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by

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A Note on a Correction Factor for the Pressure of Progressive Waves

by

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波高補正係数に関する一考察

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概　　要

水圧式波高計を使って測定された水中の圧力変動の振幅から進行波の波高を推定するためには，一般に微小振幅波理論による波圧の式が用いられ，実測値とのくいちがいを補正するために，次式で定義され，経験的に決定される n

$$n=\frac{\rho gH}{|\Delta p|}\frac{\cosh 2\pi S/L_1}{\cosh 2\pi d/L_1}$$

すなわち，微小振幅波理論によって算定される圧力変動の振幅と実際の圧力変動の振幅 $|\Delta p|$ との比が用いられている。各地の海岸における従来の観測結果によると，波高 H を使って微小振幅波理論によって算定された圧力変動の振幅は，圧力変動の振幅の実測値よりかなり大きくなっており，n は1.3〜1.5となることが報告されている。このように，算定値と実測値との間に大きな差異が見られるのは，n が波動の非線型性の効果のほかに，ランダムな波を有義波で代表させるという便宜的な方法を用いたことによる誤差を含んでいるためと思われる。

本報告では，前者の効果のみについて考察した。まず Skjelbreia の３次近似の速度ポテンシャルを使って水中圧力の一般式を求め，これをもとにして圧力変動の各調和成分の大きさと相対水深や波形勾配などの波の特性との関係について考察したほか，水面変動についても同様の検討を行った。水面変動については，さらに計算された各調和成分の大きさと，合田が水路実験で得たデータとの比較を行ない，相対水深 $d/L>1/7$ の範囲では，３次近似解は実験値とかなりよく合致することを確かめた。

以上の考察にもとづいて，進行波の非線型性の影響を考慮した波高補正係数の大きさについて具体的に評価したほか，算定値と浜田・光易・長谷の規則波を使った水路における実験値との比較を行ない，その結果，実験値は全体の傾向として算定値に則した変化をしているが，個々については実験値の方が少し大きくなることを見出した。

算定値および実験値の両者から判断して，波形の非線型性にもとづく n は，大きくても1.2〜1.25程度であって，水深と沖波々長との比が0.1より大きいときには，むしろ１より小さくなっている。したがって海岸における実測値が n=1.3〜1.5とかなり大きくなることは，波の非線型効果によっては説明できず，むしろこの影響は小さいと見るべきであって，n は現地波浪の統計的特性に大きく支配されると見るのが妥当と思われる。また，現地波浪の場合は，海底勾配の影響や表面波の干渉によって生じた束縛波が n を大きくさせる一要因となるであろうことを指摘した。

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A Note on a Correction Factor for the Pressure of Progressive Waves

by Fumiyasu HIROMOTO*

Synopsis

In order to reveal the nature of a correction factor n for the wave pressures, the effect of the nonlinearity of progressive waves on underwater pressuress was investigated. The generalized equations for wave pressures were derived from the wave theory of higher order approximation, and the magnitudes of harmonic components of wave presuress were determined as the functions of wave characteristics.

The value of n estimated from the above analysis and measured in a wave tank for regular trains of progressive waves does not exceed the value of 1.2~1.25 and even becomes smaller than 1.0 if the ratio of water depth to wavelength is larger than 0.1.

The actual values of the correction factor n measured at coastline are 1.3~1.5, however. Such the large values of n cannot be explained as due to the effect of wave nonlinearity. The statistical characteristics of actual waves are considered to govern the value of n greatly. The formation of bound waves by the interference of surface waves is also mentioned as a contributory cause for the increase of the value of n.

1. Introduction

In the field observation of sea waves, wave meters of underwater-pressure-type are often used. The measured pressures are converted to wave heights with the relationship of the small amplitude wave theory and with a correction factor n, which is determined empirically. The correction factor n is the ratio of the amplitude of pressure fluctuation calculated from the wave height by using the small amplitude wave theory to the actual amplitude $|\Delta p|$ of pressure fluctuation: i.e.

$$n=\frac{\rho g H}{|\Delta p|}\frac{\cosh 2\pi S/L_1}{\cosh 2\pi d/L_1} \qquad (1)$$

where L_1 is a wavelength determined by the small amplitude wave theory.

According to the results of the various observations hitherto made in the field, the correction factor n is reported to have a value around 1.3~1.5. Concerning the considerable disparity between the estimated and measured values of wave pressures, the following two reasons can be considered; first, the actual waves which are essentially nonlinear are approximated with the small amplitude waves, and second, the random waves in the field are tentatively represented with significant waves. Therefore, we need to investigate the effects of statistical properties of ocean waves as well

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as the effects of nonlinearity of waves. In this note, only the effect of nonlinearity of sea waves upon the correction factor n is estimated.

The precise estimation of this effect has still been left to the difficulty on account of the insufficiency in the order of approximation in the expression for finite amplitude progressive waves, but several higher order wave theories have been recently presented in succession. Accordingly, it became possible for us to make use of these results for the problem of the relationship between the surface profile and underwater pressure profile.

So, in this paper, first a general expression of wave pressure is derived from the third order solution of velocity potential obtained by Skjelbreia. Based on this result, the relationship between the pressure fluctuations and the wave properties such as relative water depth and wave steepness are discussed. Next, a similar discussion is made as to the surface elevations. Finally the magnitude of a correction factor for the wave pressure which results from the nonlinearity of wave motion is estimated.

2. Pressure Fluctuation

Skjelbreia[1] has obtained the third order velocity potential of progressive waves on the assumptions that the wave motion is irrotational and the fluid is inviscid and incompressible. With the aid of Bernoulli's equation, the general expression for wave pressures can be obtained from the velocity potential as follows:

$$\begin{aligned}\frac{p}{\rho} =\beta^{-1}\dot{C}_o^2\Big[&-\tfrac{1}{4}\lambda^2 A_{11}^2 \cosh 2\beta S+ \Big\{(\lambda A_{11}+\lambda^3 A_{11}C_1) \cosh \beta S \\ &-\lambda^3 A_{11}A_{22} \cosh 3\beta S\Big\}\cos\theta+\Big\{2\lambda^2 A_{22} \cosh 2\beta S - \tfrac{1}{4}\lambda^2 A_{11}^2\Big\}\cos 2\theta \\ &+\Big\{3\lambda^3 A_{33} \cosh 3\beta S-\lambda^3 A_{11}A_{22} \cosh \beta S\Big\} \cos 3\theta\Big] -gS+\text{const.}\end{aligned} \tag{2}$$

in which β is wave number of $2\pi/L$, $\lambda=\beta a$, and $\theta=\beta\ (x-\overline{C}t)$.

The co-ordinate system and description of the waves to be considered in this note are shown in Fig. 1. The parameter λ is related to the wave height H with the following equation:

$$\frac{\pi H}{d} = \frac{1}{d/L}\,(\lambda+\lambda^3 B_{33}) \tag{3}$$

Furthermore, wave celerity $\overline{C}$ is expressed as:

$$\beta\,\overline{C}^2 =C_o^2\,(1+\lambda^2 C_1), \quad C_o^2 =g \tanh \beta d \tag{4}$$

The coefficient A_{11}, A_{22}, ... , B_{33}, C_1 are the functions of relative water depth d/L; they are identical with those in the paper[2] of Skjelbreia and Hendrickson.

The constant in Eq. (2) should be determined from the surface condition, but it

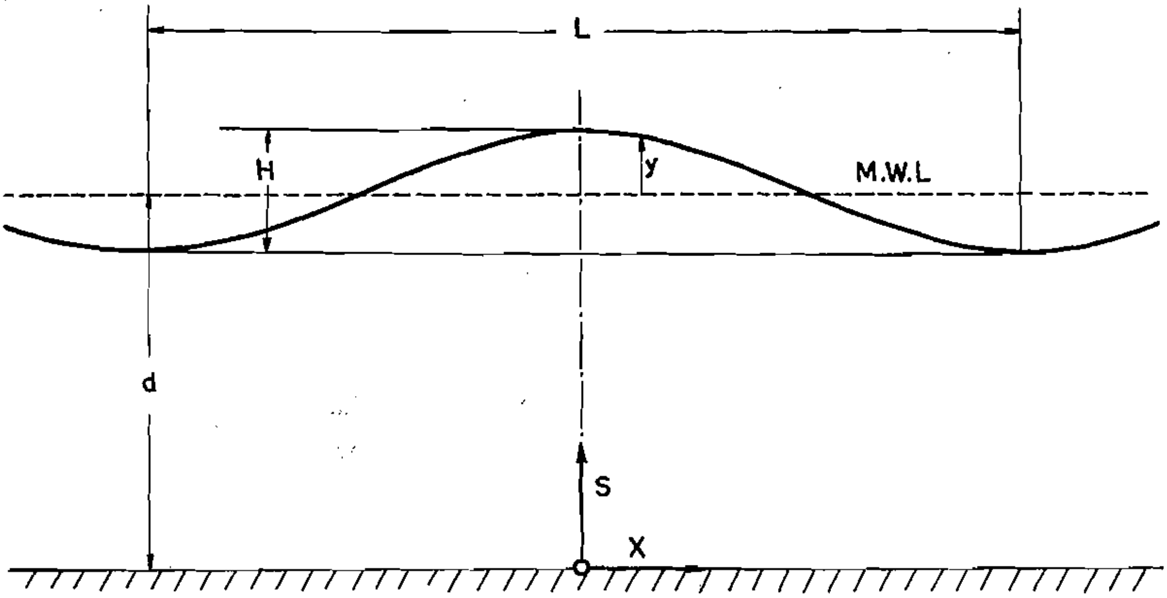

Fig. 1. Definition Sketch

is in principle identical with the constant in the second order approximation. Hence reffering to the second order solution[3)], the constant is found to be equal to (¼) $\beta^{-1}\, C_0^2\, \lambda^2\, A_{11}^2$. Consequently, if Eq. (2) is rewritten with H in Eq. (3) instead of λ, the following result is obtained:

$$\frac{2p}{\rho g H/\cosh \beta d}=\frac{2}{\rho g H/\cosh \beta d}\,(p_0+p_1\cos\theta+p_2\cos 2\theta+p_3\cos 3\theta) \qquad (5)$$

where: $$\frac{2p_0}{\rho g H/\cosh \beta d}=\frac{1}{4}\left(\frac{\pi H}{L}\right) A_{11}(1-\cosh 2\beta S)-\left(\frac{\pi H}{L}\right)^{-1}(\cosh \beta d)\ \beta S$$

$$\frac{2p_1}{\rho g H/\cosh \beta d}=\left\{1-\left(\frac{\pi H}{L}\right)^2(B_{33}-C_1)\right\}\cosh \beta S-\left(\frac{\pi H}{L}\right)^2 A_{22}\cosh 3\beta S$$

$$\frac{2p_2}{\rho g H/\cosh \beta d}=2\left(\frac{\pi H}{L}\right)\frac{A_{22}}{A_{11}}\cosh 2\beta S-\frac{1}{4}\left(\frac{\pi H}{L}\right)A_{11}$$

$$\frac{2p_3}{\rho g H/\cosh \beta d}=3\left(\frac{\pi H}{L}\right)^2\frac{A_{33}}{A_{11}}\cosh 3\beta S-\left(\frac{\pi H}{L}\right)^2 A_{22}\cosh \beta S$$

Based on the Eq. (5), the relationship between each harmonic component of pressure fluctuations and wave properties such as relative water depth and wave steepness were calculated and shown in Figs. 2 and 3. Figure 2 shows the relationship at the bottom and Fig. 3 at the still water level. From these results it can be seen that with the decrease of the relative water depth the fundamental component becomes smaller while the second and third harmonics become larger, both at the bottom and at the still water level. In the case of waves in comparatively deep water, the fundamental component increases with the increase of H/L, but the trend is reversed in the case of comparatively shallow water waves. In the case of wave pressures at the bottom, for instance, the fundamental component increases when $d/L>0.15$ and decreases when $d/L<0.15$ as the wave steepness increases; it does not depend on H/L when $d/L\approx 0.15$.

Comparing Fig. 2 with Fig. 3, we can see that the second and third harmonics in the wave pressure at the bottom are smaller than those at the still water level, and thus pressure profiles at the bottom approach to sinusoidal forms.

If we examine the effects of each harmonic component on the pressure profiles, it is seen that the second harmonic components raise the pressure profiles at the crests and troughs from the sinusoidal forms of the fundamental components and lower the portions between the crests and troughs, because of $p_2>0$ in general. But in the case of waves in comparative deep water where d/L is large, these effects of

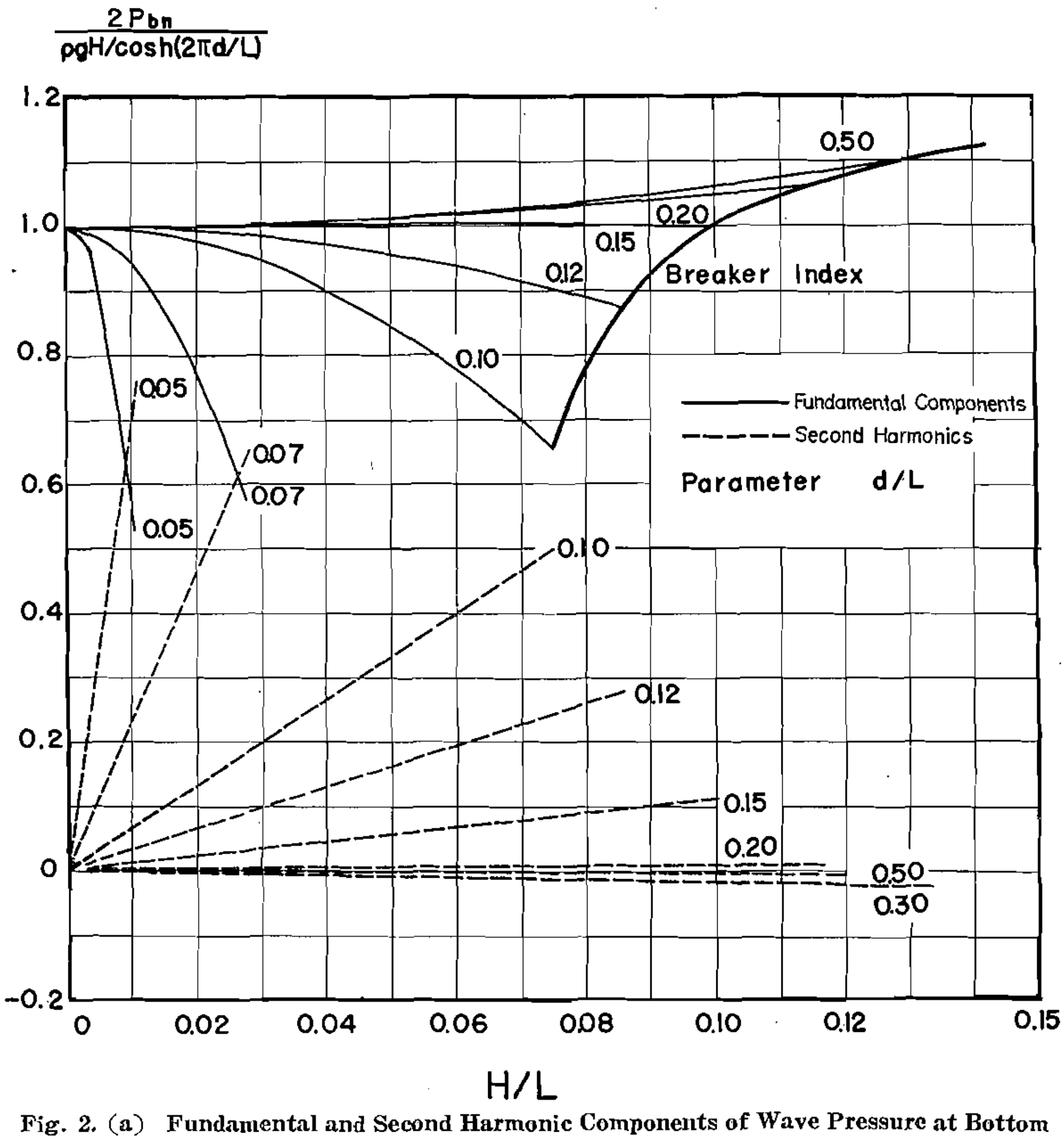

Fig. 2. (a) Fundamental and Second Harmonic Components of Wave Pressure at Bottom

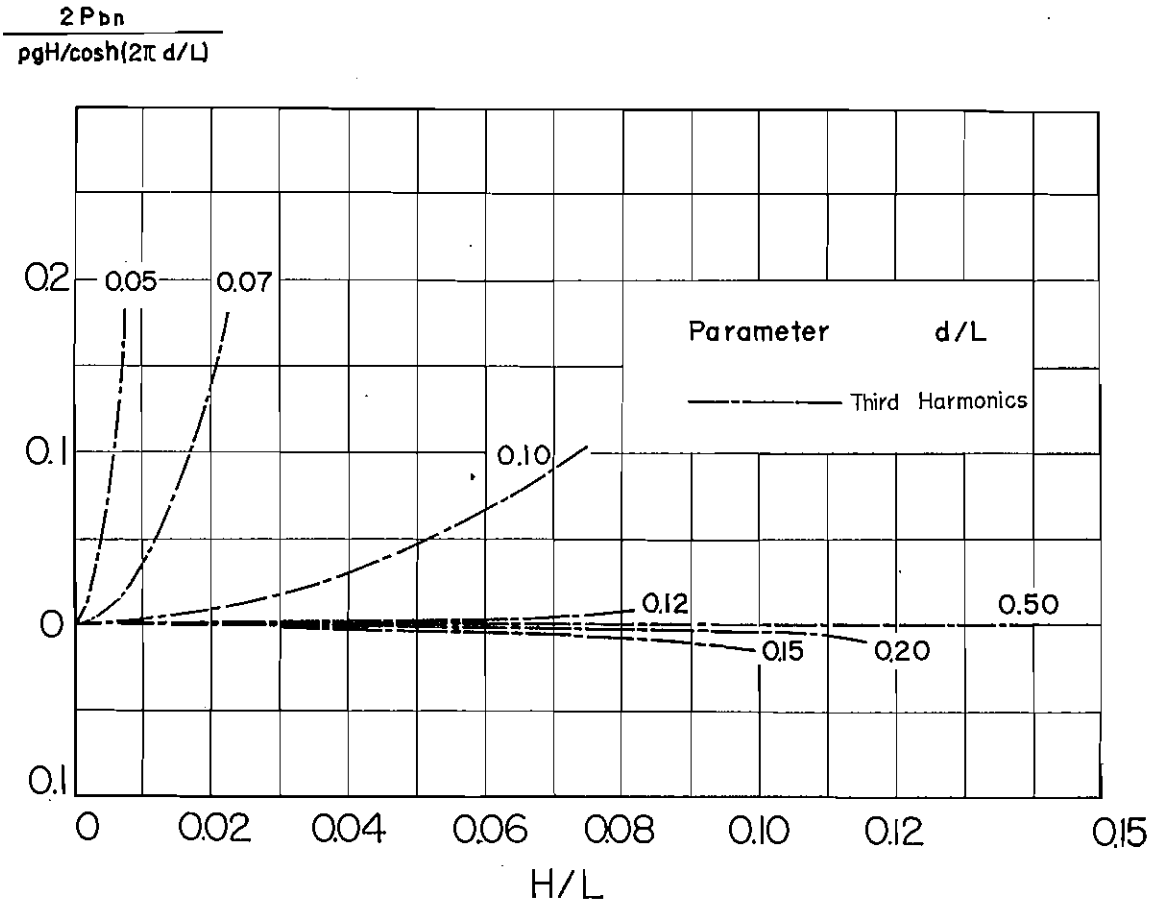

Fig. 2. (b) Third Harmonic Component of Wave Pressure at Bottom

the second harmonics are reversed near the bottom because of $p_2<0$. The third harmonics lower the crests and raise the troughs of the sinusoidal pressure forms at the bottom because of $p_3<0$. The amplitudes of third harmonics, however, are relatively small, and there are no double-humped profiles which appear often in standing waves, because the condition $\frac{\partial^2 p}{\partial \theta^2}\Big|_{\theta=0}\geq 0$, i.e., $p_1+4p_2+9p_3\leq 0$, is satisfied in no case. Consequently the maximum pressure occurs always at the phase of a wave crest.

3. Surface Elevation

The expression for the surface elevation obtained by Skjelbreia is

$$\beta y=\lambda \cos\theta+\lambda^2 B_{22}\cos 2\theta+\lambda^3 B_{33}\cos 3\theta \tag{6}$$

The above expression is rewritten with Eq. (3) as:

$$\frac{2y}{H}=\frac{2}{H}\quad(A_1\cos\theta+A_2\cos 2\theta+A_3\cos 3\theta) \tag{7}$$

where: $\frac{2A_1}{H}=1-\left(\frac{\pi H}{L}\right)^2 B_{33},\quad \frac{2A_2}{H}=\left(\frac{\pi H}{L}\right) B_{22},\quad \frac{2A_3}{H}=\left(\frac{\pi H}{L}\right)^2 B_{33}$

The amplitudes of harmonic components calculated from Eq. (7) were compared with the measured values[5] which were obtained by Goda in a wave channel (Fig. 4). Generally the measured values agree closely with the theoretical curves, but the discrepancy between the theory and experimental data increases in the range of small relative depth. As already pointed out by other investigators, Eq. (7) seems not applicable to the finite amplitude waves in the range where d/L is smaller than 1/7.

As shown in Fig. 4, the fundamental component of surface elevation decreases and second harmonic and third harmonic increase with the decrease of d/L and with the increase of H/L. The relative water depth d/L affects little the amplitude of each harmonic component if d/L is larger than about 0.3.

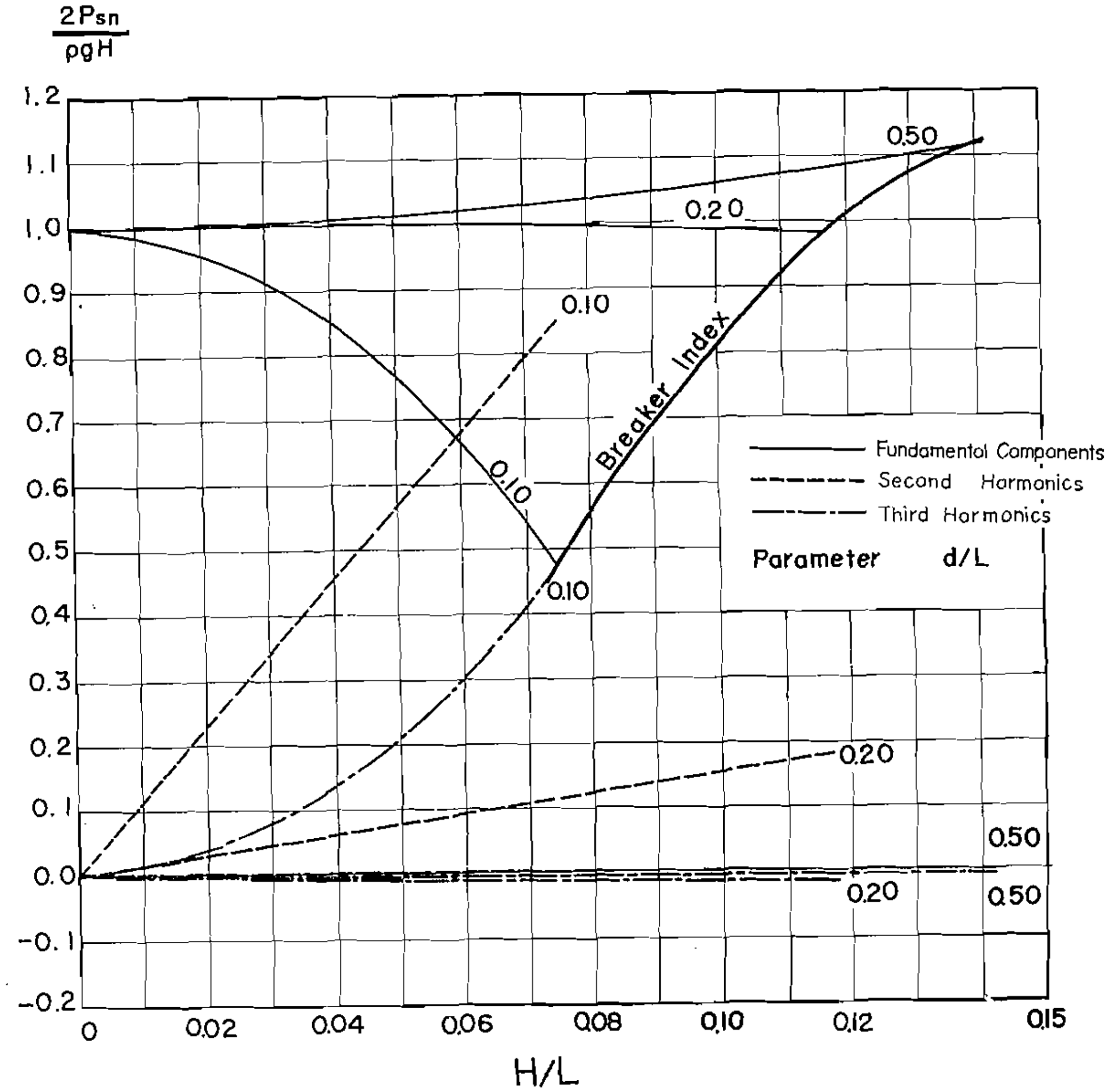

Fig. 3. **Harmonic Components of Wave Pressure at Still Water Level**

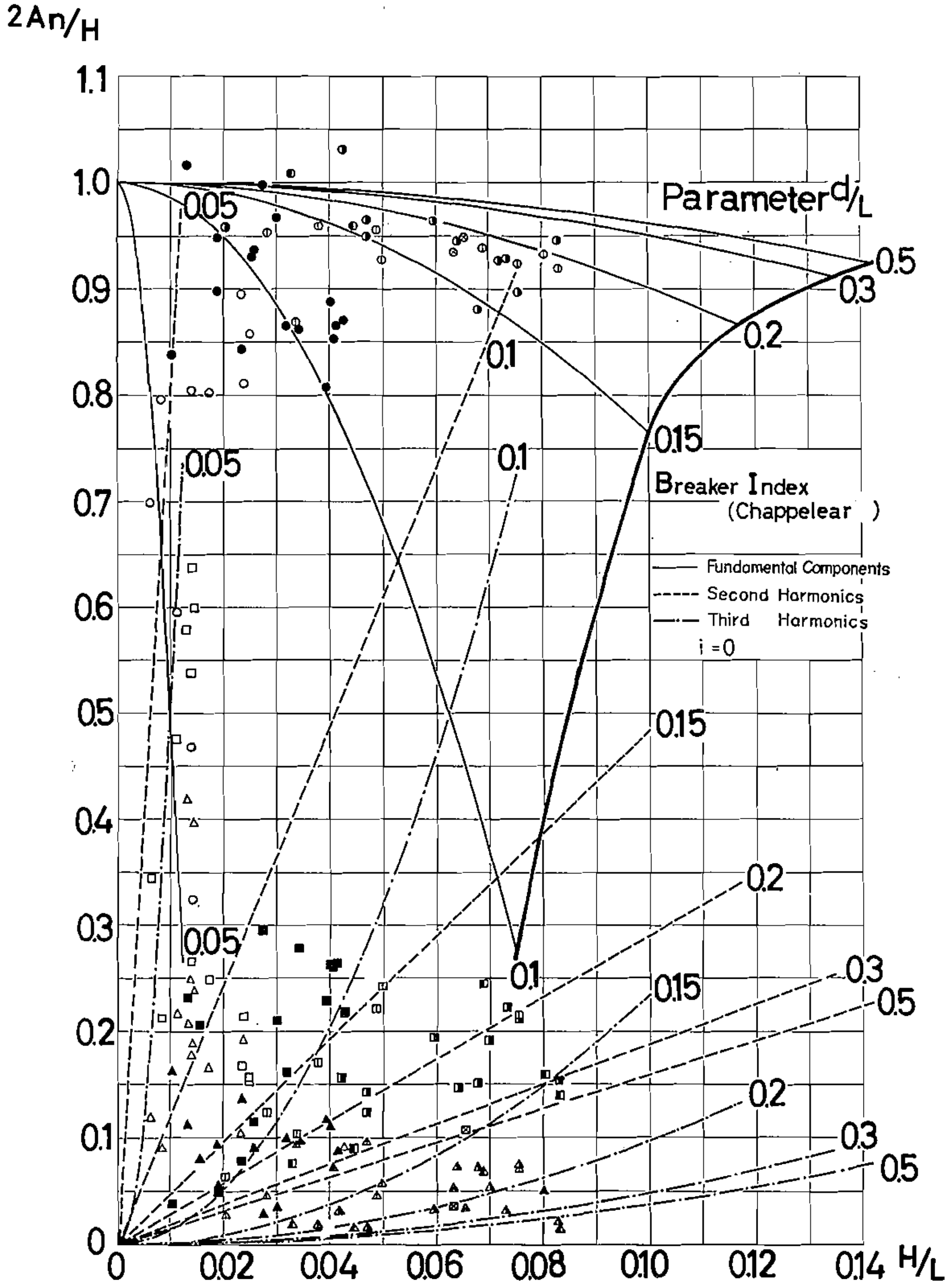

Fig. 4. Comparison of Harmonic Components of Surface Elevation with Experimental Data of Goda

d/L		Fundamental Components	Second Harmonics	Third Harmonics
0.35 ≦		⊗	⊠	◬
0.25 ≦	< 0.35	◐	◧	◭
0.175 ≦	< 0.25	◑	◨	◮
0.125 ≦	< 0.175	⦶	◫	◬
0.075 ≦	< 0.125	●	■	▲
	< 0.075	⦶	□	△

4. Correction Factor for the Pressure

As described above, the pressure fluctuation and surface elevation are affected considerably by the presence of higher order terms. Therefore, the pressure profile estimated by the small amplitude wave theory must be considerably different from the actual profile, and a correction factor n is expected to be remarkably affected by the wave properties.

From Eq. (5) the amplitude of pressure fluctuation at the bottom becomes

$$|\Delta p_b| = \frac{\rho g H}{\cosh 2\pi d/L}\left[1+\left(\frac{\pi H}{L}\right)^2\left(-2A_{22}+3\frac{A_{33}}{A_{11}}-B_{33}+C_1\right)\right] \qquad (8)$$

From this expression the correction factor due to nonlinearity of wave is obtained as:

$$n = \frac{\cosh 2\pi d/L}{\cosh 2\pi d/L_1}\left[1+\left(\frac{\pi H}{L}\right)^2\left(-2A_{22}+3\frac{A_{33}}{A_{11}}-B_{33}+C_1\right)\right]^{-1} \qquad (9)$$

Figure 5 shows the relationship between the wave properties represented with H/L_0 and d/L_0 and the correction factor n calculated on the basis of Eq. (9), where L_0 is a wave length in deep water. It is seen from Fig. 5 that the correction factor n decreases with the increase of d/L_0 and H/L_0 when d/L_0 is larger than 0.1. In the range of $d/L_0 < 0.1$, the factor n increases greatly with the decrease of d/L_0, but n is larger in the steeper wave in contrast to the trend in the range of $d/L_0 > 0.1$. The correction factor n is nearly equal to 1.0 at $d/L_0 \approx 0.1$ without being affected by H/L_0.

In Fig. 5 the data of a laboratory experiment[6] are also shown. Comparing them with the theoretical values, it can be seen that the tendency of the observed values are the same as that of calculated ones, but the observed values, as a whole, are slightly larger than the calculated ones. Unfortunately, the number of experimental data is rather small and the data are concentrated in the range of small relative depth where the accuracy of the third order solution is not good. Figure 5 also shows the experimental data on inclined beds (1/15~1/5) as well as the data on the horizontal bed. The values of the correction factor n obtained from the data on inclined beds are considerably larger than those obtained in horizontal bed, and n seems to be strongly affected by the inclination of bed. This fact suggests that the large value of n observed in the actual coastal area may be partly produced by the effect of bottom inclination.

Both the calculated and experimental values shown in Fig. 5 indicate that the

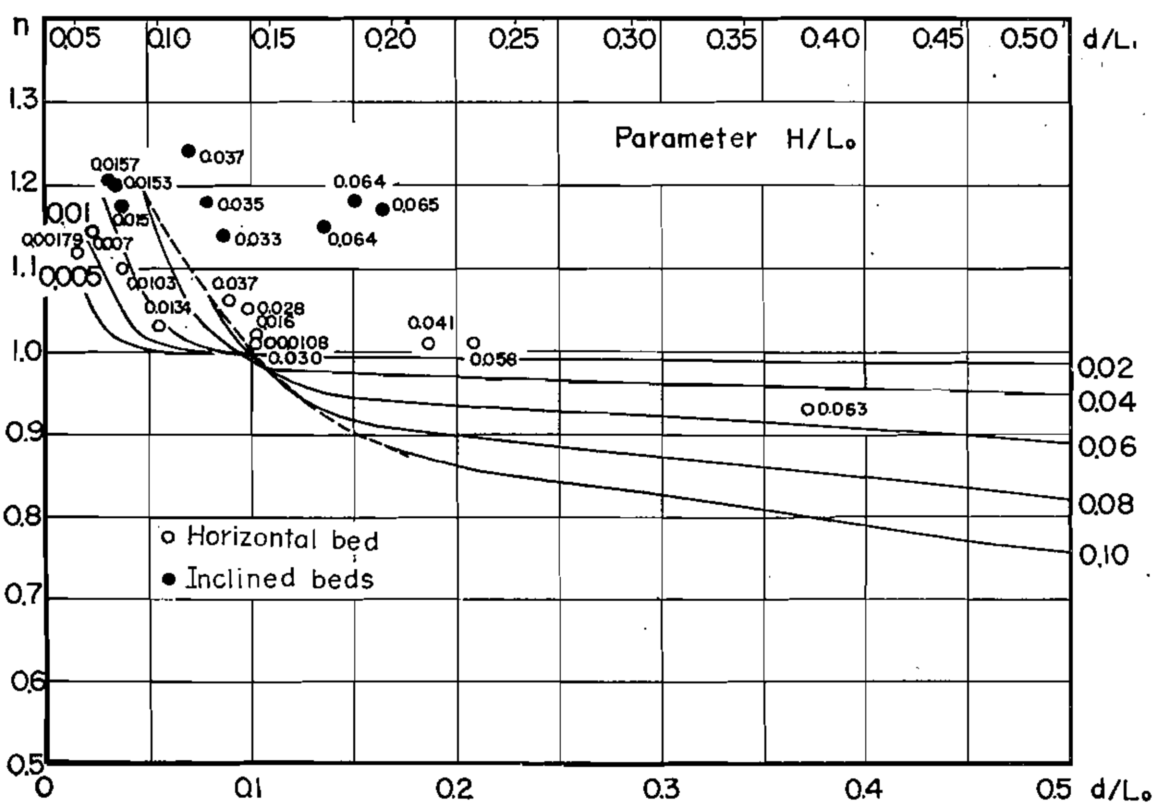

Fig. 5. Correction Factor n as a Function of wave Characteristics with Experimental Data of Hamada et. al.

correction factor for the pressure due only to the nonliner properties of regular waves is 1.2~1.25 at most and becomes smaller than 1.0 if $d/L_0>0.1$. Therefore, the fact that n measured in the actual shore is fairly large with the value of $n=1.3\sim1.5$ cannot be explained by the nonlinearity of wave motion only, even if the influence of the inclination of sea-bed is considered. The scatter of the correction factor n reported by various observers at different observation sites can partly come from the following situations; the value of n is affected by the statistical properties of ocean waves which may vary with respect to the time and place of observations. Further, the existence of bounded waves generated by the interference of surface waves may be a contributory cause for the increase of the value of n, since the value of n of standing waves is larger than that of progressive waves[7].

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