 423-439.

36) HOUMB, O. G. and OVERIK, T. [1976]: Parameterization of wave spectra and long term joint distribution of wave height and period, *Proc. BOSS '76*, Trondheim, Vol. I., pp. 144-169.

37) HOUMB, O. G. and OVERIK, T. [1977]: On the statistical properties of 115 wave records from the Norwegian continental shlef, *Div. Port and Ocean Engg., Univ. of Trondheim, Norwegian Inst. Tech.*

38) HOUMB, O. G. and VIK, I. [1977]: On the duration of sea state, *Rept. by Ship Res. Inst. Norway and Div. Port and Ocean Engg., Norwegian Inst. Tech.*

39) HOUMB, O. G. and DUE, E. [1978]: On the occurrence of wave spectra with more than one peak, *Div. Port and Ocean Engg., Univ. of Trondheim, Norwegian Inst. Tech.*

40) HOUMB, O. G., MO, L. and OVERIK, T. [1978]: Reliability tests of visual wave data and estimation of extreme sea states, *Div. Port and Ocean Engg., Univ. of Trondheim, Norwegian Inst. Tech.*

41) ISSC [1967]: *Rept. Committee 1 on Environmental Conditions, 3rd International Ship Structures Congress*, Oslo, pp. 42-48.

42) ISSC [1970]: *Rept. Committee 1 on Environmental Conditions, 4th ISSC*, Tokyo, pp. 51-61.

43) ISSC [1976]: *Rept. Committee I.1 on Environmental Conditions, 6th ISSC*, Boston, pp. I.1-53 to I.1-56.

44) JASPER, N. H. [1955]: Distribution patterns on wave heights, ship motions and hull stresses, *Proc. 1st Conf. on Ships and Waves.*

45) JASPER, N. H. [1956]: Statistical distribution patterns of ocean waves and of wave-induced ship stresses and motions, with engineering applications, *Trans. SNAME*, Vol. 64.

46) LAWSON, N. V. [1978]: Private communications to Y. Goda, July 19 and 25.

47) LAWSON, N. V. and ABERNETHY, C. L. [1975]: Long term wave statistics off Botany

Bay, *Proc. 2nd Australian Conf. on Coastal and Ocean Engg.*, pp. 167-176.

48) LAWSON, N. V. and YOULL, P. H. [1977]: Storm duration and return interval for waves off the Central N.S.W. Coast, *Proc. 3rd Australian Conf. on Coastal and Ocean Engg.*, pp. 123-130.

49) LIU, P. C. and ROBBINS, R. J. [1974]: Wave data analyses at GLERL, *Proc. Int. Symp. on Ocean Wave Measurements and Analysis*, ASCE, New Orleans, pp. 64-73.

50) LONGUET-HIGGINS, M. S. [1952]: On the statistical distribution of the heights of sea waves, *J. Marine Res.*, Vol. IX, No. 3, pp. 245-266.

51) LONGUET-HIGGINS, M. S. [1975]: On the joint distribution of the periods and amplitudes of sea waves, *J. Geophys. Res.*, Vol. 80, No. 18, June, pp. 2688-2693.

52) MITSUYASU, H. [1970]: On the growth of spectrum of wind-generated waves (2)—spectral shape of wind waves at finite fetch—, *Proc. 17th Coastal Engg. Conf. in Japan*, pp. 1-7 (*in Japanese*).

53) MITSUYASU, H., NAKAYAMA, R., and KOMORI, T. [1971]: Observations of the wind and waves in Hakata Bay, *Rept. Res. Inst. Applied Mechanics, Kyushu Univ.*, Vol. XIX, No. 62, pp. 37-74.

54) NEUMANN, G. [1953]: On ocean wave spectra and a new method of forecasting wind-generated sea, *U.S. Army Corps of Engrs.*, *Beach Erosion Board*, *Tech. Memo.*, No. 43.

55) NOLTE, K. G. [1973]: Statistical methods for determining extreme sea states, *Proc. 2nd Int. Conf. on Port and Ocean Engg. under Arctic Conditions*, Univ. of Iceland, pp. 705-742.

56) NORDENSTRøM, N. [1969]: Long term distribution of wave heights and periods, *Rept. No. 69-21-S, Det Norske Veritas.*

57) NORDENSTRøM, N. [1973]: A method to predict long-term distributions of waves and wave-induced motions and loads on ships and other floating structures, *Pub. No. 81, Det Norske Veritas.*

58) OCHI, M. K. and HUBBLE, E. N. [1976]: On six-parameter wave spectra, *Proc. 15th Int. Conf. Coastal Engg.*, Hawaii, pp. 301-328.

59) PETRUASKAS, C. and AAGAARD, P. M. [1970]: Extraporation of historical storm data for estimating design wave heights, OTC 1190, *Prepr. 2nd Annual Offshore Tech. Conf.*

60) PHILLIPS, O. M. [1958]: The equilibrium range in the spectrum of wind-generated waves, *J. Fluid Mech.*, Vol. 4, pp. 426-434.

61) PIANC [1973]: Final Report of the International Commission for the Study of Waves, *Bul. PIANC*, No. 15, pp. 52-90.

62) PIERSON, W. J. and MOSKOWITZ, L. [1964]: A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii, *J. Geophys. Res.*, Vol. 69, No. 24, pp. 5181-5190.

63) PITT, E. G., DRIVER, J. S., and EWING, J. A. [1978]: Some intercomparisons between wave records, *Inst. Oceanographic Sci.*, *Rept.* No. 43.

64) RESIO, R. T. [1973]: Extreme wave heights for Cleveland Harbor, *Proc. Civil Engg. in the Oceans*, III, Delaware, ASCE, pp. 62-78.

65) RICE, S. O. [1944]: Mathematical analysis of random noise, reprinted in *Selected Papers on Noise and Stochastic Processes*, Dover Pub., Inc., 1954, pp. 133-294.

66) RYE, H. and SVEE, R. [1976]: Parametric representation of a wind-wave field, *Proc. 15th Int. Coastal Engg. Conf.*, Hawaii, pp. 183-201.

67) SOEJIMA, T., SASAKI, H., and TAKAHASHI, T. [1975]: On the methods of observing and analyzing wave data, *Proc. Annual Symp. of Port and Harbour Res. Inst. for 1975*, pp. 1-52 (*in Japanese*).

68) SVERDRUP, H. U. and MUNK, W. H. [1947]: *Wind Sea and Swell, Theory of Relations for Forecasting*, U.S. Hydrographic Office, Pub. No. 601.

69) TAKAHASHI, T. et al. [1975]: An annual report for wave observations at chosen points (1973), *Tech. Note, Port and Harbour Res. Inst.*, No. 209 (*in Japanese*).

70) TAKAHASHI, T. et al. [1976]: An annual report for wave observations at chosen points (1974), *Tech. Note, Port and Harbour Res. Inst.*, No. 233 (*in Japanese*).

71) TAKAHASHI, T. et al. [1977]: An annual report for wave observations at chosen points (1975), *Tech. Note, Port and Harbour Res. Inst.*, No. 258 (*in Japanese*).

72) TAKAHASHI, T. et al. [1978]: An annual report for wave observations at chosen points (1976), *Tech. Note, Port and Harbour Res. Inst.*, No. 282 (*in Japanese*).

73) TAKAHASHI, T., HIROSE, S., and SASAKI, T. [1979]: Relationship between spectral width parameter and wave statistics, *Tech. Note, Port and Harbour Res. Inst.*, No. 315 (*in Japanese*).

74) THOM, H. C. S. [1973]: Extreme wave height distributions over oceans, *Proc. ASCE*, Vol. 99, No. WW3, pp. 355-374.

75) VIK, I. and HOUMB, O. G. [1976]: Wave statistics at Utsira with special reference to duration and frequency of storm, *Rept. by Ship Res. Inst. Norway and Div. Port and Ocean Engg., Norwegian Inst. Tech.*

76) WARD, E. G., EVANS, D. J., and POMPA, J. A. [1977]: Extreme wave heights along the Atlantic Coast of the United States, OTC 2846, *Prepr. 9th Annual Offshore Tech. Conf.*

77) WILSON, J. R. and BAIRD, W. B. [1972]: A discussion of some measured wave data, *Proc. 13th Int. Coastal Engg. Conf.*, Vancouver, pp. 113-130.

78) YAMAGUCHI, M. and TSUCHIYA, Y. [1979]: Statistical properties of wind waves in limited fetch, *Memoirs of Ehime Univ., Sect. III*, Vol. IX, No. 2 (*in Japanese*).

79) YAMAGUCHI, M., TSUCHIYA, Y., and SHIBANO, T. [1978]: Analysis of extreme wave statistics with hindcasted data for 80 years, *Proc. 25th Coastal Engg. Conf. in Japan*, pp. 70-74 (*in Japanese*).

Appendix: List of Symbols

a : constant defined by Eq. (2)
A : parameter in Eqs. (16) and (17), or constant in Eqs. (24) and (35)
B : parameter in Eqs. (16) and (17), or constant in Eqs. (24) and (35)
f : wave frequency
f_p : wave frequency corresponding to the peak of wave spectrum
g : acceleration of gravity
H : wave height in general
H_c : wave height parameter in Eq. (15)
H_{max} : height of highest zero up-crossing waves in a wave train
H_0 : wave height parameter in Eq. (15)
H_s : significant wave height ($=H_{1/3}$)
H_v : visually observed wave height
H_* : reference wave height
H_{rms} : root-mean-square value of zero up-crossing wave heights
$\bar{H}$: arithmetic mean of zero up-crossing wave heights
$H_{1/3}$: significant wave height or the mean of the heights of highest one-third zero up-crossing waves
k : shape parameter of Weibull distribution of Eqs. (15) and (16)
m : exponent of frequency in wave spectrum of Eq. (24), or order number ($=1, 2, \cdots, N$)
m_n : n-th moment of wave frequency spectrum about the origin
m_0 : zero-th moment of wave frequency spectrum ($=\eta_{rms}^2$)
n : exponent of frequency in wave spectrum of Eq. (24)
N : number of extreme wave data
N_c : number of maxima of surface elevation in a wave record

N_0 : number of zero up-crossings of surface elevation in a wave record
$p(\ )$: probability density of the variable inside the parentheses
$P[\ ]$: cumulative distribution or non-exceedance probability of the variable inside the brackets
$S(f)$: one-sided frequency spectrum of waves
T : wave period in general
$T_{H_{\max}}$: period of highest zero up-crossing wave in a wave train
$T_{H_{1/3}}$: significant wave period or the mean of the periods of highest one-third zero up-crossing waves
$T_{m_{0,1}}$: mean wave period defined from wave spectrum by Eq. (22)
$T_{m_{0,2}}$: mean wave period defined from wave spectrum by Eq. (21)
$T_{m_{-1,0}}$: mean wave period defined from wave spectrum by Eq. (28)
$T_{m_{-2,1}}$: mean wave period defined from wave spectrum by Eq. (28)
T_p : wave period corresponding to spectral peak ($=1/f_p$)
T_v : visually observed wave period
$\bar{T}_z$: arithmetic mean of zero up-crossing wave periods
x : non-dimensional wave height ($=H/H_*$)
X : arbitrary level of wave height parameter
$X_{m,N}$: m-th largest wave height among N extreme wave data
y : logarithm of wave height ($=\log H$)

α : parameter defined by Eq. (9)
α_0 : parameter for JONSWAP spectrum of Eq. (30)
α_* : parameter for normalized JONSWAP spectrum defined by Eq. (31)
α_1 : parameter in Eq. (19)
β : parameter defined by Eq. (10)
β_1 : parameter in Eq. (19)
β_2 : parameter of Frechet distribution of Eq. (18)
γ : shape parameter for JONSWAP spectrum
ε : spectral width parameter, either ε_S or ε_T
ε_S : spectral width parameter defined by Eq. (11)
ε_T : spectral width parameter defined by Eq. (13)
ζ : non-dimensional wave period employed in Eqs. (8) and (12)
η_{rms} : root-mean-square value of surface elevation
κ : shape parameter of Frechet distribution in Eq. (18)
λ : parameter of wave height distribution in Eq. (3)
μ : parameter of wave height distribution in Eq. (3)
μ_y : mean of $y=\log H$
ν : spectral width parameter defined by Eq. (5)
ξ : non-dimensional wave height ($=H/\sqrt{m_0}$)
π : constant, 3.14159$\cdots$
σ : parameter of JONSWAP spectrum of Eqs. (30) and (31)
σ_a, σ_b : parameter of JONSWAP spectrum of Eq. (30)
σ_y : standard deviation of $y=\log H$
τ : non-dimensional wave period ($=T/\bar{T}_z$)