REPORT

OF

PORT AND HARBOUR RESEARCH INSTITUTE

REPORT NO. 15

Analysis of Consolidation under Increasing and Decreasing Load

by

Tatsuro Okumura, Koji Moto

December, 1967

PORT AND HARBOUR RESEARCH INSTITUTE
MINISTRY OF TRANSPORT

1-1, 3 Chome, Nagase Yokosuka-City, Kanagawa-Prefecture, Japan

漸増及び漸減荷重下における圧密解析

奥　村　樹　郎
本　　浩　司

概　要：

地盤沈下の解析を行おうとすると荷重分布が粘土層中で一様でなく，しかも荷重が時間とともに変る圧密を取り扱わなければならない。

初めに筆者らは荷重分布が直線的で荷重が時間とともに一次的に増加する場合について，粘土層の歪を変数とし，熱伝導の理論を応用して，その解析解を求めた。結果は Terzaghi-Fröhlich および Schiffman の解と一致し，これらを一部拡張したものといえる。現場での計算に便利な諸図表を電子計算機によって作成した。

荷重が時間とともに減少する場合には粘土の膨張をも加味した圧密を取り扱わなければならない。筆者らは比較的単純な膨張特性を想定し，電子計算機を用いたこの場合の数値解法を考案した。計算結果の一例を 2，3 の近似解析解と比較して，かなりの差が見られた。

以上の計算方法はウェルポイントを施した基礎の沈下，プレローディング工法の計算など，応用範囲が広い。

ANALYSIS OF CONSOLIDATION UNDER INCREASING AND DECREASING LOAD

Contents

ANALYSIS OF CONSOLIDATION UNDER INCREASING AND DECREASING LOAD

*Tatsuro Okumura**
*Koji Moto***

Synopsis

In connection with the analysis of a ground subsidence problem, an analytical solution of consolidation under increasing load and a numerical method for decreasing load were developed. The analytical solution using the compression strain as a variable was simply obtained by means of the theory of conduction of heat and corresponded to the Terzaghi-Fröhlich's and Schiffman's solutions. A numerical analysis was carried out, in which the difference in values of modulus of volume compressibility and rebound, as well as, in values of coefficients of consolidation and rebound were considered. The result of this numerical computation was compared with some analytical approximations.

1. Introduction

One of engineering problems encountered in settlement analysis is how to treat the time dependent load. Terzaghi and Fröhlich (1936) have solved this problem analytically in their early publication. Afterward Schiffman (1963) dealt with the same subject and gave the tabulated results for computation of the excess pore water pressure at any point in a clay layer. In the Schiffman's work the stress increment within a clay layer is assumed to be constant with depth, so the solution is not readily applicable for the case of well-point or ground subsidence, where water heads at the upper and lower boundaries of a clay layer are not necessarily equal.

The analytical solution of one dimensional consolidation with the linear stress distribution is simply developed by means of the theory of conduction of heat using compression strain as a variable. Tabulated forms for computing the compression strain, the excess pore pressure, the degree of consolidation and the settlement of a clay layer are given.

When the water head of subsiding stratum recovers or the surface loading is partly removed during consolidation, it seems necessary to consider the compound phenomenon of consolidation and rebound within a clay layer. Terzaghi and Fröhlich have solved approximately some simple cases assuming the coefficient of rebound as infinite. In general, however, these cases can not be solved analytically.

The authors show a numerical method of analysis for such cases assuming that a ratio of the coefficient of rebound to the coefficient of consolidation is constant regardless of the magnitude of preconsolidation load and over consolida-

* Chief, Soil Test and Investigation Section, Soils Division
** Member, Computation Center, Design Standard Division

tion ratio. A numerical example of the settlement analysis by this method is worked out and is compared with results by some analytical approximations.

2. Consolidation under increasing load

According to the Mikasa's theory (1963), it is more convenient to analyze consolidation in terms of strain, instead of the excess pore water pressure, since the problem of pore pressure generation can be replaced by the problem of boundary conditions.

If the change in the thickness of a clay layer and the influence of its own weight are neglected, and the coefficient of consolidation c_v is assumed constant, the fundamental differential equation of one dimensional consolidation in terms compression strain ε is given as

$$\frac{\partial \varepsilon}{\partial t}=c_v\frac{\partial^2 \varepsilon}{\partial z^2} \tag{1}$$

Putting the boundary and initial conditions of

$$\left.\begin{aligned}&\varepsilon(0, t)=\phi_1(t)\\&\varepsilon(2H, t)=\phi_2(t)\\&\varepsilon(z, 0)=f(z)\end{aligned}\right\} \tag{2}$$

in which

$2H$: thickness of a clay layer

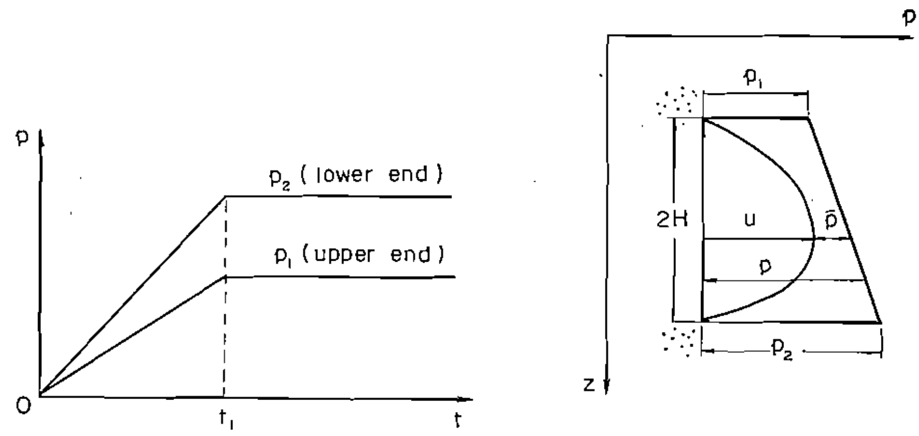
the solution of Eq. (1) is given, by the theory of conduction of heat (Carslow and Jaeger, 1959), as follows

$$\begin{aligned}\varepsilon=\frac{1}{H}\sum_{n=1}^{\infty} e^{-c_v n^2\Pi^2 t/4H^2}\sin\frac{n\Pi z}{2H}\Bigg[\int_0^{2H} f(x)\sin\frac{n\Pi x}{2H}\,dx\\+\frac{c_v n\Pi}{2H}\int_0^t e^{c_v n^2\Pi^2\lambda/4H^2}\{\phi_1(\lambda)-(-1)^n\phi_2(\lambda)\}\,d\lambda\Bigg]\end{aligned} \tag{3}$$

Assuming that the stress distribution is linear with depth and also load increases at a constant rate up to time t_1, and remains constant thereafter, as shown in Fig. 1, the boundary and initial conditions becomes

$$\left.\begin{aligned}&\varepsilon(0, t)=\begin{cases} m_v p_1 t/t_1 & (0\leqq t\leqq t_1)\\ m_v p_1 & (t\geqq t_1)\end{cases}\\&\varepsilon(2H, t)=\begin{cases} m_v p_2 t/t_1 & (0\leqq t\leqq t_1)\\ m_v p_2 & (t\geqq t_1)\end{cases}\\&\varepsilon(z, 0)=\begin{cases} 0 & (0\leqq t\leqq t_1)\\ \varepsilon(z, t_1) & (t\geqq t_1)\end{cases}\end{aligned}\right\} \tag{4}$$

where the modulus of volume compressibility m_v is assumed constant. Then we have the following solutions,



(a) (b)

Fig. 1 Loading condition (1)

$$\varepsilon=\frac{m_v}{T_1}\Bigg[T\left\{p_1+(p_2-p_1)\frac{z}{2H}\right\} -\frac{p_1+p_2}{2}\cdot\frac{16}{\Pi^3}\sum_{n=1,3,5}^{\infty}\frac{1}{n^3}(1-e^{-n^2\Pi^2T/4})\sin\frac{n\Pi z}{2H} +\frac{p_2-p_1}{2}\cdot\frac{16}{\Pi^3}\sum_{n=2,4,6}^{\infty}\frac{1}{n^3}(1-e^{-n^2\Pi^2T/4})\sin\frac{n\Pi z}{2H}\Bigg] \quad (0\leqq t\leqq t_1) \qquad (5a)$$

$$\varepsilon=\frac{m_v}{T_1}\Bigg[T_1\left\{p_1+(p_2-p_1)\frac{z}{2H}\right\}-\frac{p_1+p_2}{2}\cdot\frac{16}{\Pi^3}\sum_{n=1,3,5}^{\infty}\frac{1}{n^3}\{e^{-n^2\Pi^2(T-T_1)/4} -e^{-n^2\Pi^2T/4}\}\sin\frac{n\Pi z}{2H}+\frac{p_2-p_1}{2}\cdot\frac{16}{\Pi^3}\sum_{n=2,4,6}^{\infty}\frac{1}{n^3}\{e^{-n^2\Pi^2(T-T_1)/4} -e^{-n^2\Pi^2T/4}\}\sin\frac{n\Pi z}{2H}\Bigg] \quad (t\geqq t_1) \qquad (5b)$$

in which

T : time factor ($T=c_vt/H^2$)

T_1 : time factor at time t_1 ($T_1=c_vt_1/H^2$)

The above infinite series have been calculated with digital computer TOSBAC-3400 by the six figures. The results are shown Table 1 and Fig. 2, as a function of the time factor and relative depth $z/2H$. Eq. (5) may then be rewritten in the following form,

$$\varepsilon=\frac{m_v}{T_1}\left[T\left\{p_1+(p_2-p_1)\frac{z}{2H}\right\}-\frac{p_1+p_2}{2}F_1(T,z/2H)+\frac{p_2-p_1}{2}F_2(T,z/2H)\right] \quad (0\leqq t\leqq t_1) \qquad (6a)$$

$$\varepsilon=\frac{m_v}{T_1}\Bigg[T_1\left\{p_1+(p_2-p_1)\frac{z}{2H}\right\}-\frac{p_1+p_2}{2}\{F_1(T,z/2H)-F_1(T-T_1,z/2H)\} +\frac{p_2-p_1}{2}\{F_2(T,z/2H)-F_2(T-T_1,z/2H)\}\Bigg] \quad (t\geqq t_1) \qquad (6b)$$

in which $F(T,z/2H)$ is referred to the coefficient of strain and expressed in the forms

Table—1(a) Coefficient of strain $F_1(T, z/2H)$

T \ $z/2H$	0.05 / 0.95	0.10 / 0.90	0.15 / 0.85	0.20 / 0.80	0.25 / 0.75	0.30 / 0.70	0.35 / 0.65	0.40 / 0.60	0.45 / 0.55	0.50
0	0	0	0	0	0	0	0	0	0	0
0.0010	0.000995	0.001000	0.001000	0.001000	0.001000	0.001000	0.001000	0.001000	0.001000	0.001000
0.0015	0.001471	0.001500	0.001500	0.001500	0.001500	0.001500	0.001500	0.001500	0.001500	0.001500
0.0020	0.001926	0.002000	0.002000	0.002000	0.002000	0.002000	0.002000	0.002000	0.002000	0.002000
0.0030	0.002769	0.002995	0.003000	0.003000	0.003000	0.003000	0.003000	0.003000	0.003000	0.003000
0.0040	0.003538	0.003978	0.004000	0.004000	0.004000	0.004000	0.004000	0.004000	0.004000	0.004000
0.0050	0.004247	0.004942	0.004998	0.005000	0.005000	0.005000	0.005000	0.005000	0.005000	0.005000
0.0060	0.004907	0.005886	0.005994	0.006000	0.006000	0.006000	0.006000	0.006000	0.006000	0.006000
0.0070	0.005526	0.006806	0.006985	0.006999	0.007000	0.007000	0.007000	0.007000	0.007000	0.007000
0.0080	0.006112	0.007704	0.007978	0.007998	0.008000	0.008000	0.008000	0.008000	0.008000	0.008000
0.0090	0.006669	0.008579	0.008949	0.008996	0.009000	0.009000	0.009000	0.009000	0.009000	0.009000
0.0100	0.007201	0.009432	0.009920	0.009992	0.010000	0.010000	0.010000	0.010000	0.010000	0.010000
0.0150	0.009575	0.013408	0.014630	0.014933	0.014991	0.014999	0.014999	0.015000	0.015000	0.015000
0.0200	0.011614	0.016986	0.019086	0.019769	0.019952	0.019992	0.019999	0.020000	0.020000	0.020000
0.0300	0.015083	0.023293	0.027298	0.029032	0.029694	0.029915	0.029979	0.029996	0.029999	0.030000
0.0400	0.018035	0.028806	0.034736	0.037728	0.039105	0.039679	0.039896	0.039969	0.039992	0.039996
0.0500	0.020650	0.033761	0.041568	0.045907	0.048149	0.049223	0.049698	0.049891	0.049961	0.049978
0.0600	0.023022	0.038300	0.047915	0.053631	0.056833	0.058518	0.059348	0.059728	0.059884	0.059926
0.0700	0.025208	0.042512	0.053863	0.060961	0.065179	0.067556	0.068822	0.069453	0.069735	0.069814
0.0800	0.027247	0.046458	0.059480	0.067945	0.073212	0.076340	0.078107	0.079045	0.079487	0.079616
0.0900	0.029164	0.050183	0.064812	0.074626	0.080958	0.084878	0.087195	0.088484	0.089120	0.089310
0.1000	0.030980	0.053720	0.069900	0.081037	0.088439	0.093178	0.096083	0.097760	0.098614	0.098873
0.1500	0.038937	0.069314	0.092524	0.109865	0.122507	0.131458	0.137556	0.141448	0.143600	0.144286
0.2000	0.045616	0.082468	0.111756	0.134615	0.152080	0.165064	0.174340	0.180519	0.184047	0.185193
0.3000	0.056483	0.103916	0.143227	0.175294	0.200930	0.220854	0.235684	0.245914	0.251904	0.253876
0.4000	0.064912	0.120566	0.167683	0.206951	0.239003	0.264404	0.283636	0.297088	0.305044	0.307676
0.5000	0.071492	0.133563	0.186777	0.231672	0.268741	0.298427	0.321106	0.337082	0.346577	0.349727
0.6000	0.076632	0.143717	0.201695	0.250985	0.291975	0.325009	0.350383	0.368332	0.379031	0.382585
0.7000	0.080649	0.151650	0.213350	0.266076	0.310129	0.34578	0.373258	0.392749	0.404388	0.408259
0.8000	0.083787	0.157849	0.222457	0.277867	0.324314	0.362008	0.391131	0.411827	0.424201	0.428318
0.9000	0.086238	0.162693	0.229573	0.287079	0.335396	0.374689	0.405097	0.426733	0.439682	0.443992
1.0000	0.088154	0.166477	0.235133	0.294278	0.344056	0.384596	0.416008	0.438380	0.451777	0.456239
1.5000	0.093006	0.176062	0.249214	0.312509	0.365989	0.409690	0.443645	0.467880	0.482413	0.487256
2.0000	0.094419	0.178853	0.253315	0.317819	0.372376	0.416998	0.451693	0.476471	0.491335	0.496289
3.0000	0.094951	0.179903	0.254857	0.319815	0.374778	0.419745	0.454720	0.479701	0.494689	0.499685
4.0000	0.094996	0.179992	0.254988	0.319984	0.374981	0.419978	0.454976	0.479975	0.494974	0.499973
5.0000	0.095000	0.179999	0.254999	0.319999	0.374998	0.419998	0.454998	0.479998	0.494998	0.499998
6.0000	0.095000	0.180000	0.255000	0.320000	0.375000	0.420000	0.455000	0.480000	0.495000	0.500000
∞	0.095	0.18	0.255	0.32	0.375	0.42	0.455	0.48	0.495	0.5

Table—1(b) Coefficient of strain $F_2(T, z/2H)$

T \ $z/2H$	0.05 0.95	0.10 0.90	0.15 0.85	0.20 0.80	0.25 0.75	0.30 0.70	0.35 0.65	0.40 0.60	0.45 0.55
0	0	0	0	0	0	0	0	0	0
0.0010	0.000895	0.000800	0.000700	0.000600	0.000500	0.000400	0.000300	0.000200	0.000100
0.0015	0.001321	0.001200	0.001050	0.000900	0.000750	0.000600	0.000450	0.000300	0.000150
0.0020	0.001726	0.001600	0.001400	0.001200	0.001000	0.000800	0.000600	0.000400	0.000200
0.0030	0.002469	0.002395	0.002100	0.001800	0.001500	0.001200	0.000900	0.000600	0.000300
0.0040	0.003138	0.003178	0.002800	0.002400	0.002000	0.001600	0.001200	0.000800	0.000400
0.0050	0.003747	0.003942	0.003498	0.003000	0.002500	0.002000	0.001500	0.001000	0.000500
0.0060	0.004307	0.004686	0.004194	0.003600	0.003000	0.002400	0.001800	0.001200	0.000600
0.0070	0.004826	0.005406	0.004885	0.004199	0.003500	0.002800	0.002100	0.001400	0.000700
0.0080	0.005312	0.006104	0.005571	0.004798	0.004000	0.003200	0.002400	0.001600	0.000800
0.0090	0.005769	0.006779	0.006249	0.005396	0.004500	0.003600	0.002700	0.001800	0.000900
0.0100	0.006201	0.007432	0.006920	0.005992	0.005000	0.004000	0.003000	0.002000	0.001000
0.0150	0.008075	0.010408	0.010130	0.008933	0.007491	0.005999	0.004500	0.003000	0.001500
0.0200	0.009614	0.012986	0.013086	0.011769	0.009952	0.007992	0.005999	0.004000	0.002000
0.0300	0.012083	0.017293	0.018298	0.017032	0.014694	0.011915	0.008979	0.005996	0.002999
0.0400	0.014035	0.020806	0.022736	0.021728	0.019105	0.015679	0.011896	0.007970	0.003992
0.0500	0.015650	0.023761	0.026568	0.025907	0.023149	0.019223	0.014698	0.009892	0.004967
0.0600	0.017022	0.026300	0.029915	0.029631	0.026834	0.022519	0.017350	0.011736	0.005909
0.0700	0.018208	0.028518	0.032863	0.032961	0.030180	0.025559	0.019831	0.013479	0.006806
0.0800	0.019247	0.030458	0.035480	0.035946	0.033215	0.028349	0.022132	0.015110	0.007650
0.0900	0.020164	0.032183	0.037813	0.038629	0.035966	0.030900	0.024252	0.016622	0.008435
0.1000	0.020979	0.033721	0.039903	0.041045	0.038460	0.033228	0.026199	0.018017	0.009162
0.1500	0.023952	0.039353	0.047606	0.050029	0.047823	0.042054	0.033647	0.023394	0.011977
0.2000	0.025729	0.042731	0.052248	0.055477	0.053540	0.047480	0.038254	0.026736	0.013733
0.3000	0.027468	0.046037	0.056798	0.060824	0.059161	0.052824	0.042798	0.030037	0.015468
0.4000	0.028115	0.047268	0.058493	0.062816	0.061255	0.054816	0.044493	0.031267	0.016115
0.5000	0.028357	0.047727	0.059125	0.063559	0.062036	0.055559	0.045125	0.031727	0.016357
0.6000	0.028447	0.047898	0.059360	0.063836	0.062327	0.055836	0.045360	0.031898	0.016447
0.7000	0.028480	0.047962	0.059448	0.063939	0.062436	0.055939	0.045448	0.031962	0.016480
0.8000	0.028493	0.047986	0.059481	0.063977	0.062476	0.055977	0.045481	0.031986	0.016493
0.9000	0.028497	0.047995	0.059493	0.063992	0.062491	0.055992	0.045493	0.031995	0.016497
1.0000	0.028499	0.047998	0.059497	0.063997	0.062497	0.055997	0.045497	0.031998	0.016499
∞	0.0285	0.048	0.0595	0.064	0.0625	0.056	0.0455	0.032	0.0165

Note: When $z/2H = 0.5$, $F_2(T, z/2H) = 0$

$z/2H < 0.5$, $F_2(T, z/2H) \geqq 0$

$z/2H > 0.5$, $F_2(T, z/2H) \leqq 0$

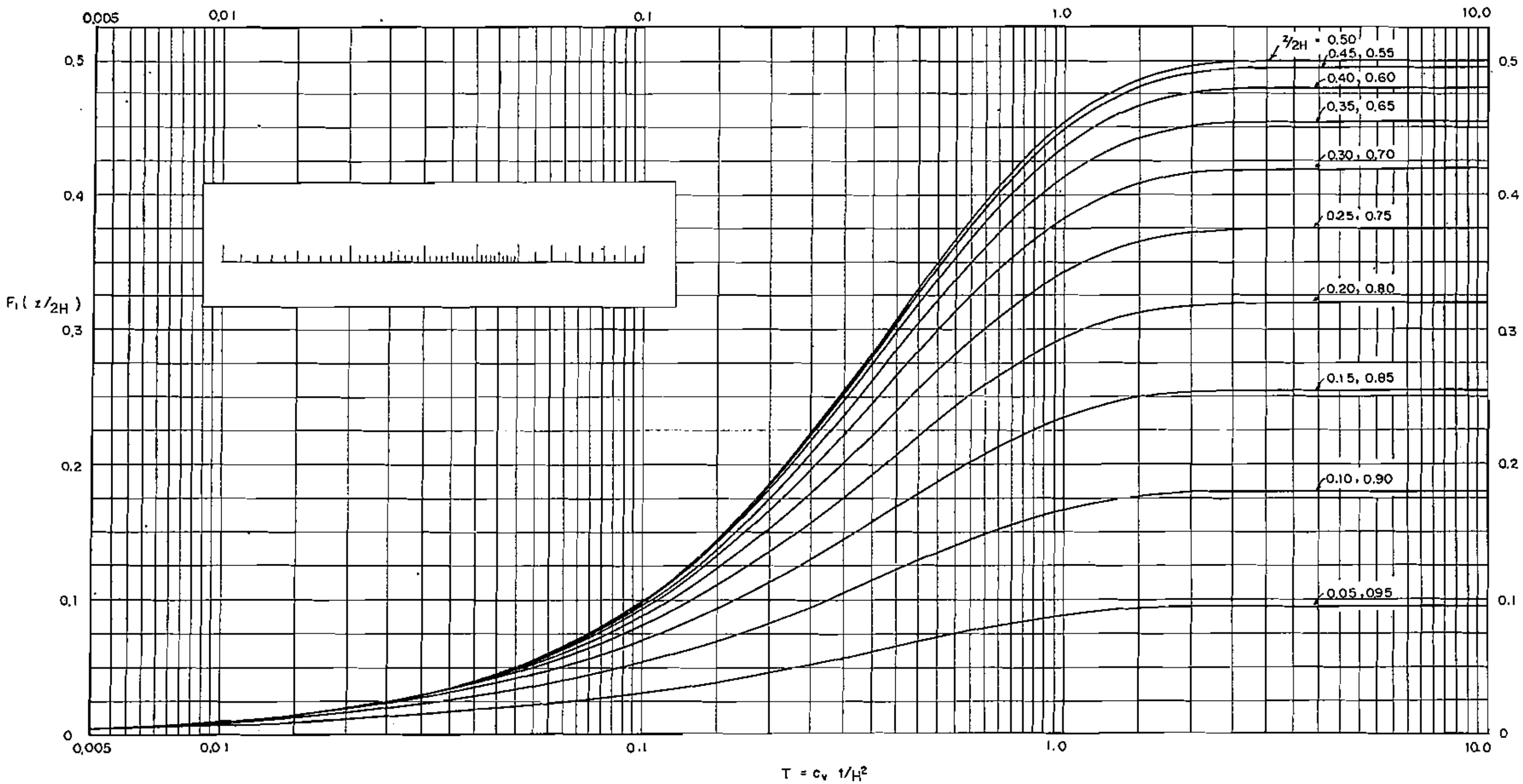


Fig. 2(a) Coefficient of strain $F_1(T, z/2H)$

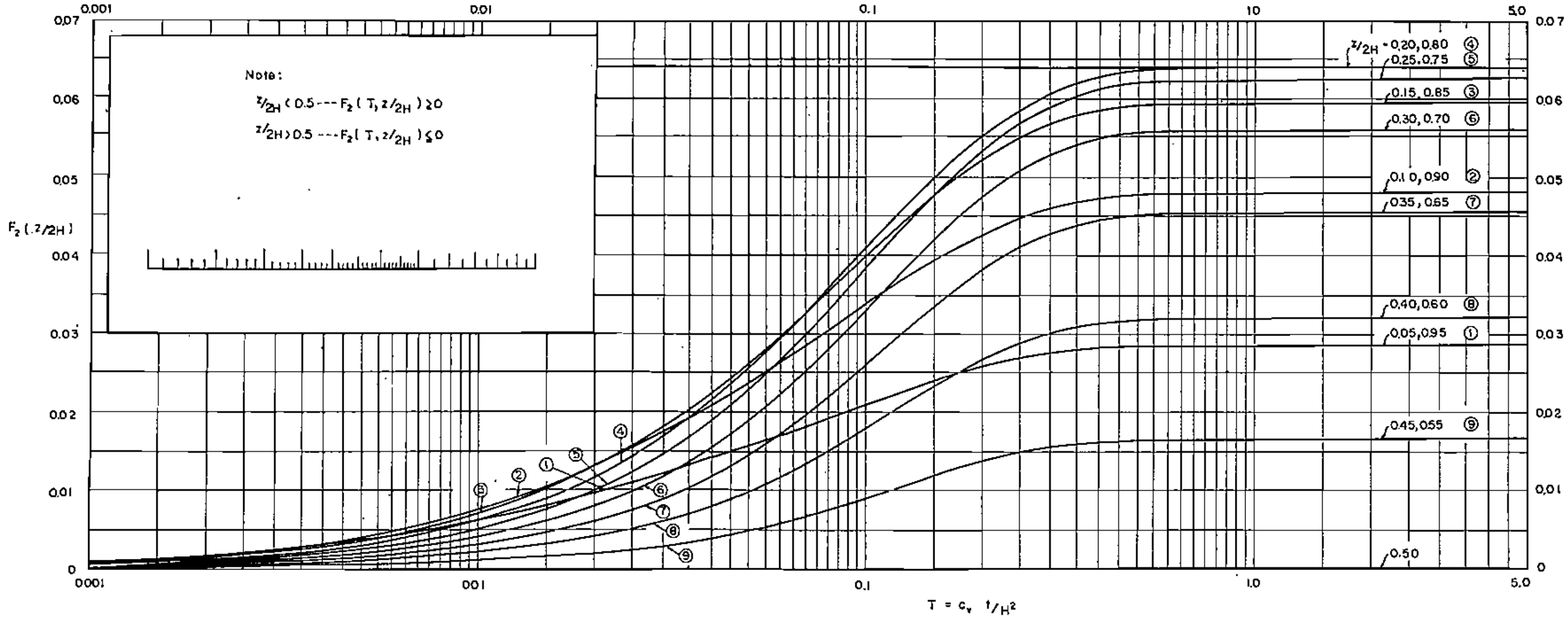


Fig. 2(b) Coefficient of strain $F_2(T, z/2H)$

$$F_1(T, z/2H)=\frac{16}{\Pi^3}\sum_{n=1,3,5}^{\infty}\frac{1}{n^3}(1-e^{-n^2\Pi^2T/4})\sin\frac{n\Pi z}{2H} \tag{7a}$$

$$F_2(T, z/2H)=\frac{16}{\Pi^3}\sum_{n=2,4,6}^{\infty}\frac{1}{n^3}(1-e^{-n^2\Pi^2T/4})\sin\frac{n\Pi z}{2H} \tag{7b}$$

If p_2 is equal to p_1, i.e. in the case of uniform stress distribution, Eq. (6) becomes

$$\varepsilon=\frac{m_v p_1}{T_1}\{T-F_1(T, z/2H)\} \qquad (0\leqq t\leqq t_1) \tag{8a}$$

$$\varepsilon=\frac{m_v p_1}{T_1}[T_1-\{F_1(T, z/2H)-F_1(T-T_1, z/2H)\}] \qquad (t\geqq t_1) \tag{8b}$$

Relationship between the strain ε and the excess pore water pressure u is

$$\varepsilon=m_v(p-u)$$
$$=\left\{\begin{array}{ll} m_v\left[\frac{t}{t_1}\left\{p_1+(p_2-p_1)\frac{z}{2H}\right\}-u\right] & (0\leqq t\leqq t_1) \\ m_v\left\{p_1+(p_2-p_1)\frac{z}{2H}-u\right\} & (t\geqq t_1) \end{array}\right\} \tag{9}$$

where p is total pressure at a depth in consideration. The solution in terms of the excess pore pressure is then written as

$$u=\frac{1}{T_1}\left\{\frac{p_1+p_2}{2}F_1(T, z/2H)-\frac{p_2-p_1}{2}F_2(T, z/2H)\right\} \qquad (0\leqq t\leqq t_1) \tag{10a}$$

$$u=\frac{1}{T_1}\left[\frac{p_1+p_2}{2}\{F_1(T, z/2H)-F_1(T-T_1, z/2H)\}\right.$$
$$\left.-\frac{p_2-p_1}{2}\{F_2(T, z/2H)-F_2(T-T_1, z/2H)\}\right] \qquad (t\geqq t_1) \tag{10b}$$

These correspond to the Tarzaghi-Fröhlich's solution (1936). And the condition of $p_1=p_2$ gives

$$u=\frac{p_1}{T_1}F_1(T, z/2H) \qquad (0\leqq t\leqq t_1) \tag{11a}$$

$$u=\frac{p_1}{T_1}\{F_1(T, z/2H)-F_1(T-T_1, z/2H)\} \qquad (t\geqq t_1) \tag{11b}$$

These correspond to the Schiffman's solution (1963).

Denoting the degree of consolidation U as the ratio of the mean effective pressure $\bar{p}$ to the mean total pressure, the expression for the degree of consolidation is

$$U=1-\frac{\int_0^{2H} u\,dz}{\int_0^{2H} p\,dz}$$

$$
=\begin{cases} 1-\dfrac{1}{T}\left(\dfrac{1}{3}-\dfrac{32}{\Pi^4}\sum_{n=1,3,5}^{\infty}\dfrac{1}{n^4}e^{-n^2\Pi^2T/4}\right) & (0\leqq t\leqq t_1) \\ 1-\dfrac{1}{T_1}\cdot\dfrac{32}{\Pi^4}\sum_{n=1,3,5}^{\infty}\dfrac{1}{n^4}\{e^{-n^2\Pi^2(T-T_1)/4}-e^{-n^2\Pi^2T/4}\} & (t\geqq t_1) \end{cases} \tag{12}
$$

and U is independent of magnitude of pressures p_1 and p_2.

If we write

$$
U_0(T)\equiv 1-\frac{1}{T}\left(\frac{1}{3}-\frac{32}{\Pi^4}\sum_{n=1,3,5}^{\infty}\frac{1}{n^4}e^{-n^2\Pi^2T/4}\right) \tag{13}
$$

Eq. (12) becomes:

$$
U=\begin{cases} U_0(T) & (0\leqq t\leqq t_1) \\ \dfrac{1}{t_1}[tU_0(T)-(t-t_1)U_0(T-T_1)] & (t\geqq t_1) \end{cases} \tag{14}
$$

$U_0(T)$ shall be called the coefficient of degree of consolidation. Values of $U_0(T)$ computed by TOSBAC-3400 are shown in Table 2 and Fig. 3. Using the coefficient of degree of consolidation, the expression for settlement becomes

$$
S=\int_0^{2H}\varepsilon dz
$$

$$
=\begin{cases} 2Hm_v\cdot\dfrac{p_1+p_2}{2}\cdot\dfrac{t}{t_1}U_0(T) & (0\leqq t\leqq t_1) \\ 2Hm_v\cdot\dfrac{p_1+p_2}{2}\;\dfrac{1}{t_1}[tU_0(T)-(t-t_1)U_0(T-T_1)] & (t\geqq t_1) \end{cases} \tag{15}
$$

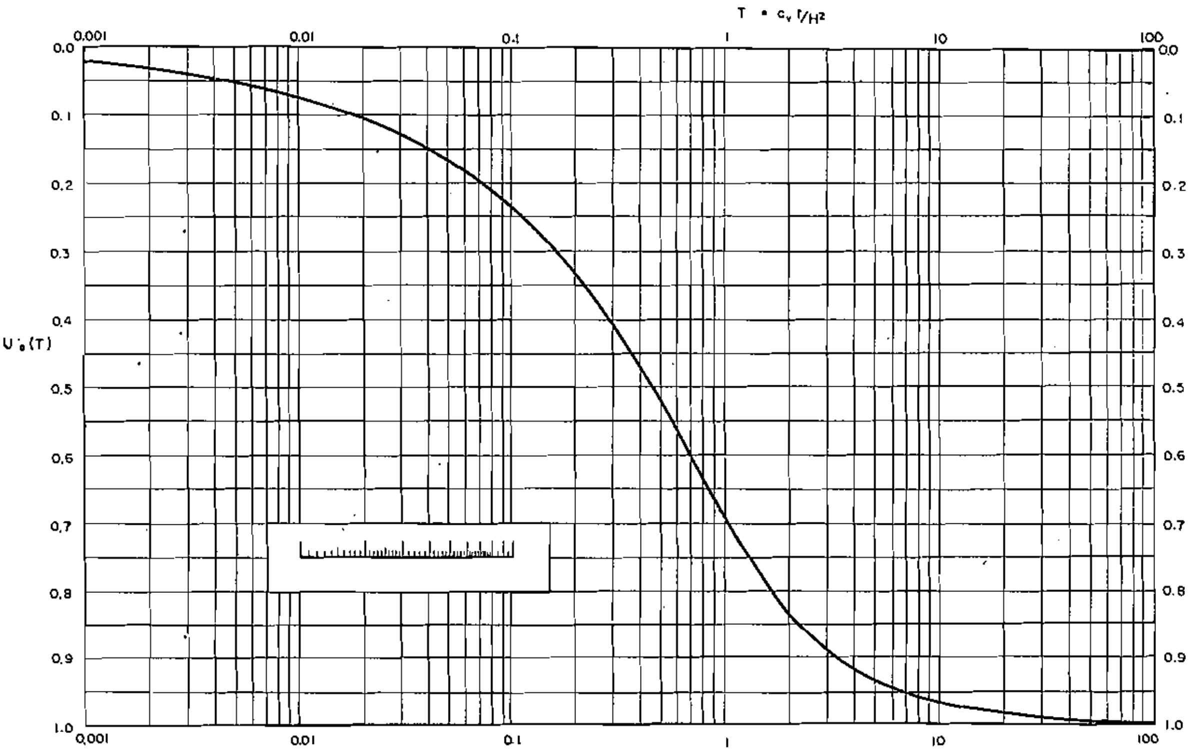


Fig. 3 Coefficient of degree of consolidation $U_0(T)$

Table—2 Coefficient of degree of consolidation $U_0(T)$

T	$U_0(T)$	T	$U_0(T)$
0	0	1	0.694526
0.0010	0.023788	1.5	0.783186
0.0015	0.029135	2	0.834515
0.0020	0.033642	3	0.888956
0.0030	0.041203	4	0.916671
0.0040	0.047577	5	0.933334
0.0050	0.053192	6	0.944444
0.0060	0.058269	7	0.952381
0.0070	0.062938	8	0.958333
0.0080	0.067284	9	0.962963
0.0090	0.071365	10	0.966667
0.0100	0.075225	15	0.977778
0.0150	0.092132	20	0.983333
0.0200	0.106385	30	0.988889
0.0300	0.130294	40	0.991667
0.0400	0.150451	50	0.993333
0.0500	0.168209	60	0.994444
0.0600	0.184264	70	0.995238
0.0700	0.199207	80	0.995833
0.0800	0.212769	90	0.996296
0.0900	0.225676	100	0.996667
0.1000	0.237883	150	0.997778
0.1500	0.291339	200	0.998333
0.2000	0.336350	300	0.998889
0.3000	0.411249	400	0.999167
0.4000	0.472765	500	0.999333
0.5000	0.524667	600	0.999444
0.6000	0.569026	700	0.999524
0.7000	0.607244	800	0.999583
0.8000	0.640376	900	0.999630
0.9000	0.669247	∞	1

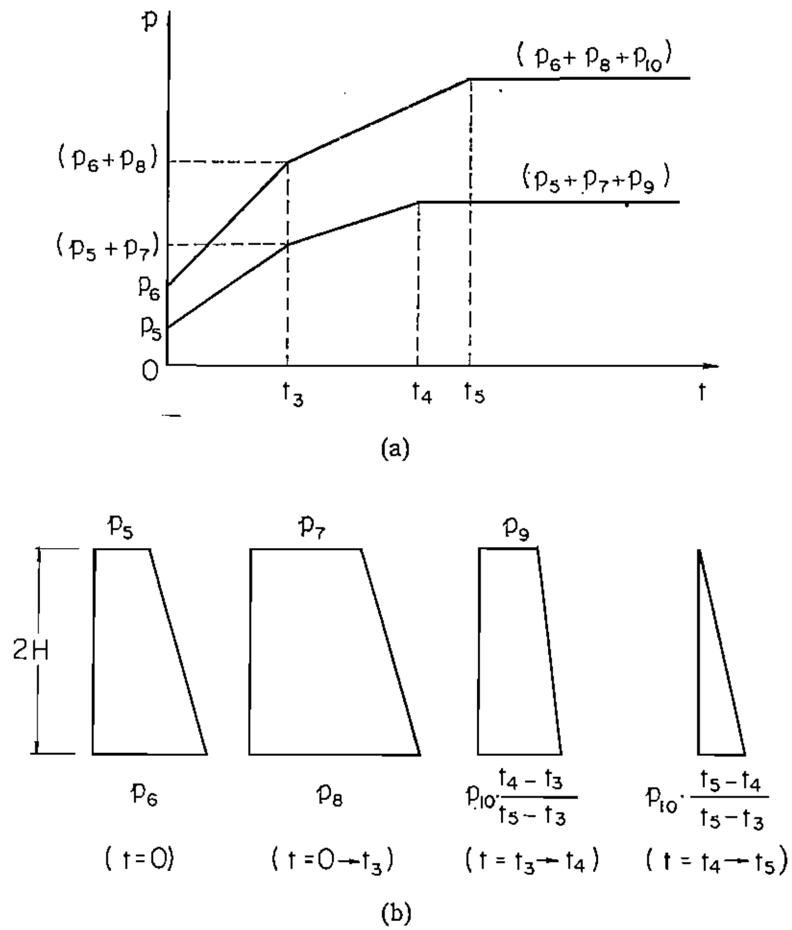


Fig. 4 Loading condition (2)

It may be said that the settlement in this particular case is expressed as the product of the thickness of clay layer, the modulus of volume compressibility, the mean total pressure at that time and the degree of consolidation.

The law of superposition holds in this case, then more complicated cases can be analyzed. For example, under the loading condition shown in Fig. 4, the excess pore pressure is represented as follows

$$\left.\begin{aligned}
&u=u_1+u_2' && (0\leqq t\leqq t_3)\\
&u=u_1+u_2+u_3' && (t_3\leqq t\leqq t_4)\\
&u=u_1+u_2+u_3+u_4' && (t_4\leqq t\leqq t_5)\\
&u=u_1+u_2+u_3+u_4 && (t\geqq t_5)
\end{aligned}\right\} \tag{16}$$

in which

$$\left.\begin{aligned}
2u_1 &=(p_5+p_6)G_1(T, z/2H)-(p_6-p_5)G_2(T, z/2H)\\
2u_2' &=\frac{1}{T_3}\{(p_7+p_8)F_1(T, z/2H)-(p_8-p_7)F_2(T, z/2H)\}\\
2u_2 &=\frac{1}{T_3}[(p_7+p_8)\{F_1(T, z/2H)-F_1(T-T_3, z/2H)\}\\
&\quad -(p_8-p_7)\{F_2(T, z/2H)-F_2(T-T_3, z/2H)\}]\\
2u_3' &=\frac{1}{T_4-T_3}\left[\left\{p_9+\frac{p_{10}(t_4-t_3)}{t_5-t_3}\right\}F_1(T-T_3, z/2H)\right.\\
&\quad \left.-\left\{\frac{p_{10}(t_4-t_3)}{t_5-t_3}-p_9\right\}F_2(T-T_3, z/2H)\right]\\
2u_3 &=\frac{1}{T_4-T_3}\left[\left\{p_9+\frac{p_{10}(t_4-t_3)}{t_5-t_3}\right\}\{F_1(T-T_3, z/2H)-F_1(T-T_4, z/2H)\}\right.\\
&\quad \left.-\left\{\frac{p_{10}(t_4-t_3)}{t_5-t_3}-p_9\right\}\{F_2(T-T_3, z/2H)-F_2(T-T_4, z/2H)\}\right]\\
2u_4' &=\frac{p_{10}}{T_5-T_3}\{F_1(T-T_4, z/2H)-F_2(T-T_4, z/2H)\}\\
2u_4 &=\frac{p_{10}}{T_5-T_3}\{F_1(T-T_4, z/2H)-F_1(T-T_5, z/2H)-F_2(T-T_4, z/2H)\\
&\quad +F_2(T-T_5, z/2H)\}
\end{aligned}\right\} \tag{17}$$

In the above equations, T_3, T_4, T_5 are the time factor corresponding to time t_3, t_4, t_5 respectively. And $G_1(T, z/2H)$ and $G_2(T, z/2H)$ are given as

$$\left.\begin{aligned}
G_1(T, z/2H) &=\frac{4}{\Pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}e^{-n^2\Pi^2T/4}\sin\frac{n\Pi z}{2H}\\
G_2(T, z/2H) &=\frac{4}{\Pi}\sum_{n=2,4,6}^{\infty}\frac{1}{n}e^{-n^2\Pi^2T/4}\sin\frac{n\Pi z}{2H}
\end{aligned}\right\} \tag{18}$$

The solution u_1 in Eq. (17) for the instantaneous loading is derived by Eq. (3)

Table—3(a) Coefficient of pore pressure $G_1(T, z/2H)$

$z/2H$ / T	0.05 / 0.95	0.10 / 0.90	0.15 / 0.85	0.20 / 0.80	0.25 / 0.75	0.30 / 0.70	0.35 / 0.65	0.40 / 0.60	0.45 / 0.55	0.50
0	1	1	1	1	1	1	1	1	1	1
0.0010	0.974653	0.999992	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.0015	0.932111	0.999739	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.0020	0.886154	0.998435	0.999998	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.0030	0.803294	0.990177	0.999893	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.0040	0.736448	0.974653	0.999204	0.999992	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.0050	0.682690	0.954500	0.997300	0.999937	0.999999	1.000000	1.000000	1.000000	1.000000	1.000000
0.0060	0.638690	0.932111	0.993830	0.999739	0.999995	1.000000	1.000000	1.000000	1.000000	1.000000
0.0070	0.601975	0.909031	0.988770	0.999277	0.999976	1.000000	1.000000	1.000000	1.000000	1.000000
0.0080	0.570805	0.886154	0.982294	0.998435	0.999923	0.999998	1.000000	1.000000	1.000000	1.000000
0.0090	0.543944	0.863963	0.974653	0.997131	0.999806	0.999992	1.000000	1.000000	1.000000	1.000000
0.0100	0.520500	0.842701	0.966105	0.995322	0.999593	0.999978	0.999999	1.000000	1.000000	1.000000
0.0150	0.436297	0.751787	0.916736	0.979079	0.996108	0.999468	0.999947	0.999996	1.000000	1.000000
0.0200	0.382925	0.682690	0.866386	0.954500	0.987581	0.997300	0.999535	0.999937	0.999993	0.999999
0.0300	0.316909	0.585784	0.779329	0.897530	0.958773	0.985694	0.995733	0.998908	0.999754	0.999911
0.0400	0.276326	0.520500	0.711156	0.842701	0.922900	0.966104	0.986667	0.995300	0.998437	0.999186
0.0500	0.248170	0.472911	0.657218	0.794096	0.886152	0.942211	0.973104	0.988440	0.995069	0.996869
0.0600	0.227170	0.436297	0.613523	0.751783	0.851070	0.916682	0.956517	0.978547	0.989129	0.992215
0.0700	0.210732	0.407018	0.577316	0.714932	0.818490	0.891008	0.938119	0.966150	0.980560	0.984947
0.0800	0.197411	0.382918	0.546724	0.682626	0.788524	0.865920	0.918728	0.951800	0.969592	0.975161
0.0900	0.186330	0.362626	0.520438	0.654059	0.761000	0.841733	0.898857	0.935976	0.956583	0.963156
0.1000	0.176918	0.345223	0.497521	0.628560	0.735651	0.818542	0.878825	0.919071	0.941922	0.949305
0.1500	0.144471	0.284043	0.414234	0.531316	0.632525	0.716093	0.781136	0.827414	0.855042	0.864222
0.2000	0.123869	0.244248	0.357830	0.461647	0.553176	0.630401	0.691806	0.736327	0.763286	0.772312
0.3000	0.095256	0.188119	0.276266	0.357505	0.429843	0.491521	0.541065	0.577302	0.599386	0.606804
0.4000	0.074262	0.146691	0.215498	0.278987	0.335597	0.383934	0.422815	0.451286	0.468651	0.474488
0.5000	0.058006	0.114584	0.168339	0.217947	0.262188	0.299972	0.330370	0.352633	0.366213	0.370777
0.6000	0.045321	0.089526	0.131526	0.170288	0.204856	0.234380	0.258133	0.275530	0.286142	0.289709
0.7000	0.035411	0.069950	0.102767	0.133053	0.160063	0.183131	0.201691	0.215284	0.223576	0.226363
0.8000	0.027668	0.054655	0.080296	0.103960	0.125064	0.143089	0.157590	0.168211	0.174690	0.176867
0.9000	0.021618	0.042704	0.062739	0.081228	0.097718	0.111801	0.123132	0.131430	0.136493	0.138194
1.0000	0.016891	0.033367	0.049021	0.063467	0.076351	0.087355	0.096208	0.102692	0.106648	0.107977
1.5000	0.004919	0.009717	0.014275	0.018483	0.022235	0.025439	0.028017	0.029905	0.003057	0.031444
2.0000	0.001433	0.002830	0.004157	0.005382	0.006475	0.007408	0.008159	0.008709	0.009044	0.009157
3.0000	0.000122	0.000240	0.000353	0.000456	0.000549	0.000628	0.000692	0.000739	0.000767	0.000777
4.0000	0.000010	0.000020	0.000030	0.000039	0.000047	0.000053	0.000059	0.000063	0.000065	0.000066
5.0000	0.000001	0.000002	0.000003	0.000003	0.000004	0.000005	0.000005	0.000005	0.000006	0.000006
6.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000001	0.000001

Table—3(b) Coefficient of pore pressure $G_2(T, z/2H)$

T \ $z/2H$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55
0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0010	0.874653	0.799992	0.700000	0.600000	0.500000	0.400000	0.300000	0.200000	0.100000
0.0015	0.832111	0.799739	0.700000	0.600000	0.500000	0.400000	0.300000	0.200000	0.100000
0.0020	0.786154	0.798435	0.699998	0.600000	0.500000	0.400000	0.300000	0.200000	0.100000
0.0030	0.703294	0.790177	0.699893	0.600000	0.500000	0.400000	0.300000	0.200000	0.100000
0.0040	0.636448	0.774653	0.699204	0.599992	0.500000	0.400000	0.300000	0.200000	0.100000
0.0050	0.582690	0.754500	0.697300	0.599937	0.499999	0.400000	0.300000	0.200000	0.100000
0.0060	0.538690	0.732111	0.693830	0.599739	0.499995	0.400000	0.300000	0.200000	0.100000
0.0070	0.501975	0.709031	0.688770	0.599277	0.499976	0.400000	0.300000	0.200000	0.100000
0.0080	0.470805	0.686154	0.682294	0.598435	0.499923	0.399998	0.300000	0.200000	0.100000
0.0090	0.443944	0.663963	0.674653	0.597131	0.499806	0.399992	0.300000	0.200000	0.100000
0.0100	0.420500	0.642701	0.666105	0.595322	0.499593	0.399978	0.299999	0.200000	0.100000
0.0150	0.336297	0.551787	0.616736	0.579079	0.496108	0.399468	0.299947	0.199996	0.100000
0.0200	0.282925	0.482690	0.566386	0.554500	0.487581	0.397300	0.299535	0.199937	0.099993
0.0300	0.216909	0.385784	0.479329	0.497530	0.458773	0.385694	0.295733	0.198910	0.099769
0.0400	0.176326	0.320500	0.411156	0.442701	0.422900	0.366106	0.286676	0.195344	0.098638
0.0500	0.148170	0.272911	0.357218	0.394097	0.386156	0.342230	0.273183	0.188736	0.096078
0.0600	0.127170	0.236297	0.313525	0.351791	0.351100	0.316789	0.256867	0.179611	0.092121
0.0700	0.110732	0.207021	0.277328	0.314970	0.318612	0.291373	0.239143	0.168831	0.087127
0.0800	0.097415	0.182932	0.246767	0.282753	0.288877	0.266851	0.221036	0.157200	0.081511
0.0900	0.086343	0.162670	0.220561	0.254384	0.261814	0.243668	0.203223	0.145331	0.075627
0.1000	0.076956	0.145335	0.197809	0.229253	0.237244	0.222033	0.186126	0.133652	0.069735
0.1500	0.045264	0.085955	0.118002	0.138266	0.144855	0.137263	0.116379	0.084332	0.044261
0.2000	0.027397	0.052093	0.071657	0.084175	0.088434	0.084036	0.071432	0.051867	0.027258
0.3000	0.010187	0.019376	0.026667	0.031348	0.032960	0.031345	0.026663	0.019371	0.010184
0.4000	0.003796	0.007221	0.009938	0.011683	0.012284	0.011683	0.009938	0.007221	0.003796
0.5000	0.001415	0.002691	0.003704	0.004354	0.004579	0.004354	0.003704	0.002691	0.001415
0.6000	0.000527	0.001003	0.001381	0.001623	0.001706	0.001623	0.001381	0.001003	0.000527
0.7000	0.000197	0.000374	0.000515	0.000605	0.000636	0.000605	0.000515	0.000374	0.000197
0.8000	0.000073	0.000139	0.000192	0.000225	0.000237	0.000225	0.000192	0.000139	0.000073
0.9000	0.000027	0.000052	0.000072	0.000084	0.000088	0.000084	0.000072	0.000052	0.000027
1.0000	0.000010	0.000019	0.000027	0.000031	0.000033	0.000031	0.000027	0.000019	0.000010
1.5000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Note: When $z/2H=0.5$, $G_2(T, z/2H)=0$

$z/2H<0.5$, $G_2(T, z/2H)\geqq 0$

$z/2H>0.5$, $G_2(T, z/2H)\leqq 0$

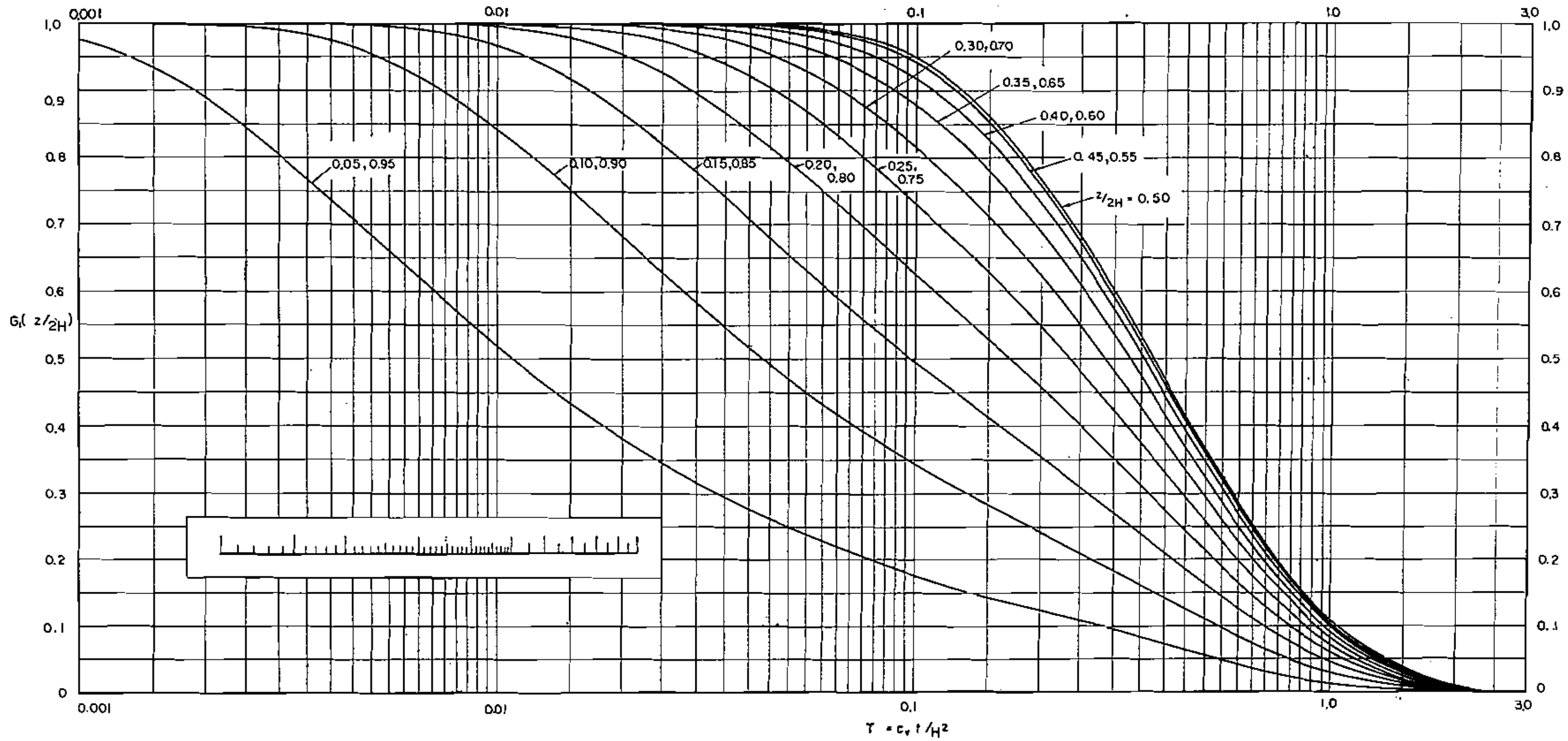


Fig. 5(a) Coefficient of pore pressure $G_1(T, z/2H)$

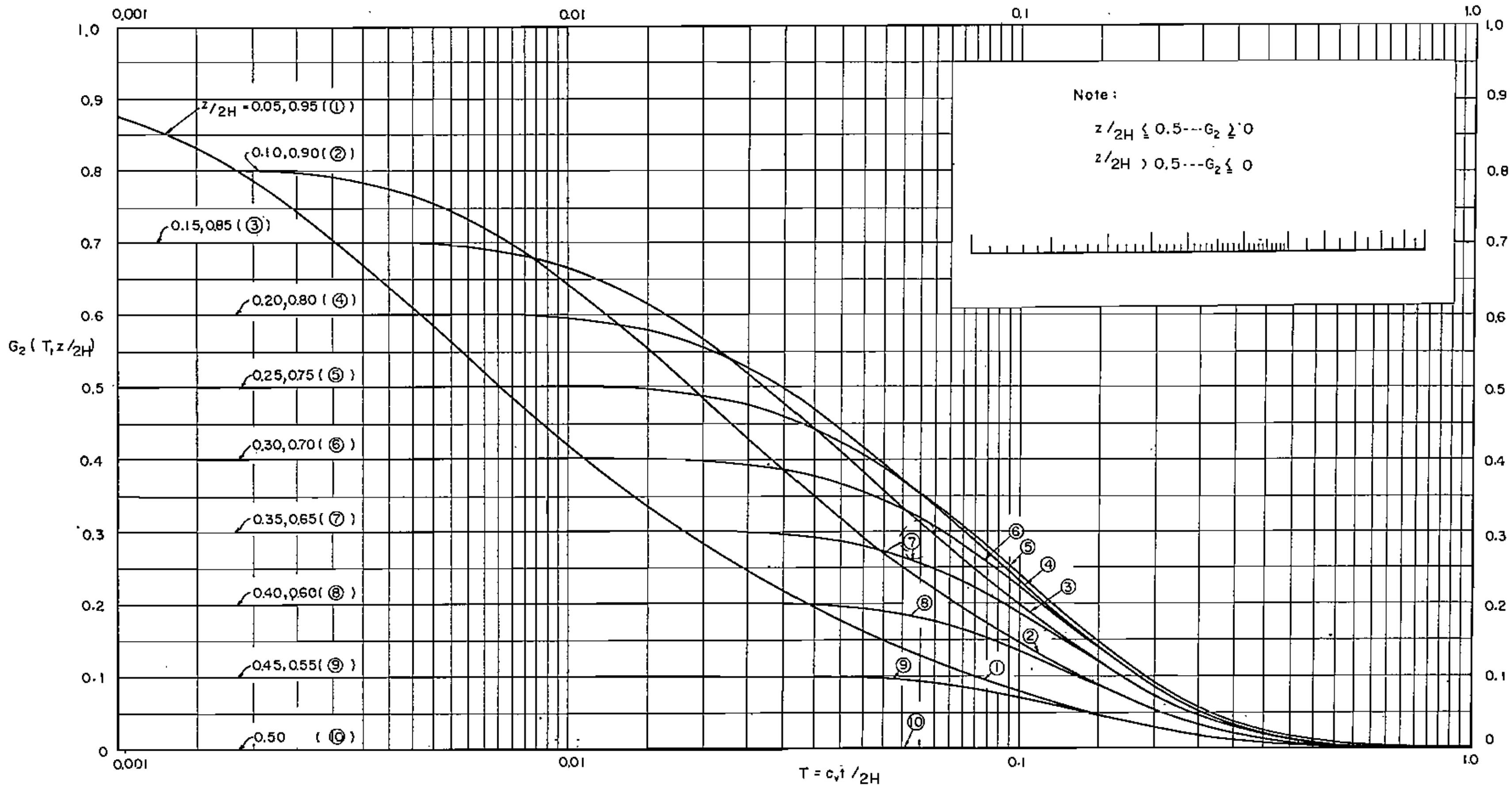


Fig. 5(b) Pore pressure function $G_2(T, z/2H)$

Table—4
Degree of consolidation
$U_1(T)$

T	$U_1(T)$
0.0010	0.035682
0.0015	0.043702
0.0020	0.050463
0.0030	0.061804
0.0040	0.071365
0.0050	0.079788
0.0060	0.087404
0.0070	0.094407
0.0080	0.100925
0.0090	0.107047
0.0100	0.112838
0.0150	0.138198
0.0200	0.159577
0.0300	0.195441
0.0400	0.225676
0.0500	0.252313
0.0600	0.276395
0.0700	0.298541
0.0800	0.319154
0.0900	0.338513
0.1000	0.356823
0.1500	0.436950
0.2000	0.504088
0.3000	0.613236
0.4000	0.697882
0.5000	0.763950
0.6000	0.815565
0.7000	0.855893
0.8000	0.887403
0.9000	0.912023
1.0000	0.931260
1.5000	0.979982
2.0000	0.994170
3.0000	0.999506
4.0000	0.999958
5.0000	0.999996
6.0000	1.000000

and corresponds to the Terzaghi-Fröhlich's solution. Infinite series $G_1(T, z/2H)$ and $G_2(T, z/2H)$ have been computed by TOSBAC-3400 and shown in Table 3 and Fig. 5.

The settlement of layer under the loading condition shown in Fig. 4 is

$$\left.\begin{aligned} S&=2Hm_v(s_1+s_2') && (0\leqq t\leqq t_3)\\ S&=2Hm_v(s_1+s_2+s_3') && (t_3\leqq t\leqq t_4)\\ S&=2Hm_v(s_1+s_2+s_3+s_4') && (t_4\leqq t\leqq t_5)\\ S&=2Hm_v(s_1+s_2+s_3+s_4) && (t\geqq t_5) \end{aligned}\right\} \qquad (19)$$

in which

$$\left.\begin{aligned} 2s_1&=(p_5+p_6)U_1(T)\\ 2s_2'&=(p_7+p_8)\frac{t}{t_3}U_0(T)\\ 2s_2&=(p_7+p_8)\frac{1}{t_3}\{tU_0(T)-(t-t_3)U_0(T-T_3)\}\\ 2s_3'&=\left(\frac{p_9}{t_4-t_3}+\frac{p_{10}}{t_5-t_3}\right)(t-t_3)U_0(T-T_3)\\ 2s_3&=\left(\frac{p_9}{t_4-t_3}+\frac{p_8}{t_5-t_3}\right)\{(t-t_3)U_0(T-T_3)\\ &\qquad -(t-t_4)U_0(T-T_4)\}\\ 2s_4'&=\frac{p_{10}(t-t_4)}{t_5-t_3}U_0(T-T_4)\\ 2s_4&=\frac{p_{10}}{t_5-t_3}\{(t-t_4)U_0(T-T_4)\\ &\qquad -(t-t_5)U_0(T-T_5)\} \end{aligned}\right\} \qquad (20)$$

$$U_1(T)=1-\frac{8}{\Pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n_2}e^{-n^2\Pi^2T/4} \qquad (21)$$

$U_1(T)$ is so-called degree of consolidation in the Terzaghi's theory of consolidation, values of which are tabulated in Table 4.

3. Consolidation under decreasing load

When a load decreases before a corresponding consolidation of a clay layer is completed, the following situation may occur: the effective pressure in a part of the layer continues to increase, whereas it decreases in the other part, hence the latter part of soil tends to swell or rebound. In such a case the compound phenomenon of consolidation and rebound should be considered.

The modulus of volume expansibility m_v' of the clay will be small by about one figure compared with the modulus of volume compressibility, and it will be dependent on the magnitude of pre-consolidation load and the over consolidation ratio. As far as the authors are aware, however, no established relationship between the above factors has been reported. Similarly much ambiguity exists in the coefficient of rebound c_v', which is a counterpart of the coefficient of consolidation c_v. If the coefficient of permeability does not change, however, the coefficient of rebound will become m_v/m_v' times the coefficient of consolidation. Then in this section it will be assumed,

$$\left.\begin{aligned} m_v' &= m_v/l \\ c_v' &= lc_v \end{aligned}\right\} \tag{22}$$

where l is a constant for particular soil.

In the consolidation phenomenon including rebound, the strain depends upon the stress history, so the choice of the strain as the dependent variable may not be relevant. The fundamental differential equation of consolidation including rebound, under an assumption of linear stress distribution, is given as a function of excess pore water pressure,

$$\frac{\partial u}{\partial t}=\begin{Bmatrix} c_v \\ lc_v \end{Bmatrix}\frac{\partial^2 u}{\partial z^2}+\frac{\partial p}{\partial t} \tag{23}$$

The choice of the coefficient depends on the rate of change in effective pressure, and may be written as follows,

$$\left.\begin{aligned} &\frac{\partial \bar{p}}{\partial t}\geqq 0 \quad \text{i.e.} \quad \frac{\partial^2 u}{\partial z^2}\leqq 0 \cdots c_v \\ &\frac{\partial \bar{p}}{\partial t}<0 \quad \text{i.e.} \quad \frac{\partial^2 u}{\partial z^2}>0 \cdots lc_v \end{aligned}\right\} \tag{24}$$

Under the loading condition shown in Fig. 6, the fundamental equation, the boundary conditions and the initial condition are represented respectively as follows

$$\frac{\partial u}{\partial t}=\begin{Bmatrix} c_v \\ lc_v \end{Bmatrix}\frac{\partial^2 u}{\partial z^2}-\frac{1}{t_2}\left\{p_3+(p_4-p_3)\frac{z}{2H}\right\} \tag{25a}$$

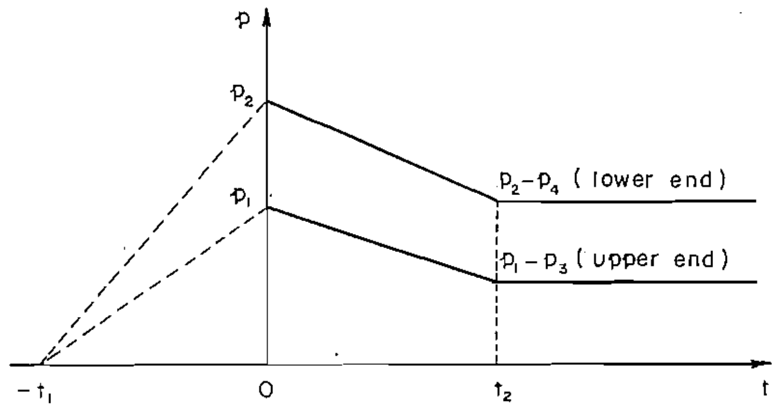


Fig. 6 Loading condition (3)

$$\left.\begin{aligned} u(0,t)&=0\\ u(2H,t)&=0 \end{aligned}\right\} \tag{25b}$$

$$u(z,0)=\frac{1}{T_1}\left\{\frac{p_1+p_2}{2}F_1(T_1,z/2H)-\frac{p_2-p_1}{2}F_2(T_1,z/2H)\right\} \tag{25c}$$

Since the above equation can not be solved analytically, a numerical method will be used. The whole layer of clay is to be divided into α slices as shown in Fig. 7. Eq. (25a) can then be written in the form of a finite difference equation as

$$\frac{\Delta u}{\Delta t}\begin{Bmatrix} c_v\\ lc_v\end{Bmatrix}\frac{u_{i-1}^j+u_{i+1}^j-2u_i^j}{\Delta z^2}-\frac{1}{t_2}\left\{p_3+(p_4-p_3)\frac{i}{\alpha}\right\} \tag{26}$$

The value of $\{c_v, lc_v\}\Delta t/\Delta z^2$, from which Δt is to be decided, should not be larger than 0.5, but an appropriate value in practice is about 0.25 (Mikasa, 1963). In practice it can not be acceptable to take different values of Δt for each slice, to which c_v or lc_v is applied. Also it is inconvenient that the values of Δt are different each other, when the settlement of several layers is computed. Then it may be advised that the value of Δt shall be so decided that $lc_v\Delta t/\Delta z^2$ is about 0.25 and the common multiple of Δt for each layer becomes an appropriate value.

The excess pore pressure at time $(t+\Delta t)$ can then be computed by the following equation, using the excess pore pressures at time t,

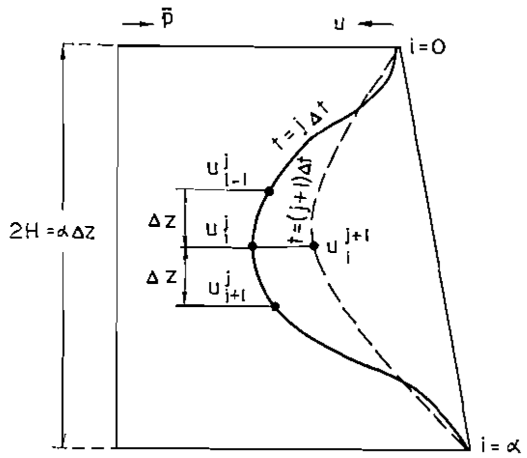


Fig. 7 Key sketch to numerical analysis

$$u_i^{j+1}=u_i^j+\begin{Bmatrix}\beta/l\\ \beta\end{Bmatrix}(u_{i-1}^j+u_{i+1}^j-2u_i^j)-\frac{1}{t_2}\left\{p_3+(p_4-p_3)\frac{i}{\alpha}\right\}\Delta t \tag{27}$$

in which

$$\beta=lc_v\Delta t/\Delta z^2\fallingdotseq 0.25 \tag{28}$$

i: grid number in z-axis, from 1 to $(\alpha-1)$
j: grid number in t-axis, from 0 to $t_2/\Delta t$

$$\left.\begin{aligned} u_0^j&=u_\alpha^j=0\\ u_i^0&=\frac{1}{T_1}\left\{\frac{p_1+p_2}{2}F_1(T_1,i/\alpha)-\frac{p_2-p_1}{2}F_2(T_1,i/\alpha)\right\} \end{aligned}\right\} \tag{29}$$

From Eq. (24) the choice of β/l or β becomes

$$\left.\begin{aligned} F&\leqq 0 \cdots \beta/l\\ F&>0 \cdots \beta \end{aligned}\right\} \tag{30}$$

in which

$$F=u_{i-1}^j+u_{i+1}^j-2u_i^j \tag{31}$$

In some cases, however, it will be possible for the effective stress in any slices to increase after having decreased. Such situations may be encountered when the rate of loading changes after a period of time, as shown in Fig. 6. In this case Eq. (30) alone is not sufficient. Assuming that the consolidation properties for reconsolidation are the same to those for rebound as far as the effective pressure does not exceed the pre-consolidation load, the choice may be extended approximately as $F\leqq 0$ and

$$3\bar{p}_i^j-\bar{p}_i^{j-1}\geqq 2\bar{p}_{i\,\max} \quad \cdots \; \beta/l \tag{32a}$$

$$3\bar{p}_i^j-\bar{p}_i^{j-1}<2\bar{p}_{i\,\max} \quad \cdots \; \beta \tag{32b}$$

$$F>0 \qquad\qquad \cdots \; \beta \tag{32c}$$

in which the effective pressure, $\bar{p}_i^j$, can be calculated by the following equation:

$$\begin{aligned}\bar{p}_i^j&=p_i^j-u_i^j\\&=p_1-\frac{p_3\Delta t}{t_2}\cdot j+\left\{p_2-p_1-\frac{\Delta t}{t_2}(p_4-p_3)j\right\}\frac{i}{\alpha}-u_i^j\end{aligned} \tag{33}$$

and $\bar{p}_{i\,\max}$ is the maximum value of $\bar{p}_i^j$ in $t\leqq\Delta t(j-1)$.

Applying the trapezoid formula, the settlement of the layer S^j is represented as follows

$$S^j=\frac{1}{2}(\varepsilon_0^j+\varepsilon_\alpha^j)\Delta z+\sum_{i=1}^{\alpha-1}\varepsilon_i^j\Delta z \tag{34}$$

in which the strain, ε_i^j should be different corresponding to the three states of slice, i.e. rebound, reconsolidation and normal consolidation, as follows

$$\varepsilon_i^j=m_v\bar{p}_i^j \qquad \text{\{in the slice obeying Eq. (32a)\}} \tag{35a}$$

$$\varepsilon_i^j=m_v\bar{p}_{i\,\max}-\frac{m_v}{l}(\bar{p}_{i\,\max}-\bar{p}_i^j) \tag{35b}$$

{in the slice obeying Eqs. (32b) and (32c)}

Since the strain, ε_0^j and ε_α^j, for the upper and lower boundary faces of the layer are always in the over-consolidated state, Eq. (34) may then be written as

$$S^j=m_v\Delta z\left[\frac{p_1+p_2}{2}-\frac{j\Delta t}{2lt_2}(p_3+p_4)+\sum_{i=1}^{\alpha-1}\left\{\begin{matrix}\bar{p}_i^j\\ \left(1-\frac{1}{l}\right)\bar{p}_{i\,\max}+\frac{\bar{p}_i^j}{l}\end{matrix}\right\}\right] \tag{36}$$

The third term of the right hand side of the above equation should be

$$\left.\begin{matrix}\sum_{i=1}^{\alpha-1}\bar{p}_i^j & \cdots \text{ for condition of Eq. (32a)}\\ \sum_{i=1}^{\alpha-1}\left\{\left(1-\frac{1}{l}\right)\bar{p}_{i\,\max}+\frac{\bar{p}_i^j}{l}\right\} & \cdots \text{ for condition of Eqs. (32b) and (32c)}\end{matrix}\right\} \tag{37}$$

The degree of consolidation, defined in the same way to that under increasing load, is represented by the trapezoid formula as follows

$$U^j=1-\frac{2}{\alpha\{p_1+p_2-(p_3+p_4)j\,\Delta t/t_2\}}\sum_{i=1}^{\alpha-1}u_i^j \tag{38}$$

When the negative pore pressure is prevalent within the layer, the degree of consolidation may become more than 100%.

After the load has become constant as shown in Fig. 6, the differential equation of consolidation including rebound is represented

$$\frac{\partial u}{\partial t}=\begin{Bmatrix} c_v \\ lc_v \end{Bmatrix}\frac{\partial^2 u}{\partial z^2} \tag{39}$$

And the fundamental equation for numerical analysis becomes

$$u_i^{j+1}=u_i^j+\begin{Bmatrix} \beta/l \\ \beta \end{Bmatrix}(u_{i-1}^j+u_{i+1}^j-2u_i^j) \tag{40}$$

in which

i: from 1 to $(\alpha-1)$

j: from $(t_2/\Delta t)$ to infinity

$$u_0^j=u_\alpha^j=0$$

$u_i^{t_2/\Delta t}$: by Eq. (27)

The effective pressure, the settlement and the degree of consolidation of the layer become

$$\bar{p}_i^j=p_1-p_3+(p_2-p_1-p_4+p_3)\frac{i}{\alpha}-u_i^j \tag{41}$$

$$S^j=m_v\Delta z\left[\frac{p_1+p_2}{2}-\frac{p_3+p_4}{2l}+\sum_{i=1}^{\infty}\begin{Bmatrix} \bar{p}_i^j \\ \left(1-\frac{1}{l}\right)\bar{p}_{i\,\max}+\frac{\bar{p}_i^j}{l} \end{Bmatrix}\right] \tag{42}$$

$$U^j=1-\frac{2}{\alpha(p_1+p_2-p_3-p_4)}\sum_{i=1}^{\alpha-1}u_i^j \tag{43}$$

Selection of term in { } in Eqs. (40) and (42) should be made after Eqs. (32) and (37).

The above method may be applied to the problem of more complicated condition.

4. Example

An example of the foregoing methods of analysis will be shown. Soil properties and loading conditions are assumed as follows,

$2H=40$ m	$p_1=26.4$ t/m^2	$p_3=15.4$ t/m^2
$m_v=8\times10^{-4}$ m^2/t	$p_2=44.0$ t/m^2	$p_4=30.8$ t/m^2
$c_v=1.44\times10^{-2}$ m^2/day	$t_1=3100$ day	$t_2=1460$ day

	m_v'	c_v'	Note
Case A	$m_v/10$	$10c_v$	numerical method
Case B	$m_v/50$	$50c_v$	〃
Case C	m_v	c_v	for the whole layer, analytical approximation
Case D	$m_v/10$	$10c_v$	〃 , 〃
Case A′	m_v	c_v	for increasing load, analytical method

Case A and B correspond to the foregoing numerical method and are computed by TOSBAC-3400.

Cases C and D are sort of analytical approximations for decreasing load. In Case C it is assumed that the soil properties are not affected by the mode of change in loading. The calculation formula for the settlement in this case is represented

$$\left.\begin{aligned} S&=2Hm_v(s_1-s_2') \qquad (t_1\leqq t\leqq t_1+t_2)\\ S&=2Hm_v(s_1-s_2) \qquad (t\geqq t_1+t_2)\end{aligned}\right\} \tag{43}$$

in which

$$\left.\begin{aligned} 2s_1&=(p_1+p_2)\frac{1}{t_1}\{tU_0(T)-(t-t_1)U_0(T-T_1)\}\\ 2s_2'&=(p_3+p_4)\frac{t-t_1}{t_2}U_0(T-T_1)\\ 2s_2&=(p_3+p_4)\frac{1}{t_2}\{(t-t_1)U_0(T-T_1)-(t-t_1-t_2)U_0(T-T_1-T_2)\}\end{aligned}\right\} \tag{44}$$

In Case D it is assumed that m_v' and c_v' under decreasing load are $m_v/10$ and $10c_v$, respectively, for the whole layer, and the calculation formula of the settlement becomes

$$\left.\begin{aligned} S=2Hm_v\Big[&\frac{p_1+p_2}{2}\cdot\frac{1}{lt_1}[\{t_1+l(t-t_1)\}U_0\{T_1+l(T-T_1)\}\\ &-l(t-t_1)U_0\{l(T-T_1)\}+(l-1)t_1U_0(T_1)]\\ &-\frac{p_3+p_4}{2}\cdot\frac{l(t-t_1)}{l^2t_2}U_0\{l(T-T_1)\}\Big] \qquad (t_1\leqq t\leqq t_1+t_2)\\ S=2Hm_v\Big[&\frac{p_1+p_2}{2}\cdot\frac{1}{lt_1}[\{t+(l-1)t_2\}U_0\{T+(l-1)T_2\}\\ &-\{t-t_1+(l-1)t_2\}U_0\{T-T_1+(l-1)T_2\}+(l-1)t_1U_0(T_1)]\\ &-\frac{p_3+p_4}{2}\cdot\frac{1}{l^2t_2}[\{t-t_1+(l-1)t_2\}U_0\{T-T_1+(l-1)T_2\}\\ &-(t-t_1-t_2)U_0(T-T_1-T_2)]\Big] \qquad (t\geqq t_1+t_2)\end{aligned}\right\} \tag{45}$$

The results of computation are shown in Fig. 8. As shown in the figure, the settlements estimated by the analytical approximation are found to be less than those by the numerical method.

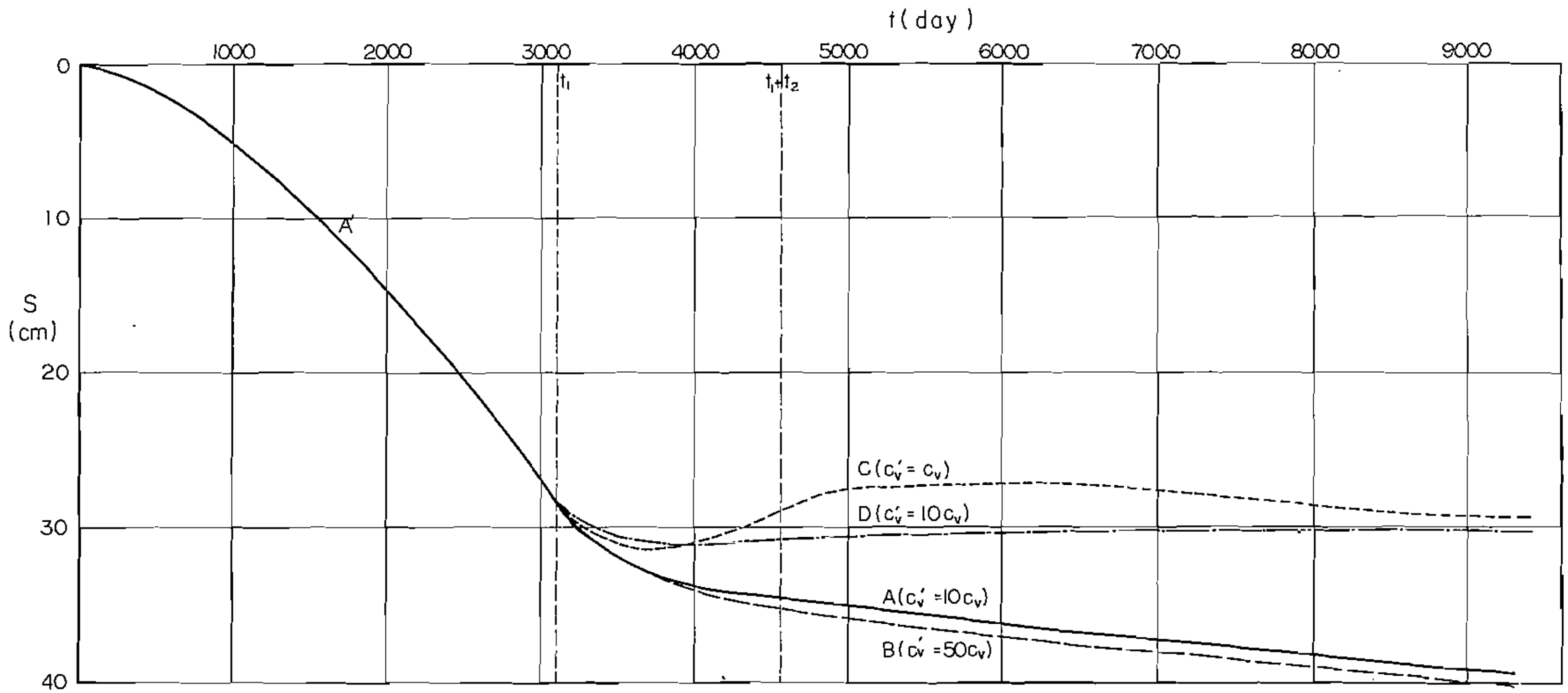


Fig. 8 Results of settlement computation by various methods

5. Summary

The results of the study described above may be summarised as follows;

1) An analytical solution of the consolidation under increasing load with the linear stress distribution within a clay layer was developed and the tabulated forms for computation of the compression strain, the excess pore water pressure, the degree of consolidation and the settlement were given, which will reduce much of labour in practical computation.
2) A numerical method of analysis of the consolidation under decreasing load was shown, assuming that the ratio of the coefficient of rebound to the coefficient of consolidation was constant regardless of the magnitude of preconsolidation load and over consolidation ratio. Numerical examples showed that the analytical approximations assuming consolidation properties as constant within the whole layer of soil, gave smaller settlement than those by the numerical method.

6. Acknowledgement

The authors wish to express their gratitude to Dr. Akio Nakase, Chief of the Soil Mechanics Laboratory, for his kind instruction.

References

1) Carslaw, H. S. and J. C. Jaeger (1959) Conduction of Heat in Solids (Oxford).
2) Mikasa, M. (1963) Consolidation of Soft Clay—A New Consolidation Theory and its Application—(Research Institute of Kashima Construction Co. Ltd., Publication Office) in Japanese.
3) Schiffman, R. L. (1963) Field Applications of Soil Consolidation under Time-dependent Loading and Varing Permeability (Highway Research Board, Bull. 248).
4) Terzaghi, K. and O. K. Fröhlich (1936) Theorie der Setzung von Tonschichten (Vienna, Deuticke).

List of Symbols

c_v : coefficient of consolidation

c_v' : coefficient of rebound

$f(z)$: function of depth z

$F=u_{i-1}^j+u_{i+1}^j-2u_i^j$

$F(T, z/2H)$: coefficient of strain (See Fig. 2 and Table 1)

$$F_1(T, z/2H)=\frac{16}{\Pi^3}\sum_{n=1,3,5}^{\infty}\frac{1}{n^3}(1-e^{-n^2\Pi^2T/4})\sin\frac{n\Pi z}{2H}$$

$$F_2(T, z/2H)=\frac{16}{\Pi^3}\sum_{n=2,4,6}^{\infty}\frac{1}{n^3}(1-e^{-n^2\Pi^2T/4})\sin\frac{n\Pi z}{2H}$$

$G(T, z/2H)$: coefficient of pore pressure (See Fig. 5 and Table 3)

$$G_1(T, z/2H)=\frac{4}{\Pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}e^{-n^2\Pi^2T/4}\sin\frac{n\Pi z}{2H}$$

$$G_2(T, z/2H)=\frac{4}{\Pi}\sum_{n=2,4,6}^{\infty}\frac{1}{n}e^{-n^2\Pi^2T/4}\sin\frac{n\Pi z}{2H}$$

$2H$: thickness of the clay layer

i : grid number in z-axis

j : grid number in t-axis

l : ratio of m_v to m_v' and of c_v' to c_v

m_v : modulus of volume compressibility

m_v' : modulus of volume expansibility

p : total pressure at a depth

$\bar{p}$: effective pressure at a depth

p_i^j : total pressure at the grid point (i, j)

$\bar{p}_i^j$: effective pressure at the grid point (i, j)

$\bar{p}_{i\,max}$: maximum value of $\bar{p}_i^j$ in $t\leqq\Delta t(j-1)$

p_1 : increasing pressure at the upper end of the layer (See Fig. 1)

p_2 : increasing pressure at the lower end of the layer (See Fig. 1)

p_3 : decreasing pressure at the upper end of the layer (See Fig. 6)

p_4 : decreasing pressure at the lower end of the layer (See Fig. 6)

p_5, p_6, p_7, p_8, p_9, p_{10}: (See Fig. 4)

s_1, s_2', s_2, s_3', s_3, s_4', s_4: (See Eqs. (19), (20), (43) and (44))

S : settlement

S_j : settlement at time $j\Delta t$

t : time

Δt : time increment

t_1 : period of load increasing (See Fig. 1)
t_2 : period of load decreasing (See Fig. 6)
t_3, t_4, t_5: (See Fig. 4)
T : time factor ($T=c_v t/H^2$)
T_1 : time factor at time t_1 ($T_1=c_v t_1/H^2$)
T_2, T_3, T_4, T_5: time factor at time t_2, t_3, t_4, t_5
u : excess pore water pressure
Δu : pore pressure increment
u_i^j : pore pressure at the grid point (i, j)
u_1 : pore pressure for an instantaneous loading
u_2', u_2, u_3', u_3, u_4', u_4: (See Eqs. (16) and (17))
U : degree of consolidation
U^j : degree of consolidation at time $j\Delta t$
$U_0(T)$: coefficient of consolidation (See Fig. 3 and Table 2)

$$=1-\frac{1}{T}\left(\frac{1}{3}-\frac{32}{\Pi^4}\sum_{n=1,3,5}^{\infty}\frac{1}{n^4}e^{-n^2\Pi^2T/4}\right)$$

$U_1(T)$: degree of consolidation for an instantaneous loading (See Table 4)

$$=1-\frac{8}{\Pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}e^{-n^2\Pi^2T/4}$$

z : depth
Δz : depth increment
α : divided number of the layer
$\beta=lc_v\Delta t/\Delta z^2$
ε : compression strain
ε_i^j : compression strain at the grid point (i, j)
$\phi(t)$: function of time t