運輸省港湾技術研究所

港湾技術研究所報告

REPORT OF THE PORT AND HARBOUR RESEARCH INSTITUTE

MINISTRY OF TRANSPORT

VOL. 16 NO. 3 SEPT. 1977

NAGASE, YOKOSUKA, JAPAN

港湾技術研究所報告　(REPORT OF R.H.R.I.)

第16巻　第3号 (Vol. 16, No. 3), 1977年9月 (Sept. 1977)

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REPORT OF THE PORT AND
HARBOUR RESEARCH INSTITUTE
Vol. 16 No. 3 (Sept. 1977)

1. Diffraction of sea waves by rigid or cushion type breakwaters

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Synopsis

The computation of wave diffraction is very important to determine the alignment of breakwaters properly. Considering the irregularity of sea waves, Nagai has presented the computation method as for irregular waves with a directional spectrum. His method is a powerful tool in the resolution of practical wave diffraction problems, but it is not applicable to the following cases:

1) for cushion type breakwaters with arbitrary reflection coefficients
2) when the incident waves are reflected by the rear side of a breakwater
3) when the diffracted waves by a breakwater are again diffracted by the other one.

In this paper, new practical computing method is presented, which resolves the remaining problems in Nagai's method. Then, the method is applicable to any type of two breakwaters, either rigid or cushion type. As for regular waves, the method is verified in the comparison with the numerical analysis of wave propagation, which is presented by Ito and Tanimoto. In the results of the comparison, it has become clear that the method is available. Furthermore, the method has been verified by the experiments for the diffraction of the unidirectional irregular waves, and confirmed to be applicable to practical problems within an acceptable accuracy.

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港湾技術研究所報告
第16巻 第3号 (1977.9)

1. 完全反射堤及び消波堤による波の回折計算

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要 旨

波の回折現象は，防波堤の最適な配置計画を行う上で必ず考慮しなければならない重要な要素となっている。模型実験による回折波の算定が最も信頼性があるけれども，海の波のような方向スペクトルを有する波を発生させる造波機がないことや想定されるあらゆるケースについて実験するには費用と時間がかかり過ぎるなどの問題がある。そのため，方向スペクトルを有する波を用いた回折波の数値計算は非常に強力な武器になっている。

このような観点から不規則波の回折計算法が開発されている。しかし，今までの計算法は，次のような場合には計算できないため適用範囲が限られている。

1) 防波堤が消波堤である場合
2) 防波堤による反射波が港内に侵入してくる場合
3) 防波堤によって回折された波が再度もう一方の防波堤で回折される二次回折の場合

本研究では，このような残された問題を解決するために，まず，半無限完全反射堤の理論解の物理的意味を考察したうえで，この理論解を半無限消波堤に適用できるように修正した。この修正の過程では，防波堤による消波のメカニズムは無視し，単に防波堤の反射率だけを導入している。この消波堤に対する解を重ね合わせることによって，開口防波堤に対する回折波の解を求めた。これによって，開口防波堤が消波堤でも，任意配置の防波堤でも計算できることになった。

本計算法の妥当性を検証するために，規則波の回折に関して数値波動解析法と比較した結果，港口部を除いて両計算法はよく一致した値を示している。さらに，不規則波に対する本計算法の妥当性を調べるため，単一方向不規則を用いた回折実験を行った。その結果，本計算法は，実用上十分な精度で実験値と一致し，完全反射堤にも消波堤にも適用できることがわかった。このことは，方向スペクトルを有する波に対する本計算法の適用可能性を示唆するものである。

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Contents

1. Introduction

The water wave diffraction phenomena are of great importance when engineers determine most suitable alignment of breakwaters, which are constructed in order to prevent sea waves from entering a harbour. If the alignment is not determined properly, ships moored inside, apparently in the lee of breakwater, might be damaged by wave action, or encounter a great difficulty during loading or unloading due to great ship motions. The effect of breakwaters must be verified by determing the diffracted waves through model tests or by computing the diffracted waves.

Though the model tests seem to be most reliable, there is the difficulty of producing irregular waves with a directional spectrum such as sea waves even by using an irregular wave generator, and furthemore it is rather time consuming to carry out tests for many different breakwater alignments. Therefore. numerical computation of diffracted waves is a powerful tool compared to model tests.

Penny and Price[1] are the researchers who first derived the solution of water wave diffraction by a rigid semi-infinite breakwater. The solution is the same as that of Sommerfeld, who solved diffraction of light due to a semi-infinite plate. Then, they applied the solution to the analysis of water waves passing through a gap in a breakwater normal to a wave approach direction.

Puttnam and Arthur[2] have checked experimentally the solution for a rigid semi-infinite breakwater in deep water. Experimental wave heights agree approximately with the theory in the sheltered region, but were considerably less than the theoretical values in the unsheltered region.

Blue and Johnson[3] also verified experimentally the solution for gaps, which had widths 1.41 to 4.87 times larger than the wave length. There was a close agreement between the experiments and the theory, that proved the usefulness of the theory for the computation of diffracted waves.

Wiegel[4] presented diffraction diagrams for a semi-infinite breakwater. Morihira and Okuyama[5] also presented diagrams for a semi-infinite breakwater and a gap in a breakwater normal to incident wave direction.

Then, Takai[6] derived the solution of diffracted wave due to a gap for oblique incident waves by introducing the phase difference of incident wave between the tips of the breakwaters. Furthermore, he computed the diffraction due to not-alined breakwaters[7].

The above papers have dealt with regular wave diffraction. However, real sea waves are quite irregular and may be expressed as the superposition of infinite number of component waves which approach along different directions and with different heights and periods. Taking into account the wave irregularity, Mobarek and Wiegel[8] carried out experiments for irregular wave diffraction due to a semi-infinite breakwater in order to determine whether or not the diffraction theory could be applied with sufficient accuracy even to irregular waves with a directional spectrum. The results of the laboratory study showed that a knowledge of the directional spectra can be used together with the diffraction theory to predict the energy spectra of waves in the lee of the breakwater with an accuracy that is probably acceptable for many engineering problems.

Referring to the conclusion of Mobarek and Wiegel, Nagai[9] computed the diffraction of irregular waves with a directional spectrum, and presented useful diffraction diagrams by defining diffraction coefficients of irregular waves as a ratio of significant wave height of the diffracted waves to that of the incident waves. As frequency spectra, he used Pierson-Moskowitz's spectrum or Bretschneider's spectrum modified by Mitsuyasu, and as wave directional distibution function, used SWOP type or $\cos^{2l}\theta$, where θ is an oblique angle to predominant direction of the wave approach and l is a positive integer.

The practical form of directional wave spectrum, however, has not been established yet. Therefore, Goda and Suzuki[10] considered to use the most reliable and practical form among the presented various spectra. They adopted Bretschneider's spectrum as a frequency spectrum, and Mitsuyasu's function as the wave directional distribution. The function has been derived from many fields wave measurements. Goda and Suzuki re-computed wave diffraction by using the adopted spectrum form and presented many practical diffraction diagrams. In their computation, Nagai's program was utilized by transforming only wave spectrum. It, however, can not be applied to the following cases;

1) for cushion type of breakwaters with arbitrary reflection coefficients.
2) when the incident waves are reflected by the rear side of a breakwater.
3) when the diffracted waves by a breakwater are again diffracted by the other one.

In this paper, new practical computing method is presented, which resolves the remaining problems in Nagai's method. Then, the method is applicable to any alignment of two breakwates or any type of breakwaters such as cushion type. It is verified by the experiments and confirmed to be available to a practical use with an acceptable accuracy.

2. Computing manner for irregular waves

2.1. Application to irregular waves

The sea surface deformation $\zeta\ (x_0,\ y_0,\ t)$ at a point of $(x_0,\ y_0)$ outside a harbour may be expressed as the superposition of an infinite number of component waves as follows:

$$\zeta(x_0, y_0, t)=\sum_{n,m}\sqrt{S(f_n,\ \theta_m)\delta f_n\delta\theta_m}\ \exp\{i(k_nx_0\cos\theta_m+k_ny_0\sin\theta_m+2\pi f_nt+\varepsilon_{nm})\} \qquad (1)$$

Where $S\ (f_n,\ \theta_m)$ denotes a directional wave spectrum at a wave frequency f_n and an approaching wave angle θ_m, ε_{nm} is a phase difference of $n\cdot m$-th component waves, and δf_n and $\delta\theta_m$ are small bands of wave frequency and wave angle, respectively. Wave number k_n has the following relation to f_n:

$$4\pi^2 f_n^2=gk_n\tanh k_nh \qquad (2)$$

where g and h are the gravitational acceleration and a water depth. Equation (2) implies that k_n has a single real value for a given value of f_n.

Since the wave diffraction theory is linear, the water surface deformation $\zeta_d\ (x,\ y,\ t)$ at a point of $(x,\ y)$ inside the harbour can be given as the following linear relation to the incident waves:

$$\zeta_d(x, y, t)=\sum_{n,m}\sqrt{S(f_n,\ \theta_m)\delta f_n\delta\theta_m}\ \phi(x, y, f_n,\ \theta_m)\exp\{i(k_nx_0\cos\theta_m+k_ny_0\sin\theta_m+2\pi f_nt+\varepsilon_{nm})\} \qquad (3)$$

where $\phi\ (x,\ y, f_n,\ \theta_m)$ is a transfer function which expresses the diffraction of a component wave with the frequency f_n and the direction θ_m.

The frequency spectrum $S_d(f_n)$ of $\zeta_d(x, y, t)$ is easily derived as follows:

$$S_d(f_n)=\sum_m S(f_n, \theta_m)\psi(x, y, f_n, \theta_m)\psi^*(x, y, f_n, \theta_m)\delta\theta_m \tag{4}$$

where $\psi^*(x, y, f_n, \theta_m)$ is a conjugate of $\psi(x, y, f_n, \theta_m)$.

First, we define the diffraction coefficient for irregular waves as a ratio of significant wave height of the diffracted waves to that of the incident waves. If the wave height probability distribution of irregular wave train follows Rayleigh distribution, the following relation is derived between the significant wave height $H_{1/3}$ and the frequency spectrum $S_\zeta(f)$:

$$H_{1/3}=4.0\sqrt{\int_{-\infty}^{\infty} S_\zeta(f)df} \tag{5}$$

According to wave simulation analysis by Goda[11], the wave height distribution well agrees with the Rayleigh distribution when individual wave height is defined by the zero-up crossing method. The analysises of other reserchers[12] also pertain Goda's conclusion. Furthermore, the statistical analysis of field wave data by Goda[13] concluded that the proportional coefficient in Eq. (5) is about 3.8 in deep sea rather than 4.0. Whether the coefficient is 4.0 or 3.8, the diffraction coefficient K_d is given, according to the previous definition, as the following formula:

$$K_d=\sqrt{\sum_n S_d(f_n)\delta f_n / \sum_{n,m} S(f_n, \theta_m)\delta f_n\delta\theta_m} \tag{6}$$

We define the diffracted wave period coefficient as a ratio of significant wave period of the diffracted waves to that of the incident waves. According to Rice's theory, the mean wave period $\bar{T}$ of a irregular wave train can be calculated from the frequency spectrum as follows:

$$\bar{T}=\sqrt{\int_{-\infty}^{\infty} S_\zeta(f)df / \int_{-\infty}^{\infty} f^2 S_\zeta(f)df} \tag{7}$$

The mean wave period calculated by Eq. (7) seems to have some difference from the real mean period. However, considering that there is no other effective method to estimate the mean wave period, we determined to utilize Eq. (7). Basing on the analysis of field wave data, the following relation is given between the mean wave period and the significant wave periods $T_{1/3}$:

$$\bar{T}=0.9T_{1/3} \tag{8}$$

Then, we can calculate the wave period coefficient K_{dT} by using the relations of Eq. (7) and (8) as follows:

$$K_{dT}=\sqrt{\left\{\frac{\sum_n S_d(f_n)df_n}{\sum_n f_n^2 S_d(f_n)df_n}\right\} / \left\{\frac{\sum_{n,m} S(f_n, \theta_m)\delta f_n\delta\theta_m}{\sum_{n,m} f_n^2 S(f_n, \theta_m)\delta f_n\delta\theta_m}\right\}} \tag{9}$$

Thus, if the forms of $S(f, \theta)$ and $\psi(x, y, f, \theta)$ are given, K_d and K_{dT} can be computed by Eq. (6) and (9).

2.2. Wave spectrum

It is necessary to determine the incident wave spctrum in order to compute the diffraction coefficients and the wave period coefficients. The following form is assumed as a sea wave spectrum:

$$S(f, \theta)=S_\zeta(f)G(f, \theta) \tag{10}$$

where $S(f, \theta)$ is a directional spectrum, $S_\zeta(f)$ is a frequency spectrum and $G(f, \theta)$ is a wave directional function. Here, wave spectra are assumed to be one-sided spectra defined within the frequency region that $0<f<\infty$, while wave spectra in the previous section of **2.1** are two-sided spectra defined in the region that $-\infty<f<\infty$. One-sided spectra show twice larger values than two-sided spectra for $f>0$.

The relation between $S(f, \theta)$ and $S_\zeta(f)$ is given as

$$S_\zeta(f)=\int_{-\pi}^{\pi} S(f,\theta)d\theta \tag{11}$$

Therefore, the wave directional fonction must satisfy the following eqation:

$$\int_{-\pi}^{\pi} G(f,\theta)d\theta=1 \tag{12}$$

Equation (11) means that the observed sea surface deformation appears as a superposition of uni-directional irregular wave train with various approach angles.

Many Oceanographers have investigated the forms of the wave frequency spectrum $S_\zeta(f)$ or the directional function $G(f, \theta)$ through the analysis of field data or theoretical treatments.

(1) Wave frequency spectrum $S_\zeta(f)$

Pireson and Moskowitz[14] presented the following form of wave freqency spectrum through a dimensional analysis:

$$S(f)=0.0081\frac{g^2}{(2\pi)^4}f^{-5}\exp\{-0.74(\omega_0/2\pi f)^4\} \tag{13}$$

where g (m/sec^2) is gravitational acceleration, $\omega_0=g/U_{19.5}$, and $U_{19.5}$ (m/sec) is a wind velocity at 19.5m above sea surface.

Their spectrum was derived as a frequency spectrum in a fully developed sea. Its form, however, is not practical because the generally unknown factor of $U_{19.5}$ is introduced. Therefore, Nagai[9] transformed it to the following practical form by using the relations of Eqs. (5), (7) and (8):

$$S(f)=0.12129\frac{H_{1/3}^2}{T_{1/3}^4}f^{-5}\exp\{-0.48515(T_{1/3}f)^{-4}\} \tag{14}$$

Bretschneider[15] also presented the wave frequency spectrum derived from the analysis of many field wave data. As the spectrum does not satisfy the basic relation of wave spectrum to water surface deformation ζ:

$$\overline{\zeta^2}=\int_0^\infty S_\zeta(f)df \tag{15}$$

Mitsuyasu[16] changed the coefficients in the spectrum to the suitable values which satisfy Eq. (15). The transformed spectrum is given as follows:

$$S_\zeta(f)=0.2572\frac{H_{1/3}^2}{T_{1/3}^4}f^{-5}\exp\{-1.0288(T_{1/3}f)^{-4}\} \tag{16}$$

Though Neumann[17] and Hasselman et. al.[18] presented the forms of wave spectra, the above three spectra are simplest and most practical, and also agree with the spectra of observed sea waves within an acceptable accuracy. Taking into acount these reasons, we determined to choose a spectrum of three spectra.

Pierson and Moskowitz spectrum of Eq. (13) itself is for fully developed sea waves and includes the unknown factor of $U_{19.5}$. Its transformed spectrum of Eq. (14) itself indicates a practical simple form. However, Rice's formula of Eq. (7) used in the process of the transformation is questionable in the application to real sea waves, as Goda[19] descrives that the mean wave periods obtained from wave records are about 20% larger on the avarage than the corresponding periods calculated from wave spectra by using Rice's theory. Considering the above questionable points of Pierson and Moskowitz's spectrum and its transformed spectrum, we adopted Bretschneider's spectrum of Eq. (16) for the computation of the wave diffraction.

(2) Wave directional functon

Cote, et. al[20]. derived the following formula as a wave directional function from stereo-photographs:

$$G(f,\theta)=\begin{cases}\frac{1}{\pi}\left\{1+\left[0.50+0.82\exp\left(-\frac{1}{2}\left\{\frac{2\pi f}{\omega_0}\right\}^4\right)\right]\cos 2\theta \right. \\ \left. +0.32\exp\left(-\frac{1}{2}\left\{\frac{2\pi f}{\omega_0}\right\}^4\right)\cos 4\theta\right\} & \left(|\theta|\leqq\frac{\pi}{2}\right) \\ 0 & \left(|\theta|>\frac{\pi}{2}\right)\end{cases} \tag{17}$$

where $\omega_0=g/U_5$, and U_5 denotes wind speed at about 5m above sea surface.

The following formula is also often used in the estimation or the forcast of wind genarated waves:

$$G(f,\theta)=\begin{cases}\frac{(2l)!!}{(2l-1)!!\theta_{\max}}\cos^{2l}\left(\frac{\pi}{2}\frac{\theta}{\theta_{\max}}\right) & (|\theta|\leqq\theta_{\max}) \\ 0 & (|\theta|>\theta_{\max})\end{cases} \tag{18}$$

where l is an integer determined from the degree of wave directional concentration, and $\theta_{\max}$ is a maximum wave angle measured from the predominant wave direction. Mitsuyasu[21),22] presented the following formula derived from many field wave data observed by a clover type buoy:

$$G(f,\theta)=\frac{1}{\pi}2^{2s-1}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}\cos^{2s}\frac{\theta}{2} \tag{19}$$

where Γ (s) is Ganma function, and s is a parameter of a wave directional concentration, which is determined by the following equation:

$$s=\begin{cases}s_{\max}\left(\frac{f}{f_p}\right)^{-2.5} & \text{for } f>f_p \\ s_{\max}\left(\frac{f}{f_p}\right)^{5} & \text{for } f\leqq f_p\end{cases} \tag{20}$$

where f_p is the wave frequency at a peak of a spectrum and given by

$$f_p=\frac{1}{1.05T_{1/3}} \tag{21}$$

Goda and Suzuki[10] presented a figure for the estimation of the mean values of $s_{\max}$ to wave steepness, and gave, as a standard value of $s_{\max}$, 10 for wind wave, 25 for swells close to wind waves and 75 for swells. In the computation, we determined to choose Mitsuyasu's wave directional function of Eq. (19), because SWOP's function of Eq. (17) includes the unknown values of U_5 and cosine type of Eq. (18) also does the unknown parameter of l.

2.3. Dividing method of frequency and wave angle

As shown in Eq. (1), the incident wave spectrum must be divided into a number of component waves in order to obtain the diffracted wave spectrum. Here, the frequency and the wave direction are assumed to be divided into N and M sections, respectively.

First, we descrive as to the frequency. The frequency is divided as each section has an equal wave energy. Then,

$$S_\zeta(f_n)\delta f_n=\int_0^\infty S_\zeta(f)df/N \tag{22}$$

and the central frequency f_n in the n-th section is given as

$$f_n=\frac{1}{0.9T_{1/3}}\sqrt{2.9124N\left[\mathrm{E_{rf}}\left(\sqrt{2\ln\left(\frac{N}{n-1}\right)}\right)-\mathrm{E_{rf}}\left(\sqrt{2\ln\left(\frac{N}{n}\right)}\right)\right]} \tag{23}$$

where $\mathrm{E_{rf}}$ ( ) is a error function. Eq. (23) is derived by Nagai for Bretschneider frequency spectrum.

Next, the width of directional angle of each section is constant and given as follows:

$$\delta\theta=\frac{\theta_{\max}-\theta_{\min}}{M} \tag{24}$$

where $\theta_{\max}$ and $\theta_{\min}$ are maximum and minimum wave angle to the predominant wave direction. As shown in **Fig. 1**, the width of each section is $\delta\theta$, and the representative angle of each section is a central angle.

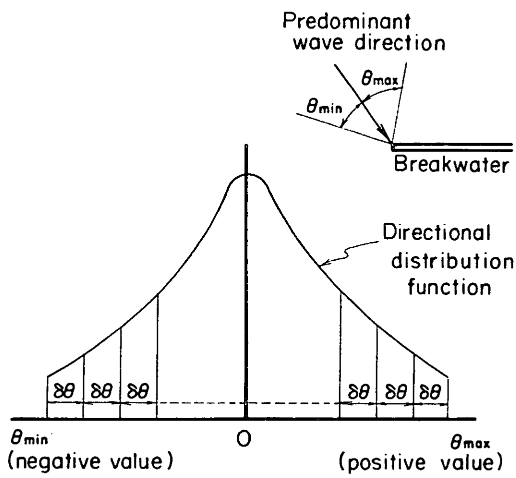

Fig. 1 Dividing of directional distribution function

It is important to take the smallest values of M and N within an acceptable accuracy in order to decrease the computing time. The dividing number N for frequency does not so much affect the values of diffraction coefficients if $N\geq5$. N is enough to be 10 in any case. On the other hand, the dividing number M for wave angle must be increased as the distance r from the opening becomes larger. According to our computation, M is enough to be 20 within $r/L<30$, but M is necessary to be 36 within $30<r/L<80$, where L is wave length of the significant wave. Within $r/L>80$, larger value of M is necessary for the precise estimation.

3. Formulation of wave diffraction and its validity

3.1 Wave diffraction due to cushion type breakwaters

When a regular wave train approaches a rigid semi-infinite breakwater with an angle of θ_m as shown **Fig. 2**, the wave diffraction formula at a point P is given as

$$\begin{aligned}\phi(r,\alpha,f_n,\theta_m)=&\frac{1}{\sqrt{2}}\exp\left[i\left\{k_n r\cos(\alpha-\theta_m)+\frac{\pi}{4}\right\}\right]\\&\times\left[\left\{C(\gamma_1)+\frac{1}{2}\right\}-i\left\{S(\gamma_1)+\frac{1}{2}\right\}\right]\\&+\frac{1}{\sqrt{2}}\exp\left[i\left\{k_n r\cos(\alpha+\theta_m)+\frac{\pi}{4}\right\}\right]\\&\times\left[\left\{C(\gamma_2)+\frac{1}{2}\right\}-i\left\{S(\gamma_2)+\frac{1}{2}\right\}\right]\end{aligned} \tag{25}$$

where

$$\left.\begin{aligned}\gamma_1&=\sqrt{4k_n r/\pi}\cos\frac{\alpha-\theta_m}{2}\\\gamma_2&=\sqrt{4k_n r/\pi}\cos\frac{\alpha+\theta_m}{2}\end{aligned}\right\} \tag{26}$$

and C (γ) and S (γ) denote Fresnel integrals as follows:

$$\left.\begin{aligned}C(\gamma)&=\int_0^\gamma\cos\frac{\pi}{2}\chi^2 d\chi\\S(\gamma)&=\int_0^\gamma\sin\frac{\pi}{2}\chi^2 d\chi\end{aligned}\right\} \tag{27}$$

Eq. (25) has been derived by Penny and Price and the function ϕ $(r, \alpha, f_n, \theta_m)$ corresponds to ϕ (x, y, f_n, θ_m) in Eq. (3). Therefore, the diffraction coefficients for irregular waves can be computed for a rigid semi-infinite breakwater if $\phi(r, \alpha, f_n, \theta_m)$ is used as a transfer function.

Beforehand, we will make clear physical meanings of ϕ $(r, \alpha, f_n, \theta_m)$ in order to modify Eq. (25) for the cushion type breakwater. First, we will consider the first term in the right hand side of Eq. (25). Since $\gamma_1>0$ in the region of $\alpha<\pi+\theta_m$, S $(\gamma_1)>0$ and C $(\gamma_1)>0$. In this case, the meaning becomes clear if the term is reformed as follows:

$$\begin{aligned}\text{The first term of Eq. (25)}=&\exp[i\{k_n r\cos(\alpha-\theta_m)\}]\\&+\frac{1}{\sqrt{2}}\exp\left[i\left\{k_n r\cos(\alpha-\theta_m)+\frac{\pi}{4}\right\}\right]\\&\times\left[\left\{C(\gamma_1)-\frac{1}{2}\right\}-i\left\{S(\gamma_1)-\frac{1}{2}\right\}\right]\end{aligned} \tag{28}$$

As $r\to\infty$, C $(\gamma_1)\to\frac{1}{2}$ and S $(\gamma_1)\to\frac{1}{2}$. Therefore, at the limit, only the first term in the right hand side of Eq. (28) remains and the second term vanishes, that is, the incident waves only exist there. Referring to this, it become clear that in the region of $\alpha<\pi+\theta_m$ the first term of Eq. (25) consists of the incident waves and the scattered waves induced by the incident waves. On the other hand, in the sheltersd region of $\alpha>\pi+\theta_m$, $\gamma_1<0$, and then C $(\gamma_1)<0$ and S $(\gamma_1)<0$. In the region, S $(\gamma_1)\to -\frac{1}{2}$ and C $(\gamma_1)\to -\frac{1}{2}$ as $r\to\infty$. Then, the first term of Eq. (25) vanishes at $r=\infty$. This proves that the first term itself indicates the scattered waves only.

Next, we will discuss the meaning of the second term in the right hand side of Eq. (25). In the region of $\alpha<\pi-\theta_m$, S $(\gamma_1)>0$ and C $(\gamma_1)>0$ since $\gamma_1>0$ by Eq. (26). As the previous treatment of the first term of Eq. (25), it is convenient in understanding to rewrite the second term as follows:

$$\text{The second term of Eq. (25)} = \exp[i\{k_n r\cos(\alpha+\theta_m)\}] + \frac{1}{\sqrt{2}}\exp\left[i\left\{k_n r\cos(\alpha+\theta_m)+\frac{\pi}{4}\right\}\right] \times\left[\left\{C(\gamma_2)-\frac{1}{2}\right\}-i\left\{S(\gamma_2)-\frac{1}{2}\right\}\right] \tag{29}$$

As $r\to\infty$ in the region, S $(\gamma_2)\to\frac{1}{2}$ and C $(\gamma_2)\to\frac{1}{2}$. Then, in Eq. (29) the first term only remains and the second term vanishes at the limit, that is, the reflected waves only exist there. This means that the second term of Eq. (25) consists of the reflected waves and the scattered waves induced by the reflected waves.

In the other region of $2\pi>\alpha>\pi-\theta_m$, S $(\gamma_2)<0$ and C $(\gamma_2)<0$ since $\gamma_2<0$. As $r\to\infty$ in the region, $C(\gamma_2)\to-\frac{1}{2}$ and S $(\gamma_2)\to-\frac{1}{2}$ and then the second term of Eq. (25) vanishes. Therefore, the second term indicates the scattered wave only in the region.

Referring to the above investigation as for Eq. (25), the whole space can be divided into following three regions:

1) The reflected wave region in $\alpha<\pi-\theta_m$, where the reflected waves, the incident waves and the scattered waves coexist.
2) The incident wave region in $\pi-\theta_m<\alpha<\pi+\theta_m$ where the incident waves and the scattered waves coexist.
3) The diffracted wave region of $\alpha>\pi+\theta_m$ where the scattered waves only exist.

Each region is shown by the region I, II, and III, in **Fig. 2**.

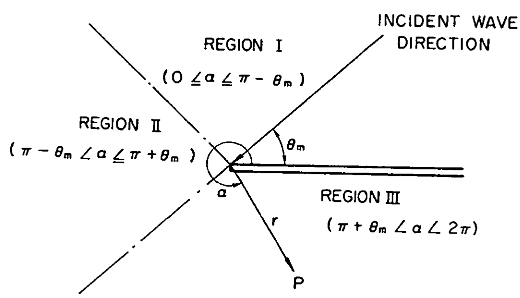

Fig. 2 Wave diffraction by a semi-infinite breakwater

Since the first and the second terms indicate the terms related to the incident waves and the reflected waves respectively, Eq. (25) can be applied to a cushion type breakwater through the following assumptions;

1) The mechanism of wave energy absorption by the cushion type breakwater is not taken into consideration.

2) The reflected wave height only decreases to K_r times of the incident wave height, where K_r represents a reflection coefficient.
3) The phase shift of the reflected wave is always kept constant in any type of breakwater.
4) The secondary waves with short periods do not appear when the wave energy is dissipated in the breakwater.

Under these assumptions, the diffraction function for a cushion type breakwater is given as follows:

$$\phi(r,\alpha,f_n,\theta_m)=\frac{1}{\sqrt{2}}\exp\left[i\left\{k_n r\cos(\alpha-\theta_m)+\frac{\pi}{4}\right\}\right]\times\left[\left\{C(\gamma_1)+\frac{1}{2}\right\}-i\left\{S(\gamma_1)+\frac{1}{2}\right\}\right]+\frac{K_r}{\sqrt{2}}\exp\left[i\left\{k_n r\cos(\alpha+\theta_m)+\frac{\pi}{4}\right\}\right]\times\left[\left\{C(\gamma_2)+\frac{1}{2}\right\}-i\left\{S(\gamma_2)+\frac{1}{2}\right\}\right] \tag{30}$$

In the case of rigid breakwater, Eq. (30) coincide with Eq. (25) because the reflection coefficient K_r is unity. If the wave energy is completely dissipated in the breakwater, that is, $K_r=0$, only both the incident waves and the scattered waves induced by the incident wave exist in the sea space.

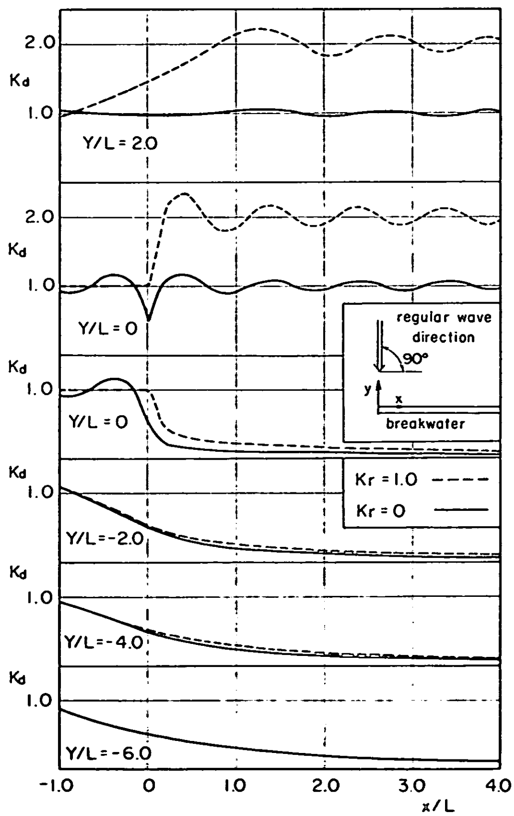

Fig. 3 Diffraction coefficient variation by a semi-infinite breakwater for regular waves

Fig. 3. shows the diffraction coefficient variation for regular waves which

approach in the direction normal to the semi-infinite breakwater. The coefficients are calculated in Eq. (30). In **Fig. 3**, the solid and the dash lines show the diffraction coefficients for a rigid breakwater (K_r=1.0) and a no-reflection breakwater (K_r=0) respectively. In front of the breakwater, the diffraction coefficients vary around K_d= 2.0 in the case of the rigid breakwater, but they do around K_d=1.0 in the case of the no-reflection breakwater. This result seems to satisfy the condition of no-reflection. At the tip of the no-reflection breakwater, the diffraction coefficients always become 0.5 in any incident wave direction. This is due to the introduction of the reflection coefficient without considering the mechanism of the wave energy absorption in the breakwater. In the lee of the breakwater, the diffraction coefficients do not greatly depend on the valve of the reflection coefficient K_r except the neighbor of the breakwater. On the lines of $y/L \leqq -2.0$, the difference between the two lines is not more than 0.05.

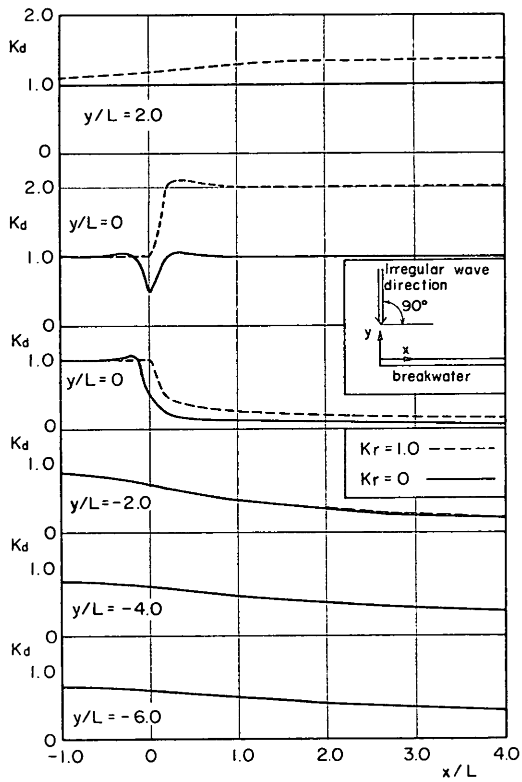

Fig. 4 Diffraction coefficient variation by a semi-infinite breakwater for irregular waves

Fig. 4 also shows the diffraction coefficient variation for irregular waves having the directional spectrum with concentration parameter of s_{max}=10 in the same case as **Fig. 3**. In front of the breakwater, the diffraction coefficients for the no-reflection breakwater are almost 1.0 except the tip of the breakwater. Therefore, the condition of no-reflection seems to be satisfied. As for the rigid breakwater, the coefficient is almost 1.4 on y/L=2.0 and 2.0 on y/L=0 in the reflected wave region. The coefficients show a great difference between two cases in front of breakwater. In the lee of the breakwater, however, they are close each other, except y/L=0. Then, it is concluded that the reflection coefficient greatly affects the value of the diffraction coefficients in front of the breakwater, but does not in the lee. Therefore, even if a breakwater is constructed in any type, the calmness in its lee will be kept almost same degree.

3.2 Wave diffraction due to a gap between breakwaters

In this section, the problems of wave intrusion into a harbour through a gap are treated. In the problems, the wave intrusion is classified into following five types:

1) when reflected waves due to a breakwater do not enter the harbour but are scattered outside (**Fig. 5 (a)**),

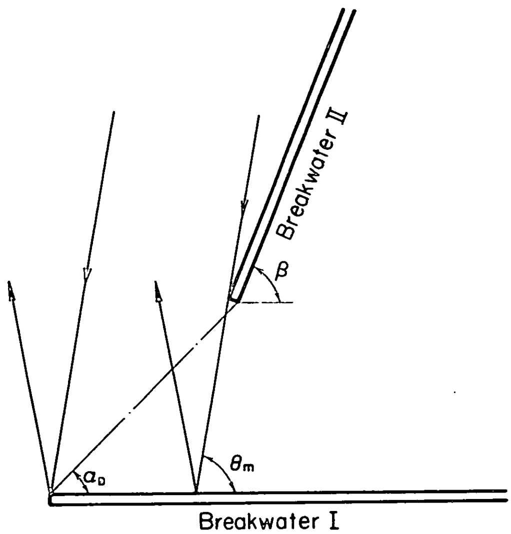

Fig. 5 (a) Case where reflected waves travel outwards

2) when reflected waves due to a breakwater are diffracted by the other one (**Fig. 5 (b)**),

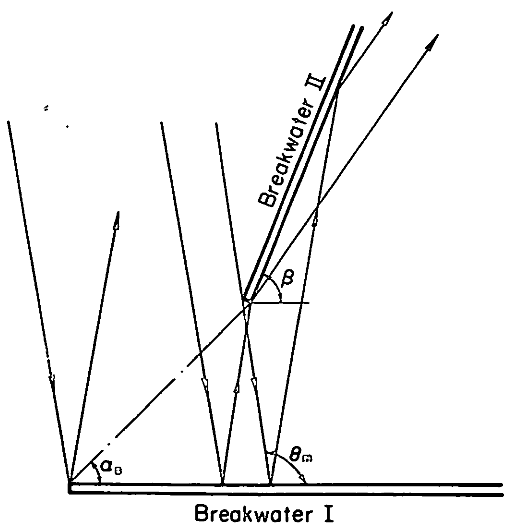

Fig. 5 (b) Case where the reflected waves by breakwater I are once more reflected by the other one

3) when reflected waves due to a breakwater enter a harbour without being diffracted by the other one (**Fig. 5 (c)**),

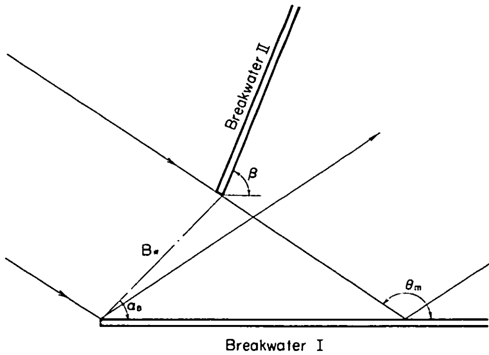

Fig. 5 (c) Case where reflected waves by breakwater I intrude inward without another reflection

4) when incident waves intrude into a harbour (**Fig. 5 (d)**) but may be reflected by a rear side of a breakwater in some breakwater alignment (**Fig. 5 (d)'**),

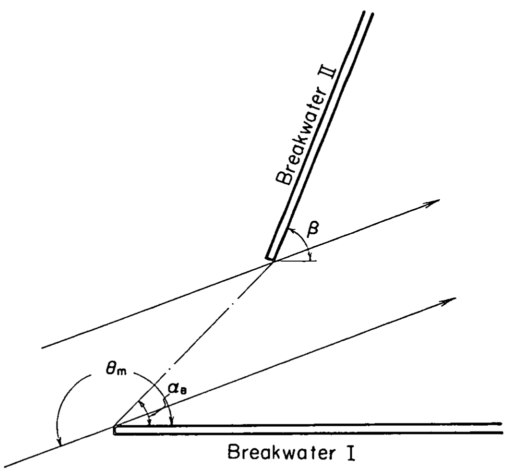

Fig. 5 (d) Case where incident waves intrude without any reflection

5) when diffracted waves due to a breakwater are again diffracted by the other one (**Fig. 5 (e)**),

As for each type of wave diffraction, an approximate formula is derived as a superposition of the solution of the wave diffraction for a semi-infinite breakwater. Here, Eq. (30) was adopted for the formulation. Since the foundamental thought of the formulation is quite same for each type, we explain the thought by making an approximate formula of the wave diffraction for one example. Type 2) is picked up as an example. This type appears only when the incident wave angle θ_m is within $\frac{\pi}{2}<\theta_m<\pi-\alpha_B$. The reflection coefficients of breakwater I and II are assumed to be $K_{r\text{I}}$ and $K_{r\text{II}}$ respectively. The alignment of the breakwaters is shown in **Fig. 5(b)**. If there were not breakwater II, the diffraction formula due to breakwater I at an arbitrary point P inside the harbour would be given as follows:

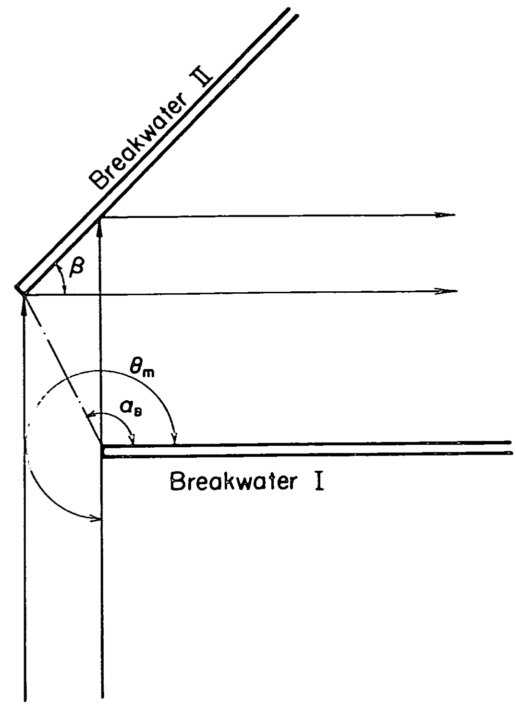

Fig. 5 (d)′ Case where incident waves are reflected by breakwater II

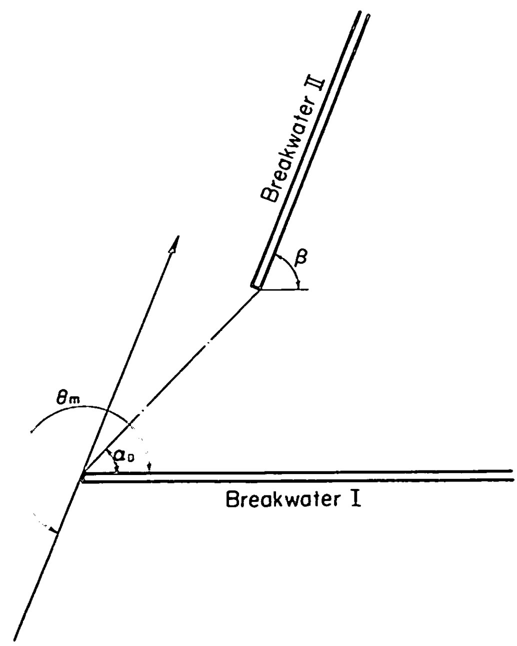

Fig. 5 (e) Case where diffracted waves by breakwater I are diffracted once more by breakwater II

$$\phi_{\mathrm{I}}(r_{\mathrm{I}}, \alpha_{\mathrm{I}}, f_n, \theta_m)=\frac{1}{\sqrt{2}}\exp\left[i\left\{k_n r_{\mathrm{I}}\cos(\alpha_{\mathrm{I}}-\theta_m)+\frac{\pi}{4}\right\}\right]\times\left[\left\{C(\gamma_{\mathrm{I}1})+\frac{1}{2}\right\}-i\left\{S(\gamma_{\mathrm{I}1})+\frac{1}{2}\right\}\right]+\frac{K_{r\mathrm{I}}}{\sqrt{2}}\exp\left[i\left\{k_n r_{\mathrm{I}}\cos(\alpha_{\mathrm{I}}+\theta_m)+\frac{\pi}{4}\right\}\right]\times\left[\left\{C(\gamma_{\mathrm{I}2})+\frac{1}{2}\right\}-i\left\{S(\gamma_{\mathrm{I}2})+\frac{1}{2}\right\}\right] \tag{31}$$

where r_{I} and α_{I} are the distance from the tip of breakwater I to P and the angle of P to breakwater I, and $\gamma_{\mathrm{I}1}$ and $\gamma_{\mathrm{I}2}$ are given as

$$\left.\begin{aligned}\gamma_{\mathrm{I}1}&=\sqrt{4k_n r_{\mathrm{I}}/\pi}\cos\frac{\alpha_{\mathrm{I}}-\theta_m}{2}\\ \gamma_{\mathrm{I}2}&=\sqrt{4k_n r_{\mathrm{I}}/\pi}\cos\frac{\alpha_{\mathrm{I}}+\theta_m}{2}\end{aligned}\right\} \tag{32}$$

The incident waves to breakwater II have a phase lag of $k_n B_w \cos(\alpha_B-\theta_m)$ to the incident wave to breakwater I, and approach with an angle of $\theta_m-\beta$ to breakwater II. Then, if there were not breakwater I, the diffraction formula due to breakwater II at the point P would be derived as

$$\phi_{\mathrm{II}}(r_{\mathrm{II}}, \alpha_{\mathrm{II}}, f_n, \theta_m)=\frac{1}{\sqrt{2}}\exp\left[i\left\{k_n r_{\mathrm{II}}\cos(\alpha_{\mathrm{II}}-\theta_m+\beta)+\frac{\pi}{4}+k_n B_w\cos(\alpha_B-\theta_m)\right\}\right]\times\left[\left\{C(\gamma_{\mathrm{II}1})+\frac{1}{2}\right\}-i\left\{S(\gamma_{\mathrm{II}1})+\frac{1}{2}\right\}\right]+\frac{K_{r\mathrm{II}}}{\sqrt{2}}\exp\left[i\left\{k_n r_{\mathrm{II}}\cos(\alpha_{\mathrm{II}}+\theta_m-\beta)+\frac{\pi}{4}+k_n B_w\cos(\alpha_B-\theta_m)\right\}\right]\times\left[\left\{C(\gamma_{\mathrm{II}2})+\frac{1}{2}\right\}-i\left\{S(\gamma_{\mathrm{II}2})+\frac{1}{2}\right\}\right] \tag{33}$$

where r_{II} and α_{II} are the distance from the tip of breakwater II to P and the angle of P to breakwater II, and $\gamma_{\mathrm{II}1}$ and $\gamma_{\mathrm{II}2}$ are given as

$$\left.\begin{aligned}\gamma_{\mathrm{II}1}&=\sqrt{4k_n r_{\mathrm{II}}/\pi}\cos\frac{\alpha_{\mathrm{II}}-\theta_m+\beta}{2}\\ \gamma_{\mathrm{II}2}&=\sqrt{4k_n r_{\mathrm{II}}/\pi}\cos\frac{\alpha_{\mathrm{II}}+\theta_m-\beta}{2}\end{aligned}\right\} \tag{34}$$

Since the reflected waves due to breakwater I become incident waves to breakwater II, their phase lag is given as $k_n B_w \cos(\alpha_B+\theta_m)$ and their approach angle to breakwater II is $(2\pi-\theta_m-\beta)$ as shown in **Fig. 5(b)**. Therefore, the diffraction formula for the reflected waves would be given as

$$\phi_{\mathrm{I,II}}(r_{\mathrm{II}}, \alpha_{\mathrm{II}}, f_n, \theta_m)=\frac{K_{r\mathrm{I}}}{\sqrt{2}}\exp\left[i\left\{k_n r_{\mathrm{II}}\cos(\alpha_{\mathrm{II}}+\theta_m+\beta)+\frac{\pi}{4}+k_n B_w\cos(\alpha_B+\theta_m)\right\}\right]\times\left[\left\{C(\gamma_{\mathrm{I\,II}1})+\frac{1}{2}\right\}-i\left\{S(\gamma_{\mathrm{I\,II}1})+\frac{1}{2}\right\}\right]+\frac{K_{r\mathrm{I}}K_{r\mathrm{II}}}{\sqrt{2}}\exp\left[i\left\{k_n r_{\mathrm{II}}\cos(\alpha_{\mathrm{II}}-\theta_m-\beta)+\frac{\pi}{4}+k_n B_w\cos(\alpha_B+\theta_m)\right\}\right]\times\left[\left\{C(\gamma_{\mathrm{I\,II}2})+\frac{1}{2}\right\}-i\left\{S(\gamma_{\mathrm{I\,II}2})+\frac{1}{2}\right\}\right] \tag{35}$$

where

$$\left.\begin{aligned}\gamma_{\mathrm{I\,II}_1}&=\sqrt{4k_n r_{\mathrm{II}}/\pi}\cos\frac{\alpha_{\mathrm{II}}+\theta_m+\beta-2\pi}{2}\\ \gamma_{\mathrm{I\,II}_2}&=\sqrt{4k_n r_{\mathrm{II}}/\pi}\cos\frac{\alpha_{\mathrm{II}}-\theta_m-\beta+2\pi}{2}\,.\end{aligned}\right\}\tag{36}$$

It is convenient to divide the area inside the harbour into 5 regions of A, B, C, D, and E as shown in **Fig. 6**. In each region, the following formula of wave diffraction can be given approximately:

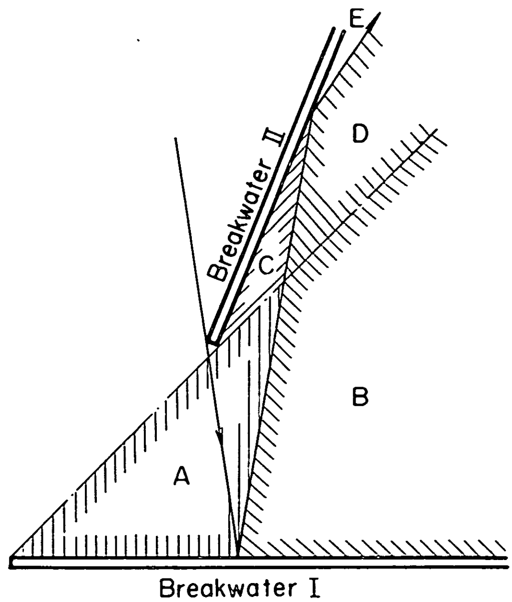

Fig. 6 Dividing inside a harbor

1) In region A

$$\begin{aligned}\phi_A=&\phi_{\mathrm{I}}(r_{\mathrm{I}},\alpha_{\mathrm{I}},f_n,\theta_m)+\phi_{\mathrm{II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)+\phi_{\mathrm{I\,II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)\\ &-\exp[ik_n r_{\mathrm{I}}\cos(\alpha_{\mathrm{I}}-\theta_m)]-K_{r\mathrm{I}}\exp[i\{k_n r_{\mathrm{II}}\cos(\alpha_{\mathrm{II}}+\theta_m+\beta)\\ &+k_n B_w\cos(\alpha_B+\theta_m)\}]\end{aligned}\tag{37}$$

2) In region B

$$\begin{aligned}\phi_B=&\phi_{\mathrm{I}}(r_{\mathrm{I}},\alpha_{\mathrm{I}},f_n,\theta_m)+\phi_{\mathrm{II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)+\phi_{\mathrm{I\,II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)\\ &-\exp[ik_n r_{\mathrm{I}}\cos(\alpha_{\mathrm{I}}-\theta_m)]-K_{r\mathrm{I}}\exp[ik_n r_{\mathrm{I}}\cos(\alpha_{\mathrm{I}}+\theta_m)]\\ &-K_{r\mathrm{I}}\exp[i\{k_n r_{\mathrm{II}}\cos(\alpha_{\mathrm{II}}+\theta_m+\beta)+k_n B_w\cos(\alpha_B+\theta_m)\}]\end{aligned}\tag{38}$$

3) In region C

$$\phi_C=\phi_{\mathrm{II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)+\phi_{\mathrm{I\,II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)\tag{39}$$

4) In region D

$$\begin{aligned}\phi_D=&\phi_{\mathrm{II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)+\phi_{\mathrm{I\,II}}(r_{\mathrm{II}},\alpha_{\mathrm{II}},f_n,\theta_m)\\ &-K_{r\mathrm{I}}\exp[i\{k_n r_{\mathrm{II}}\cos(\alpha_{\mathrm{II}}+\theta_m+\beta)+k_n B_w\cos(\alpha_B+\theta_m)\}]\end{aligned}\tag{40}$$

5) In region E

$$\phi_E=\phi_{\mathrm{II}}(r_{\mathrm{II}}, \alpha_{\mathrm{II}}, f_n, \theta_m)+\phi_{\mathrm{I\,II}}(r_{\mathrm{II}}, \alpha_{\mathrm{II}}, f_n, \theta_m)-K_{r\mathrm{I}} \exp[i\{k_n r_{\mathrm{II}} \cos(\alpha_{\mathrm{II}}+\theta_m+\beta) +k_n B_w \cos(\alpha_B+\theta_m)\}]-K_{r\mathrm{I}} K_{r\mathrm{II}} \exp[i\{k_n r_{\mathrm{II}} \cos(\alpha_{\mathrm{II}}-\theta_m-\beta) +k_n B_w \cos(\alpha_B+\theta_m)\}] \quad (41)$$

In the derivation, the influence of the diffracted waves due to breakwater I is ignored in the region of $\alpha_{\mathrm{I}}>\alpha_B$, because the waves are supposed to travel in the radial direction from the tip of breakwater I and their diffracted waves due to breakwater II are assumed to be very small in the comparison with other waves. Therefore, in the regions C,D, and E, the formulas do not include the term of ϕ_{I} $(r_{\mathrm{I}}, \alpha_{\mathrm{I}}, f, \theta_m)$. Since each region is divided in the condition of the geometrical optics, it is natural that the obtained formulas should not be continuous on the boundary lines. The difference, however, is supposed to be as small as negligible, if B_w is wide enough to satisfy the superposition condition.

The formulation in the other types are made in the same manner as in type 2) though the obtained formulas are not presented here in order to eliminate a great complicacy.

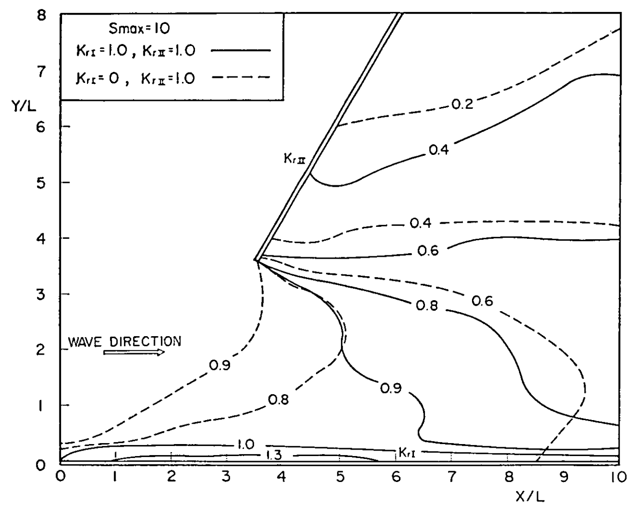

Fig. 7 Wave diffraction diagram

Fig. 7 shows the diffraction diagram of the irregular waves with $s_{\max}=10$, which is calculated by the diffraction formulas such as Eq. (37) to (41). In the figure, each solid line indicates equal diffraction coefficient in the case of the rigid breakwater ($K_{r\mathrm{I}}=1.0$, $K_{r\mathrm{II}}=1.0$) and each dash lines also does in the case where one is a cushion type breakwater ($K_{r\mathrm{I}}=0$) and the other is a rigid breakwater ($K_{r\mathrm{II}}=1.0$). Judging from the figure, the calmness inside the harbour is greatly promoted in its degree by only changing the value of the reflection coefficient of breakwater I to zero. Since the case such as **Fig. 7** is under the condition that the reflected waves intrude the harbour, the calmness can be promoted by decreasing the value of the reflection coefficients, but in other case without the reflected waves we can not expect such promotion even if the reflection coefficients of breakwaters are small. This is clear as we previously described in **3.1** that the value of the reflection coefficient does not affect the diffraction coefficient in the lee of a breakwater.

3.3 Verification of the validity of the formulation

Since the formulas, as obtained in **3.2**, are approximate, they do not satisfy the boundary conditions on the breakwater or the continuity condition on the boundary lines. In order to verify the validity of the formulation, we make comparison with the numerical analysis method of wave propagation, which has been developed by Ito and Tanimoto[23], and whose validity has already been confirmed by experiments. In their method, the adoption of irregular waves needs a great deal of computing time. Regular waves, therefore, are used for comparison. The comparisons are carried out as for three different breakwater alignments, as shown in **Fig.** 8 to **10**, where solid lines represent the diffraction coefficient variation calculated by the numerical method and small circles do the diffraction coefficients calculated by the approximate formula.

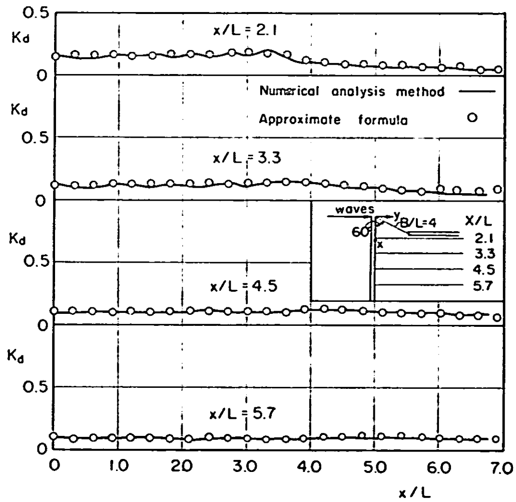

Fig. 8 Diffraction of the diffracted waves

Fig. 8 shows very close agreement between both calculated values. In this breakwater alignment, the diffracted waves due to a breakwater are once more diffracted by the other one. The diffraction coefficients are about 0.1 to 0.2, and do not show a great variation.

Fig. 9 also shows the comparison of two methods. In this case, both breakwaters are parallel to each other and the tip of a breakwater is in the lee of the other one. Since the incident waves are reflected by the rear breakwater, the diffraction coefficients on $x/L=1.6$ or 2.4 show a great variation near the gap because of the reflection. The difference between both calculated values is as great as 0.2 in maximum, but referring to the figure, it is hard to say they show bad agreement in the area with a great influence of the wave reflection, because they are close at some points. In the inner part, or in the lee of the breakwater, both values quite well agree each other.

Fig. 10 also shows the comparison in the case that the incident waves directly intrude the harbour. Both breakwaters are also parallel to each other. On the lines of $x/L=-2.0$, -1.6 and 0.8 in the unshetered region the dffraction coefficients by the approximate formula show a great discrepancy to those by the numerical analysis method. Especially, on the line of $x/L=-2.0$, the former values become gradually larger from a point of about $y/L=2.5$, while the latter values decrease from the point. Thus, on the line, the inclination itself of two diffraction coefficients is quite different each other. In the area, the diffraction coefficients by the approximate formula are different by about 20% of their values from those by the numerical analysis.

Referring to the above comparisons, the following conclusion is drawn: The approximate formula shows the estimation of the diffraction coefficients within about

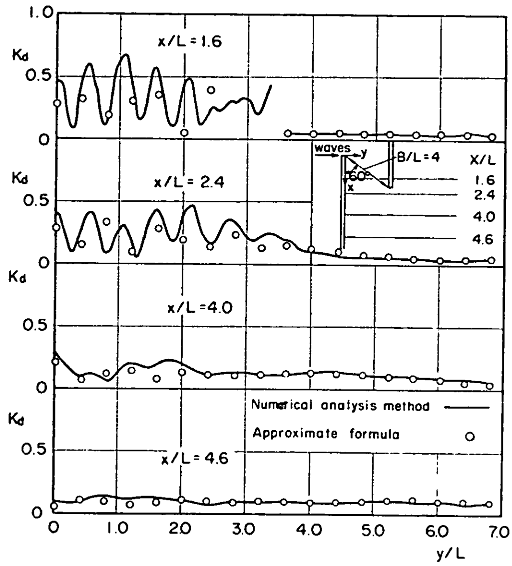

Fig. 9 Diffraction of the waves with reflected waves

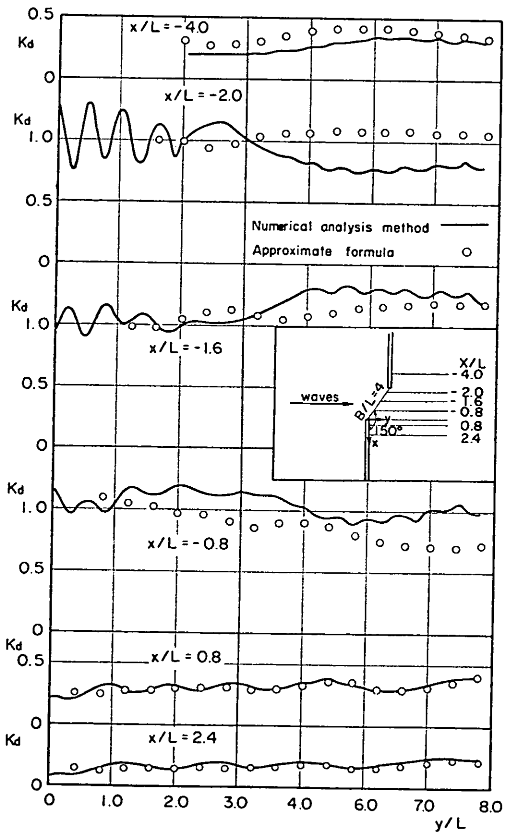

Fig. 10 Diffraction of directly intruding wave

20% error near the gap, and predicts well diffraction coefficients in the lee of breakwaters.

4. Verification of the approximate formula by the experiments

The validity of the approximate formula for the wave diffraction is confirmed for regular waves in the comparison with the numerical analysis method, but it is necessary to comfirm its validity for irregular waves. In this chapter, we presents the results of model experiments which were carried out for uni-directional irregular waves in order to verify the validity.

4.1. Measuring and recording equipments, wave tank and irregular wave generators

Capacitance type wave meters are used for the measurment of the waves. The measured analog data of the waves are stored on the magnetic tapes of a data recorder through the amplifier of the wave meter. The waves are measured simultaneously at three points for about 10 minutes. After measuring, the wave meter are moved to another three points, and the shifts of zero balance induced by the movement are adjusted by the dial on the wave meter amplifier. In the same process the experiments are continued.

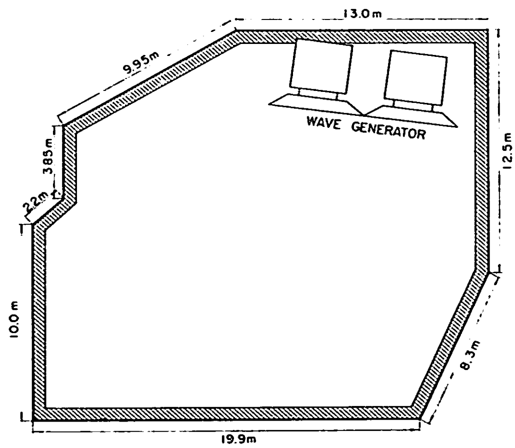

Fig. 11 Wave tank and position of wave generators

The wave tank and the position of wave generators are shown in **Fig. 11**. The gravel mounds are constructed in front of the walls of the tank in order to prevent waves from being reflected. The generators can generate uni-directional irregular waves with an arbitrary frequency spectrum. The process of the irregular wave generation is as follows:

1) The white noise signals are produced by two white noise genarators.
2) The signals are passed through ten band-pass-filters and changed to the irregular wave signals with Bretschneider's spectrum, where the dials of the filters are adjusted to the value calculated by the dial setting program.
3) The transformed signals are stored in a data recorder.
4) The signals themselves have too high frequencies to put in the wave generator. Therefore, the out-put tape speed is cut down to 1/10 or 1/20 times of the recording speed.
5) The signals at the cut-down speed are put in the wave generator control board.
6) The wave paddle moves with the signals, and the irregular waves with the expected spectrum are generated.

4.2. Experimental condition and incident waves

(1) Experimental condition

We imagined that the incident waves with the significant wave height of 2.0m and the wave period of 10 sec approached to a port with the opening length of 300m in a sea of 15m deep. We determined the port model scale to be 1/100. Then, in the model the incident significant wave height and period become 2.0cm and 1.0 sec respectively, and the depth and the opening length do 15cm and 300cm respectively.

The three kinds of breakwater alignments are adopted as shown in **Fig. 12** to **14**:

1) Case 1 that the incident waves may be reflected by the rear face of a breakwater (**Fig. 12**).

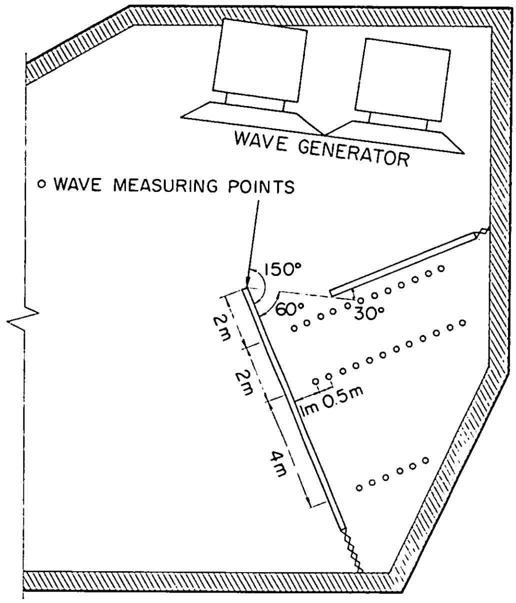

Fig. 12 Alignment of breakwaters and measuring points for case 1

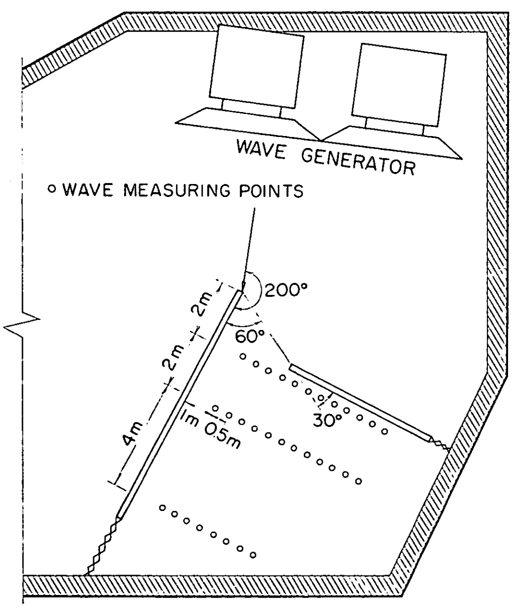

Fig. 13 Alignment of breakwaters and measuring points for case 2

2) Case 2 that the incident waves intrude the port directly (**Fig. 13**).
3) Case 3 that the diffracted waves due to a breakwater are once more diffracted by the other one (**Fig. 14**).

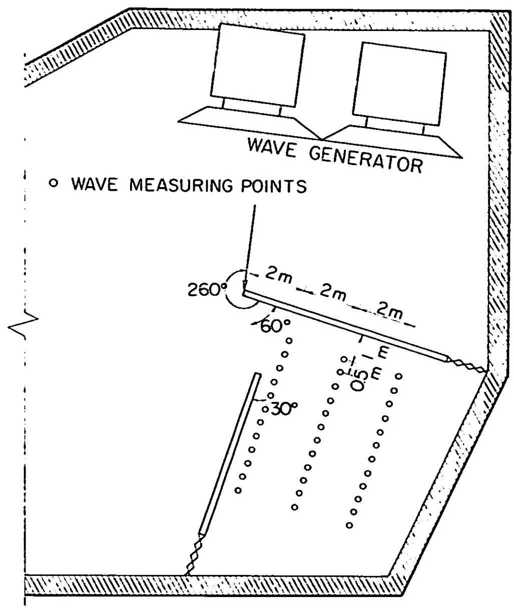

Fig. 14 Alignment of breakwaters and measuring points for case 3

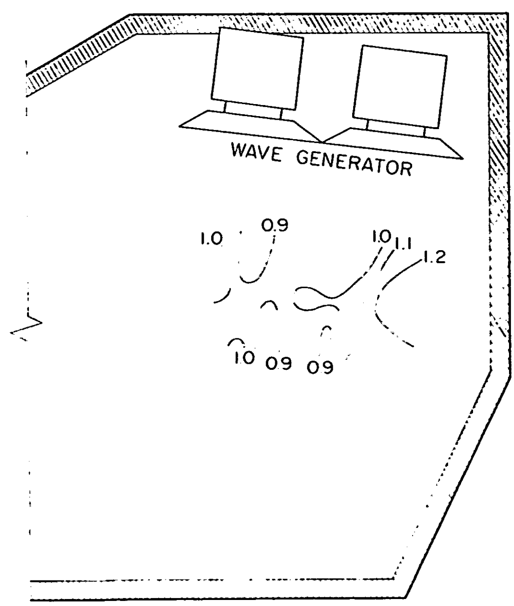

Fig. 15 Distribution of the significant wave height ratios

In these figures, small circles indicate the measuring points, whose number are about 40 and which are on three lines at 2.0m, 4.0m and 8.0m from the tip of a breakwater.

The experiments were also carried out for cushion type breakwaters in the same breakwater alignments as case 1 and 2. The breakwaters were made by gravelling around them with the slope of 1/1.5.

(2) Spectrum and statistic quantities of incident irregular waves

The incident irregular waves are measured at 18 points without breakwater models. The points are chosen around opening site of model breakwaters. **Fig. 15** shows the distribution of significant wave height ratios to the average height around the site. The ratio varies from 0.90 to 1.10 at a place to place, as shown in **Fig. 15**. Referring to **Fig. 15**, the place of the breakwater settlement was chosen. Average statistic quantities of the incident waves are shown on **Table 1**.

Table 1 Statistical quantities of the incident waves

$H_{1/10}$(cm)	$H_{1/3}$(cm)	$\overline{H}$(cm)	$\overline{T}_{1/3}$(sec)	T(sec)
2.27	1.82	1.11	1.08	0.96

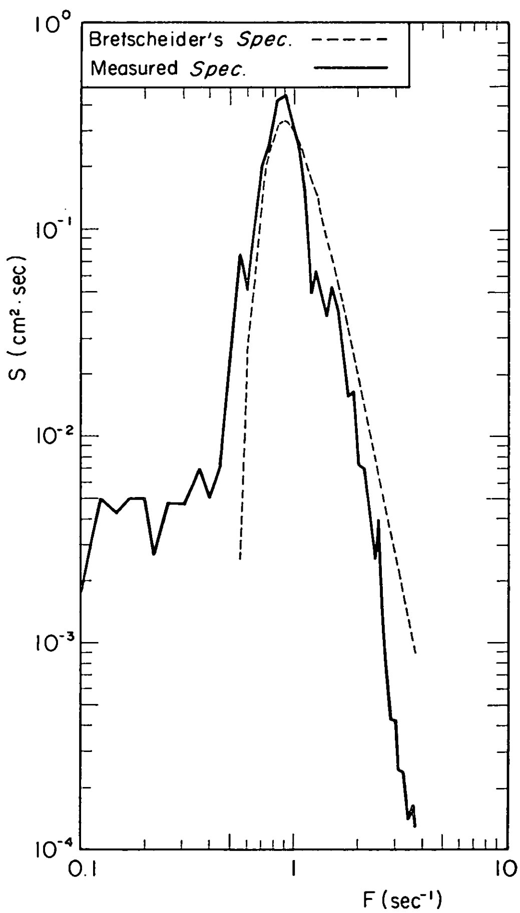

Fig. 16 Measured spectrum and Bretschneider's spectrum of incident waves

Fig. 16 shows a measured spectrum at a point and Bretschneider's spectrum calculated with the mean significant wave height and period. In **Fig. 16**, the wave energy of the former spectrum is larger than that of the latter in the part of lower frequencies than the peak one of 1.0, and less than the latter in the part of higher frequencies. The damping of the measured spectrum in the higher frequencies seems to be caused due to the dirt on water surface or viscous energy dissipation. The reflection coefficients of the cushion type breakwaters were estimated for three irregular wave trains with different significant wave heights and same wave periods. The waves were measured at two points with the distance of 20cm each other in front of the breakwater, which was set normal to the incident wave direction. The incident and reflected wave energy were computed from the measured data by the technique presented by Goda and et. al[24]. Then reflection coefficients for irregular waves were calculated as a root of the ratio of the reflected wave whole energy to the incident one. They are shown in **Table 2**. Referring to **Table 2**, we adopted 0.5 in the computation of wave diffraction as the reflection coefficient of the breakwaters.

Table 2 Reflection coefficient of the cushion type breakwater

$H_{1/3}$(cm)	$T_{1/3}$(sec)	$H_{1/3}/L$	K_r
1.81	1.12	0.0145	0.48
1.06	1.12	0.0085	0.54
0.29	1.09	0.0024	0.63

4.3. Experimental results and discussions

(1) Case 1

In this case, the incident waves reflected by a breakwater enter a harbor through an opening. The alignment of the breakwaters and measuring points for the case are shown in **Fig. 12**.

The comparisons between the experimental results and the calculated values by the approximate formula are shown in **Fig. 17** to **18**. In these figures, the solid lines, the broken lines and the small circles indicate the calculated values by Bretschneider's and the measured spectra in **Fig. 16**, and the experimental values, respectively.

Fig. 17 (a) and **(b)** show the comparisons for the diffraction coefficients and the diffracted wave period coefficients in rigid breakwaters, respectively. In **Fig. 17 (a)**, the

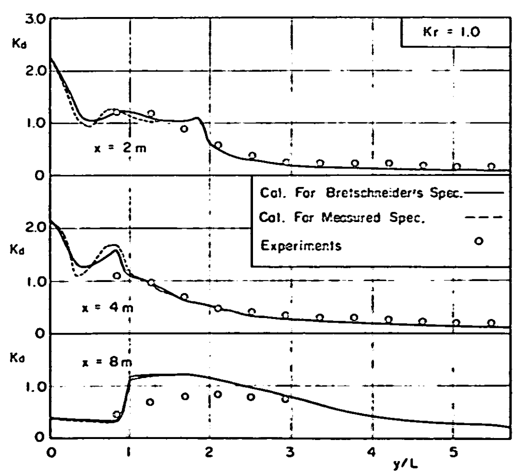

Fig. 17 (a) Diffraction coefficients for case 1 (K_r=1.0)

calculated coefficients show small differences between Bretschneider's and the measured spectra in spite of the large difference between the two spectra as shown in **Fig. 16**. This implies that diffraction coefficients are not so much affected by the difference of frequency spectral forms, if the spectra show almost same values each other around their peak frequencies. On the line of x=8.0m, the calculated values are twice or 1.5 times larger than the experimental ones in the portion of $y/L=1$ to 2. The causes of the difference are referred to the computing assumptions that the wave reflection area is determined by optical wave reflection and that wave energy is not transferred into or out of another area through the boundaries. On the other lines, the calculation shows close agreement with the experiments. Therefore, the approximate formula is applicable to practical problems with a good accuracy except some portion.

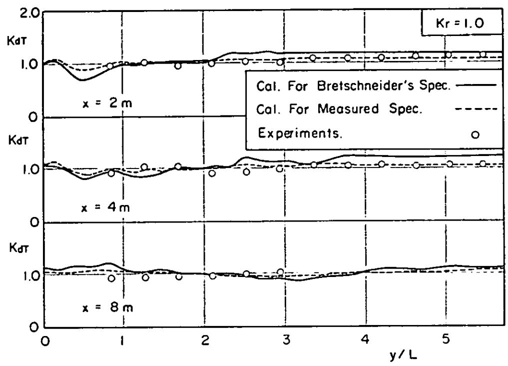

Fig. 17 (b) Periodical ratios of the diffracted waves for case 1 (K_r=1.0)

In **Fig. 17 (b)**, the wave period coefficients show some discrepancy between the calculated values by Bretschneider's and the measured spectra. The discrepancy depends upon the difference between the two spectra in high frequencies, because the frequencies influence the wave period coefficients by the square of their values, as shown in Eq. (7). The experimental values show closer agreement with the dash lines for the measured spectrum than with the solid lines for Bretschneider's one. This means that Rice's formula can be applied to the estimation of the wave period coefficients only, though it is uncertain whether the formula can precisely predict the mean wave periods.

Fig. 18 (a) and **(b)** show the distribution of the diffraction coefficients and the

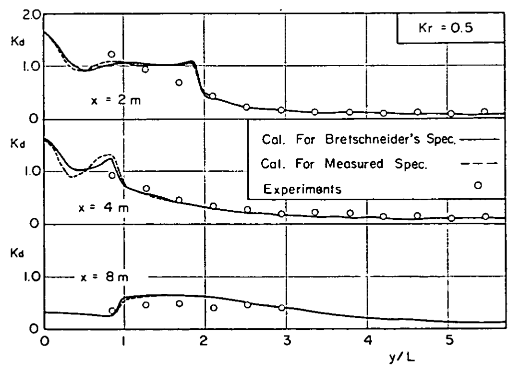

Fig. 18 (a) Diffraction coefficients for case 1 (K_r=0.5)

wave period coefficients for the cushion type breakwaters with the reflection coefficients of 0.5, respectively. In **Fig. 18 (a)**, the distribution of the diffraction coefficients shows almost the same variation as in **Fig. 17 (a)**, though their values are smaller than those in **Fig. 17 (a)**. Both the calculations show very well agreement with the experiments. Especially, in the inner habor the agreement becomes much better. Referring to **Fig. 18 (a)**, the approximate formula is also applicable to the cushion type breakwaters.

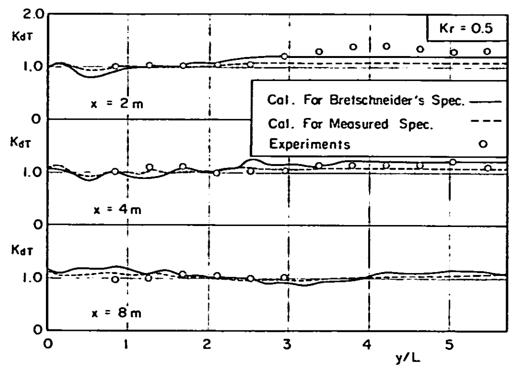

Fig. 18 (b) Periodical ratios of the diffracted waves for case 1 (K_r=0.5)

Fig. 18 (b) for the wave period coefficients shows that the experimental values are close to the calculated ones by the measured spectrum except the portion of $y/L \approx 3.5$ on x=2m. In the portion, the experimental values are larger by about 0.3 than the calculated ones. The approximate formula can estimate the wave period coefficients even for the cushion type breakwaters.

(2) Case 2

In this case, the incident waves enter the harbor through a opening without the direct reflection due to a breakwater, as shown in **Fig. 13**. **Figs. 19** and **20** show the comparisons between the calculations and the experiments for rigid and cushion type breakwaters, respectively. In these figures, each line and the small circles indicate the same as in **Fig. 17** or **18**.

In **Fig. 19 (a)**, both calculations by Bretschneider's and the measured spectra show quite close values to each other. Though the experimental values are smaller

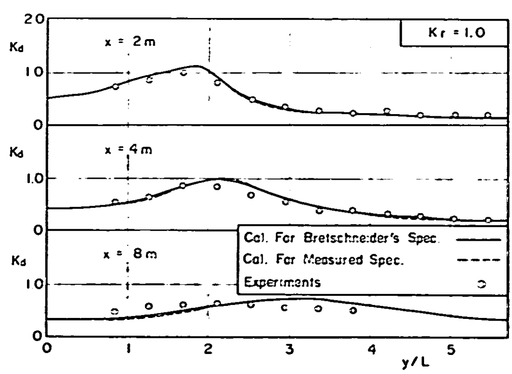

Fig. 19 (a) Diffraction coefficients for case 2 (K_r=1.0)

than the calculated ones by about 0.1 near the peak value of the wave diffraction coefficients, the discrepancy is very small. Therefore, the calculation can predict the diffracted waves very well.

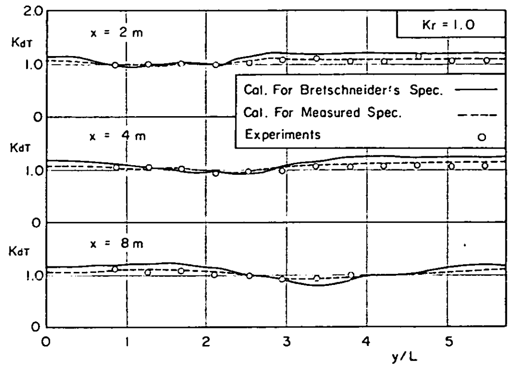

Fig. 19 (b) Periodical ratios of the diffracted waves for case 2 ($K_r=1.0$)

In **Fig. 19(b)** for the wave period coefficients, the experimental values show quite closer agreement with the calculated ones by the measured spectrum than by Bretschneider's one.

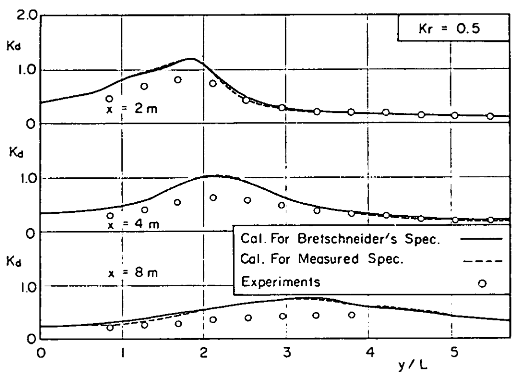

Fig. 20 (a) Diffraction coefficients for case 2 ($K_r=0.5$)

Fig. 20 is the case for the cushion type breakwater. As previously mensioned in **3.1** that the calculated diffraction coefficients are not affected by the reflection coefficients in the lee of the breakwater, the calculations in **Fig. 20 (a)** show almost same values as that in **Fig. 19 (a)**. However, the experimental diffraction coefficients are affected by the reflection coefficients, because the values in **Fig. 20(a)** are less than those in **Fig. 19 (a)** by 0.1 to 0.2 near the peak value, which exists in the unsheltered region. Therefore, the experiments show smaller diffraction coefficients than the calculations by about 0.4 in maximum except the sheltered region, where the experiments show a close agreement with the calculation. We could not make clear why the diffraction coefficients in the unsheltered region decrease their values in the experiments for the cushion type breakwaters. Anyway, according to the experimental results, we may say that it is effective to construct cushion type breakwaters on purpose of the improvement of calmness in unsheltered region.

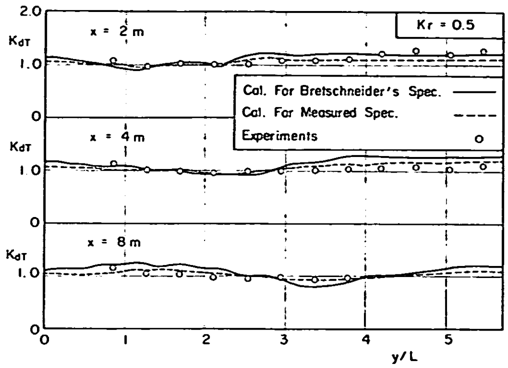

Fig. 20 (b) Periodical ratios of the diffracted waves for case 2 (K_r=0. 5)

In **Fig. 20 (b)**, the experimental wave period coefficients show much closer agreement with the calculated ones by the measured spectrum than by Bretschneider's spectrum.

In the results of the comparison between the experiments and the calculations for this case, the calculation by the approximate formula shows very good estimation in the sheltered region, but it will overestimate diffraction coefficients in the unsheltered region. The calculation of wave period coefficients shows very good estimation both in the sheltered region and in the unsheltered region, if the precise spectrum of incident waves is given.

(3) Case 3

In this case, the incident waves are diffracted by a breakwater and then once more diffracted by the other one. The diffraction coefficients are very small and 0.1 to 0.2. The experiments show close agreement with the calculations, as shown in **Fig. 21 (a)** and **(b)**. The approximate formula is applicable to the prediction of the diffracted waves. In this case, the experiments for cushion type breakwaters were not carried out, because large variation of the diffraction coefficients due to them are not expected.

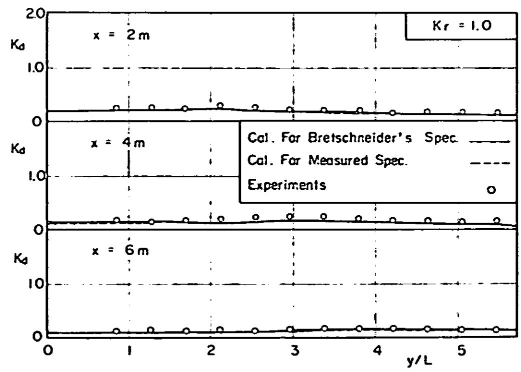

Fig. 21 (a) Diffraction coefficients for case 3 (K_r=1. 0)

The following conclusion are drawn in the comparison of the calculations with the experiments:

(1) The calculation by the presented approximate formulas gives a good estimation

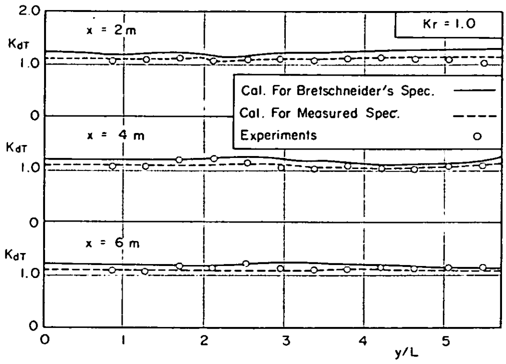

Fig. 21 (b) Periodical ratios of the diffracted waves for case 3 (K_r=1.0)

of diffraction coefficients due to cushion type breakwaters as well as rigid breakwaters.
(2) Though in Case 2 the calculated diffraction coefficients are not affected even in the unshelted region by the reflection coefficients of breakwaters, the experimental diffraction coefficients for the cushion type breakwaters decrease their values in the unsheltered region. This implies that it is effective to construct cushion type breakwaters on purpose of the improvement of calmness in unsheltered region.
(3) If the incident wave spectra show almost same values near their peak, the diffraction coefficients calculated by the spectra show almost same values without depending on the difference of their spectral forms. On the other hand, the wave period coefficients vary, depending on the spectral form in high frequencies. Therefore, precise spectral form of the incident waves is necessary in the good estimation of the wave period coefficients.
(4) For the wave period coefficients, the calculation estimates very close values to the experiments as far as the precise spectrum of incident waves is given.
(5) The close agreement between the calculations and the experiments suggests that the approximate formula is applicable to the estimation of wave diffraction for the incident waves with a directional spectrum.

5. Conclusions

The following conclusions are drawn in the study:
1) The formula of the wave diffraction due to a rigid semi-infinite breakwater is transformed to a practical form applicable to a cushion type semi-infinite breakwater.
2) According to the calculation by the transformed formula, in the lee of a semi-infinite breakwater the diffraction coefficients are not affected by the value of the reflection coefficient except the neighborhood of the breakwater.
3) The formula is applied with its superposition to the wave diffraction due to a gap, and the derived approximate formula is applicable to any alignment of breakwaters, and also to cushion type breakwaters.
4) The comparisons with the numerical analysis method of wave propagation are carried out for regular waves in order to verify the validity of the approximater formula. In the results, the calculation by the approximate formula shows a good prediction in the lee of breakwaters or in the inner harbor area, but shows some discrepancies to the numerical analysis method near a gap.
5) The experimental verification of the approximate formula is also carried out for

uni-directional irregular waves. Three cases of different breakwater alignments and two kinds of breakwaters with reflection coefficients of 1.0 and 0.5 are adopted in the experiments. In the results of the comparisons with the experimental values, the calculations by the approximate formula show a good prediction of the diffracted waves in the models. This implies that the prediction by the approximate formula is acceptable in the estimation of diffraction of incident waves with a directional spectrum.

6) The estimation of the wave period coefficients needs more precise spectral form of incident wave than that of the diffraction coefficiernts.

Acknowlegements

The authors wish to express their sincere gratitude to Dr. Yoshimi Goda, Chief of the Wave Laboratory, for his useful suggestions and discussions on the study.

The authors also wish to thank Mr. Hideyuki Sugiura of Toa Harbor Works Co., Ltd. for his assisting them in performance of the experiments and his arranging the experimental results.

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List of symbols

B_w : width of a gap
C(γ) : Fresnel integral
f : wave frequency
f_n : n-th component wave frequency
f_p : peak frequency of wave spectrum
$G(f, \theta)$: wave directional distribution function
g : gravitational acceleration
$H_{1/3}$: significant wave height
h : water depth
K_d : diffraction coefficient
K_{dT} : period ratio of diffracted wave
K_r : reflection coefficient
L : wave length of signifcant wave
M : dividing number of wave direction
N : dividing number of wave frequency
r : distance from the origin to a point
$S(f, \theta)$: directional spectrum
$S_\zeta(f)$: frequency spectrum
S(γ) : Fresnel integral
s_{max} : a parameter of wave directional concentration
$T_{1/3}$: significant wave period
$\bar{T}$: mean wave period
$U_{19.5}$: wind speed at 19.5m above the sea

α : incident wave approaching angle

γ_1 : $\gamma_1=\sqrt{4k_n r/\pi}\cos\dfrac{\alpha-\theta_m}{2}$

γ_2 : $\gamma_2=\sqrt{4k_n r/\pi}\cos\dfrac{\alpha+\theta_m}{2}$

ε_{nm} : phase difference

ζ : wave surface deformation

θ : wave approaching angle

θ_m : m-th component wave angle

$\psi(r,\alpha,f_n,\theta_m)$: transfer function of wave diffraction