REPORT

OF

PORT AND HARBOUR RESEARCH INSTITUTE

REPORT NO. 13

Laboratory Investigation on Wave Transmission over Breakwaters

by

Yoshimi GODA, Hideaki TAKEDA and Yoshiichi MORIYA

A Note on a Correction Factor for the Pressure of Progressive Waves

by

Fumiyasu HIROMOTO

April 1967

PORT AND HARBOUR RESEARCH INSTITUTE
MINISTRY OF TRANSPORT

1-1, 3 Chome, Nagase, Yokosuka-City, Kanagawa-Prefecture, Japan

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概　　要

越波の打ち込みによって防波堤の背後水域へ伝達される波高について，広範囲な実験を行ない，直立堤および混成堤について次の実験式を得た。

$$\frac{H_T}{H_I}=0.5\left[1-\sin\frac{\pi}{2\alpha}\left(\frac{R}{H_I}+\beta\right)\right]$$

たゞし，H_Tは伝達波高，H_Iは入射波高，Rは静水面上の天端高で，係数αは2.0，係数βは0.1～0.5の値である。また伝達波の波形分析から，伝達波は周期T，$T/2$，$T/3$，. . . の波列が合成されたもので，各波列はそれぞれの波速で進行することが明らかにされた。

なお附録として，造波水路における二次波峯の移動現象について実験例を紹介した。

Laboratory Investigation on Wave Transmission over Breakwaters

by Yoshimi GODA*, Hideaki TAKEDA**
and Yoshiichi MORIYA**

Synopsis

Series of laboratory tests have been conducted to investigate the wave heights transmitted over vertical wall and composite breakwaters by wave overtopping. The ratio of transmitted wave height to the incident wave height was found to be best expressed with the following equation for ordinary shapes of breakwaters with the crown height of R above the mean water level:

$$\frac{H_T}{H_I}=0.5\left[1-\sin\frac{\pi}{2\alpha}\left(\frac{R}{H_I}+\beta\right)\right]$$

where the factors α and β have the values of 2.0 and 0.1 through 0.5, respectively. The Fourier analysis of wave profiles transmitted behind a model breakwater revealed the re-generation of transmitted waves as the superposition of a number of wave trains which have the periods of T, $T/2$, $T/3, \ldots$ and propagate with the different celerities corresponding to their own periods.

1. Introduction

One of problems in the design of breakwaters is the determination of its crown height, which will prevent the transmission of waves onto the harbor basin and yet will not cause an excessive construciton cost. A raise of the crown height must be decided upon the evaluations of the gain by the decrease in wave transmission and the loss by the increase in construction cost. Such an evaluation requires a reliable information on the relation between the crown height of breakwater and the heights of transmitted waves. The information is also important for submerged breakwaters which are built to protect shores from beach erosions, because the controle of wave characteristics in front of the shores is critical for their protection.

The first laboratory data on this problem were presented by Johnson et al. (1951), primarily for submerged breakwaters. The data were rearranged by Japanese engineers so as to produce a design diagram for the wave transmission ratio, or the ratio of transmitted wave height to the incident wave height, for submerged and emerged breakwaters (J.P.H.A. 1959). Although the diagram has been utilized by Japanese engineers since then, it is not a reliable one because of large scattering of data and little information about the effect of the crown width upon the wave transmission ratio. Later, Hosai and Tominaga (1959) and Kondo and Sato (1963) presented their own laboratory data; the latter proposed an experimental formula for

*) Chief of Model Test Section, Hydraulics Division, Sc. M.
**) Member of Breakwater Laboratory, Hydraulics Division.

the wave transmission ratio. Recent experiments of Nakamura et al. (1966) concerned with submerged dikes of large widths. Their experimental conditions are rather limited, however, thus making it difficult to draw the general conclusions on the problem of wave transmission over breakwaters.

A series of experiments have been undertaken at the Port and Harbour Research Institute since 1961 in order to obtain the systematic laboratory data on the wave transmission ratio and to construct a reliable design diagram for harbour engineers. Since most of breakwaters in Japan are of composite type, tests have been made with models of vertical wall and composite breakwaters. The present report describes the results of these laboratory investigations.

2. Experimental Arrangements

The experiments differred from others at two aspects: firstly, the use of a wave basin with four inside channels, and secondly, the controle of wave characteristics, especially of relative water depth h/L throughout the experiments.

The four test channels, each 0.48 m wide and 16.5 m long, were set up in a wave basin of 20 m wide and 30 m long as shown in Fig. 1. At the shore-side ends of test channels and basin itself, wave absorbers of permeable type which were composed of rubble stones and shavings of stainless steel were provided; the water could flow through the absorbers from the channels to the basin and vice versa.

This arrangement had the following advantages over the conventional closed channel system:

1) Since the waves reflected from model breakwaters are dispersed toward the broad area of the wave basin, experiments can be continued without invoking serious multi-wave reflections in the wave basin.
2) Since the water area of the basin is large and the both ends of each test channel are open, the water level at the sea side and harbor side of a model breakwater are maintained at the same heights during a run.
3) With four test channels available, four model breakwaters can be tested at the same time for one wave condition.

The first characteristic, however, could not be fully exploited, because there still existed some secondary wave reflections from the side walls of the basin and others; the wave conditions in test channels were slightly different from one channel to others. In the later phase of experiments, a wave absorbing mound of 4 m wide and 11 m long was provided at the center of the basin and only one test channel was used in order to assure the better controle of test conditions.

As for the controle of wave characteristics, the wave period was so adjusted that the relative water depth h/L would be one of the following values:

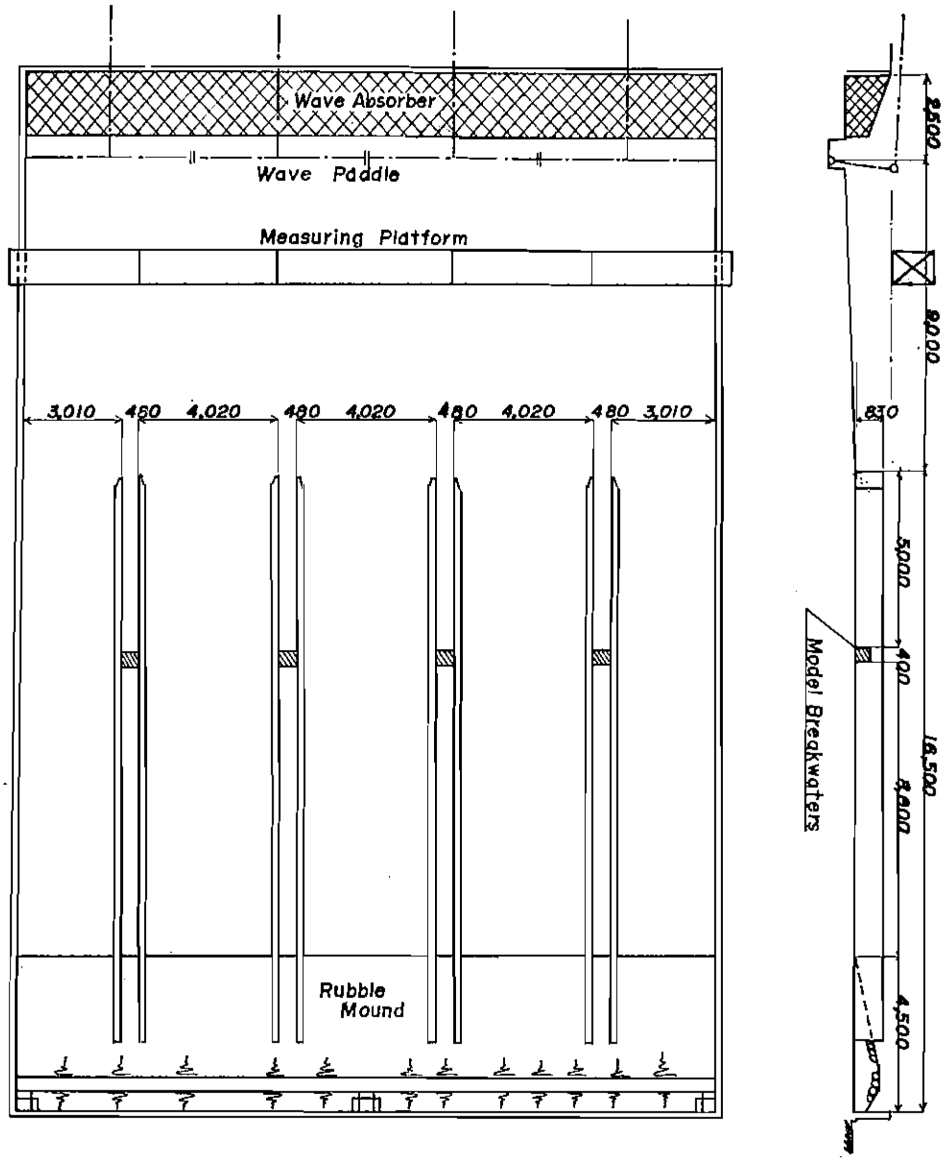

Fig. 1. Experimental Set-up

For the water depth of $h = 50$ cm, $h/L = 0.14$, 0.2, 0.3, and 0.5;
For the water depth of $h = 35$ cm, $h/L = 0.07$, 0.10, and 0.14.

The incident wave height H_I was varied gradually from 3 to 30 cm for each relative water depth. All the test conditions are listed on Tables 1 and 2.

Model breakwaters were located at a distance of 5 m from the tip of each test channel; the distance guaranteed the lengths of more than one wavelength for the sea side and more than two wavelengths for the harbor side for the measurement of wave height.

In every run, offshore and onshore wave envelopes were recorded on a pen-writing oscillograph with resistance type wave gages attached to a measuring platform which moved at a constant speed of 2 m/min. Since model breakwaters produced some reflected waves which were superimposed upon incident waves, the heights of incident and reflected waves were calculated with the conventional method based on the small amplitude wave theory; the average of the maximum and minimum wave heights located half-wavelength away was taken as the incident wave heights H_I and the one-half difference of the maximum and minimum heights was taken as the reflected wave heights H_R. Although this height H_R gives a measure of reflect waves, it is smaller than the actual height of reflect waves because of finite amplitude effect (see Goda and Kakizaki 1966). The height of transmitted waves H_T which were produced by wave overtopping was taken as the average value over the measurement distance from the rear of model breakwater to the vicinity of wave absorbers, because the transmitted wave height varied slightly from location to location as will be discussed in Section 3(5).

In the problem of wave transmission due to wave overtopping over composite breakwaters, the following seven quantities govern the phenomenon principally: the incident wave height H_I, the transmitted wave height H_T, the water depth h, the incident wavelength L, the crown height of breakwater above the mean water level R, the crown width B, and the water depth above the top of foundation mound d. All these quantities have the dimension of length. According to the Pi-theorem, the problem can be expressed as the relation among six (seven minus one) non-dimensional parameters. Among several possibilities, the following non-dimensionarization was adopted in the present study:

$$K_T = \frac{H_T}{H_I} = \phi\left(\frac{R}{H_I}, \frac{B}{h}, \frac{d}{h}, \frac{h}{L}, \frac{H_I}{h}\right) \tag{1}$$

As will be seen in the following sections, Eq. 1 is considered to be the most practical form to express the ratio of wave transmission over composite breakwaters.

3. Wave Transmission over Vertical Wall Breakwaters

(1) Model Brekwaters

Models of vertical wall breakwaters were made of wooden boxes with balance weight inside of them. The width was fixed at B=40 cm after investigations of actual breakwater designs, but the height was varied from 25 to 80 cm for ten models tested. With an application of model scale of 1/30, these dimensions are equivalent to the breakwaters of 12 m wide and 7.5 to 24 m high at the water depth of 10.5 to 15 m. With the same model scale, the incident waves are translated to the waves of 0.9 to 9 m high with periods of 4.4 to 15.2 seconds.

In addition to these model breakwaters, a thin vertical wall made of two steel plates of 9 mm thick was employed in order to investigate the effect of breakwater width upon the wave transmission ratio. The vertical wall was fixed at the channel bottom with anchor bolts and its height could be varied by changing the positions of bolts connecting the two plates.

(2) **Selection of Appropriate Parameter for Crown Height**

The first question encountered in the analysis of experimental data was the selection of the parameter for the crown height of breakwaters. Johnson et al. (1951) employed the ratio of total height to water depth, $(1+R/h)$, but the Design Manual for Harbour Construction Works in Japan (1959) uses the ratio of the crown height above the mean water level to the wave height, R/H, for a design diagram of wave transmission ratio. Of these two parameters, the one which makes the effects of other

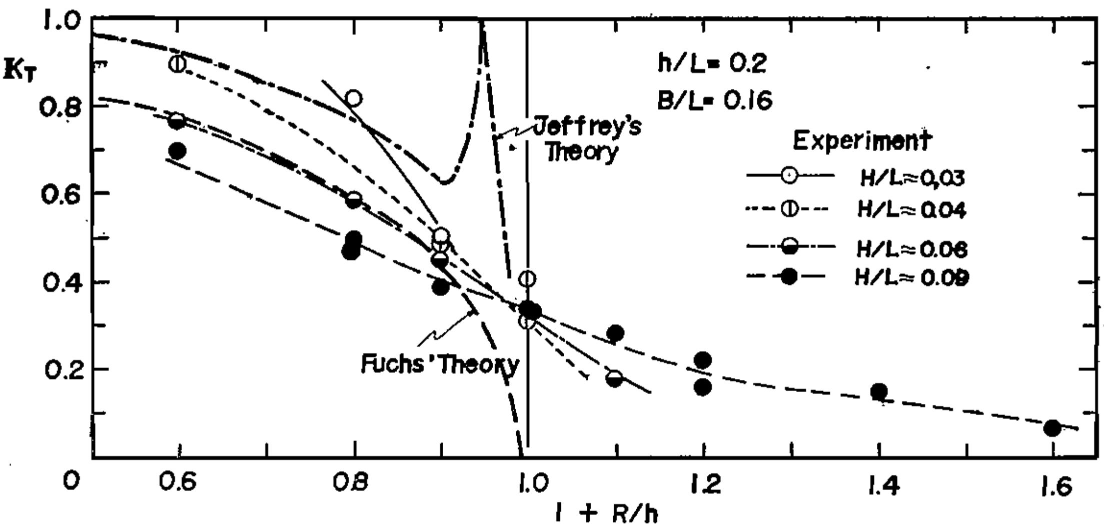

Fig. 2. Wave Transmission Ratio as a Function of Total Breakwater Height

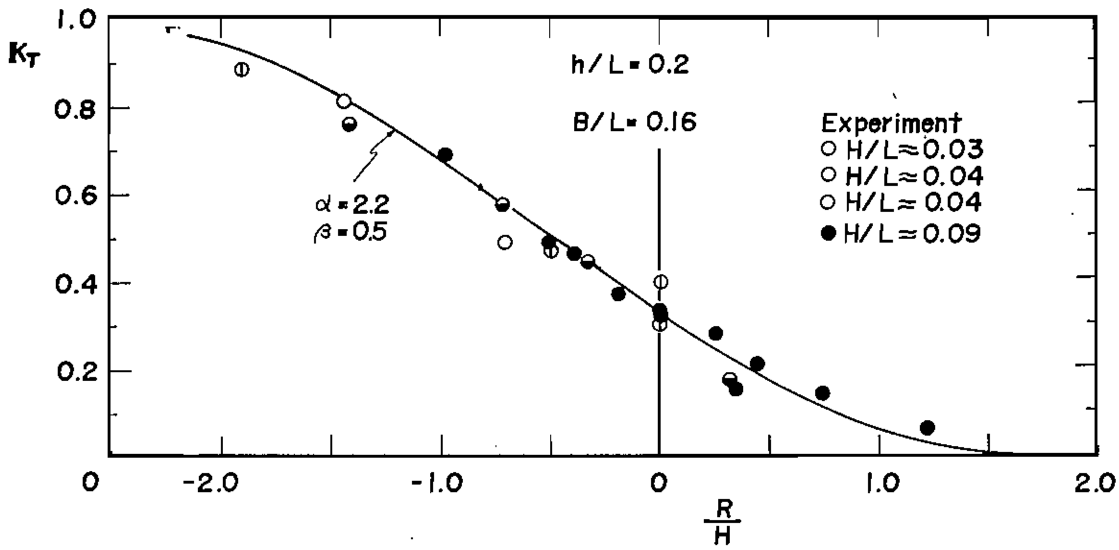

Fig. 3. Wave Transmission Ratio as a Function of Relative Crown Height

factors minimum should be selected. Figures 2 and 3 show the differences in the presentation of experimental data with the parameters of $(1+R/h)$ and R/H. The same data for the wave condition of $h/L=0.2$ (Case III of Table 1) are presented for the purpose of comparison.

In Fig. 2 with the parameter of $(1+R/h)$, the wave transmission ratio seems to be affected also by the wave steepness in such a manner that the wave transmission ratio decreases for submerged breakwaters as the wave steepness increases, but the ratio increases for emerged breakwaters as the wave steepness increases. In Fig. 3 with the parameter of R/H, however, such an effect of wave steepness does not appear. All the experimental data gather themselves around the following experimental formula:

$$K_T = 0.5\left[1-\sin\frac{\pi}{2\alpha}\left(\frac{R}{H_I}+\beta\right)\right] \tag{2}$$

in which the best fitting values of the factors α and β are 2.2 and 0.5, respectively. Equation 2 clearly indicates that the ratio R/H is the predominant factor for the phenomenon of wave transmission by wave overtopping. In Fig. 2, the theoretical values of wave transmission ratio by Fuchs and by Jeffery, both of which were introduced in the paper by Johnson et al. (1951), are shown as well as the experimental data. But the agreement with the experimental data is not good; especially the theories cannot predict the wave transmission over emerged breakwaters which have the transmission ratio of as much as 40%. The discrepancy between the theories and experiments is rather expected, because the theories presume the continuity of small amplitude waves over a breakwater, while the actual phenomenon is better explained as the wave re-generation process by the impact of overtopped or overflown water mass; the predominant role of the ratio R/H in the wave transmission ratio illustrated in Eq. 2 also indicates that the assumption of small amplitude is impractical.

(3) Variation of Wave Transmission Ratio with Respect to Wave Characteristics

All the experimental data which are listed in Table 1 were analysed in the same way as in Fig. 3 for each relative water depth. Equation 2 was then applied for each one to yield the following best fitting values of the factors α and β:

$$\left.\begin{array}{llllll} h/L=0.5 & \alpha=2.2 & \beta=0.7 & h/L=0.14 & \alpha=1.8 & \beta=0.4 \\ h/L=0.3 & \alpha=2.2 & \beta=0.5 & h/L=0.10 & \alpha=1.8 & \beta=0.5 \\ h/L=0.2 & \alpha=2.2 & \beta=0.5 & h/L=0.07 & \alpha=1.8 & \beta=0.3 \end{array}\right\} \tag{3}$$

The above results indicate a slight difference in the nature of wave transmission ratio for the relatively deep water waves of $h/L=0.2\sim0.5$ and for the relatively shallow water waves of $h/L=0.07\sim0.14$. This difference results in a little larger ratio of wave transmission for the latter waves than for the former in the case of submerged breakwaters. The difference in the wave transmission ratio is very small, however, in the case of emerged breakwaters, which are more important for the protection of harbors than submerged ones. From a practical point of view, the effect of relative water depth is considered negligible when the wave transmission ratio is expressed

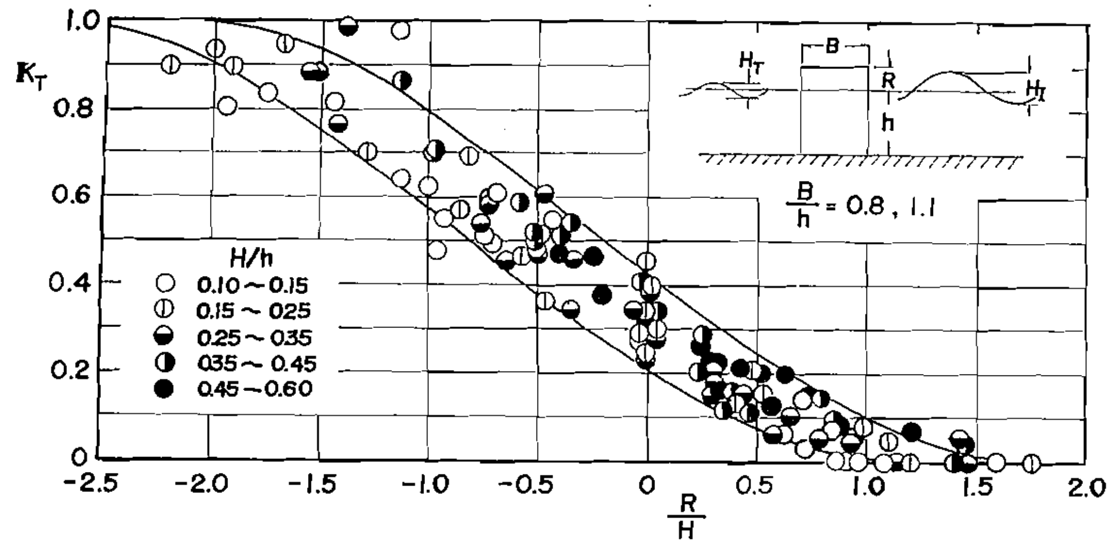

Fig. 4. Wave Transmission Ratio over Vertical Wall Breakwaters

as the function of R/H. Therefore, all the experimental data on vertical wall breakwaters of $B/H=0.8\sim1.1$ were plotted in Fig. 4 regardless of their relative water depth. By applying Eq. 2 for Fig. 4, the following values for α and β were obtained:

$$\left.\begin{array}{ll}\text{Upper limit line}: & \alpha=2.0 \quad \beta=0.2 \\ \text{Average line} \quad : & \alpha=2.0 \quad \beta=0.5 \\ \text{Lower limit line}: & \alpha=2.0 \quad \beta=0.8\end{array}\right\} \tag{4}$$

Experimental data in Fig. 4 are classified according to the ratio of wave height to water depth H/h, but the effect of H/h on the wave transmission ratio is not significant partially because of the data scattering. In order to examine the effect of the ratio H/h in detail, the case of $R=0$ (the breakwater crown is at the same height with the mean water level) was tested with small increments of wave height; the relative water depth was fixed at $h/L=0.14$. As shown in Fig. 5, the wave trans-

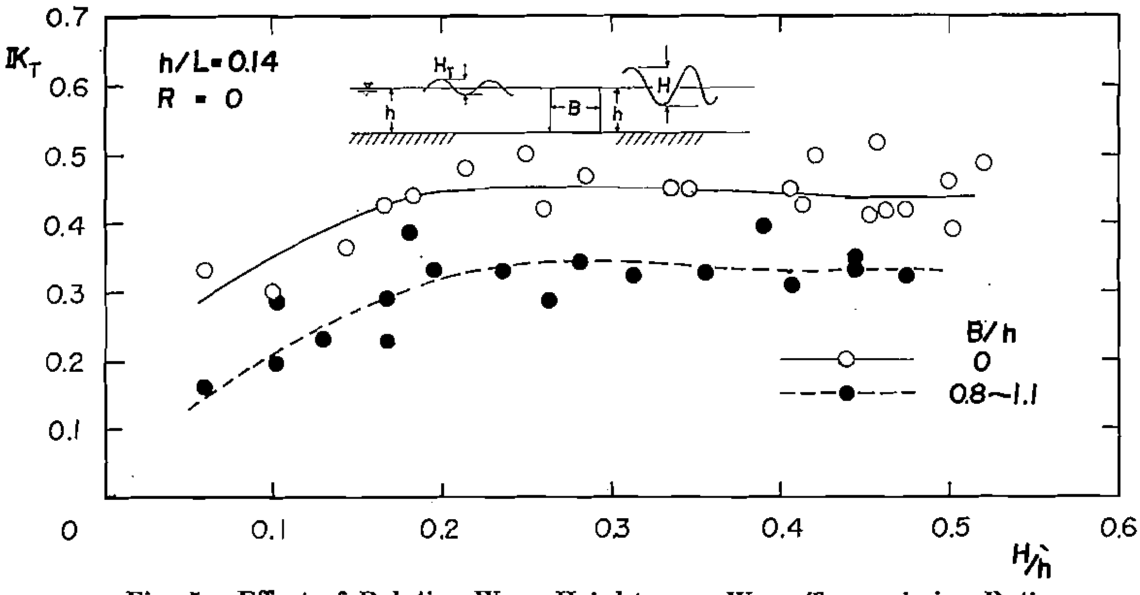

Fig. 5. Effect of Relative Wave Height upon Wave Transmission Ratio

mission ratio shows a decrease with the decrease of the relative wave height H/h in the range of $H/h=0\sim0.2$ and seems to approach $K_T=0$ at $H/h=0$. In the range of H/h larger than 0.2, however, the wave transmission ratio is almost constant. Thus, it is concluded that the relation between K_T and R/H is not affected by H/h or H/L except for the case where the wave height is so small that the approximation of small amplitude waves is well applicable.

(4) Effect of Breakwater Width

Figure 5 also shows the variation of wave transmission ratio due to the breakwater width as well as the variation with respect to the ratio H/h. Open circles indicates the data on the thin wall of 0.9 cm thick ($B/h\simeq0$) and closed circles are those on regular breakwaters of 40 cm wide ($B/h=0.8\sim1.1$). With the increase of the breakwater width, the transmission ratio clearly decreases from about 0.45 to 0.33.

The effect of breakwater width was further investigated for various crown heights as shown in Fig. 6; the relative water depth was $h/L=0.14$. It is clearly shown in

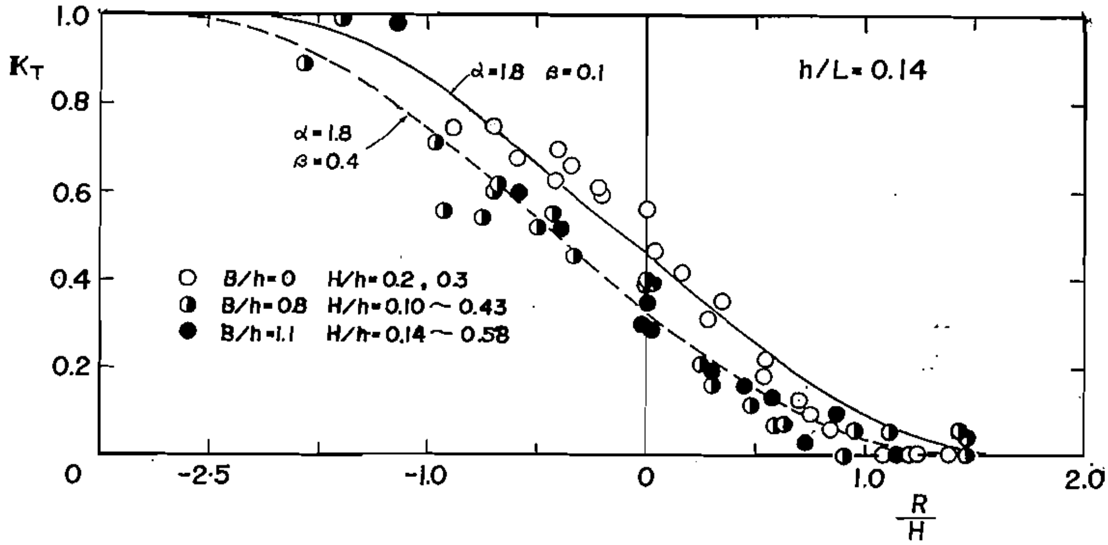

Fig. 6. Effect of Breakwater Width upon Wave Transmission Ratio

Fig. 6 that the thin breakwater of $B/h\simeq0$ produces larger ratios of wave transmission for a wide range of R/H than the wide breakwaters of $B/h=0.8\sim1.1$. The factors α and β of Eq. 2 for the data of the thin breakwater are found to be $\alpha=1.8$ and $\beta=0.1$. In comparison with the case of wide breakwaters, the thin breakwater has a decrease of 0.3 in terms of β and an increase of about 0.1 in terms of K_T.

Such differences in the wave transmission ratio due to the breakwater width can be explained from the behaviour of overtopped water mass. When the breakwater is very thin, overtopped waves are observed to drop onto the water surface behind the breakwater as a mass of water without dispersion. This drop of water mass brings a large downward momentum upon the water surface, and the momentum works effectively in the re-generation of transmitted waves. On the other hand, when the breakwater is wide, overtopped waves run over the breakwater more likely as a flow than as a wave, and then drop behind the breakwater. As a result, the overtopped waves when they hit the water surface have smaller downward momentum than in the case of thin breakwater, even though they may have larger forward momentum. The impact upon the water surface which is essential for the re-generation of transmitted waves is therefore weak in the case of wide breakwaters. In addition, a wide breakwater causes larger loss of wave energy by partial breaking and friction over it than a thin breakwater. Because of the less impact upon the water surface and the larger loss of wave energy, a wide breakwater produces smaller transmitted waves than a thin one with the same crown height.

(5) Profiles and Propagation Speed of Transmitted Waves

Generally, the profiles of transmitted waves are not the reproduction of incident waves with reduced heights. The waves transmitted over a breakwater show the harmonic components of intensified amplitudes; in the extreme cases, they are broken down into many short waves. The profiles of transmitted waves are less stable than incident waves, transforming themselves while propagating, even though the time history of water level at a fixed location repeats the same variation.

In order to grasp the general tendency on the profiles and periodicity of transmitted waves, all the records of transmitted waves over vertical wall breakwaters (B/h=0.8~1.1) were examined and classified in the following three categories, although the classification is somewhat arbitrary:

(i) With little distortion in wave profiles and no change in wave period . . . ○
(ii) With some distrosions in wave profiles and partial changes in wave period. . △
(iii) With apparent decreases in wave period . . . . ×

The dashed line in Fig. 7 illustrates the zone of distorsion where the transmitted waves may experience changes in their periodicity from that of incident waves. In comparison with wide breakwaters, thin breakwaters cause less distorsion in the periodicity of transmitted waves.

Figure 8 demonstrates the transmission of transmittied waves during the propagation; the simultaneous time histories of the water levels in front of the vertical wall (0.9 cm thick) and at the distance of 1.5, 3.0, and 4.5 m behind the wall are shown as well as the amplitudes of harmonic components of those wave profiles obtained by the Fourier analysis. The water depth was h=50 cm, and the waves were T=1.8 sec, L=3.57 m and H_I=14 cm. The crown height of the wall was $R=-5$ cm (submerged). A sharp drop of the water level in front of the vertical wall is probably caused by the overflowing from the harbor side toward the sea side at the time of wave trough.

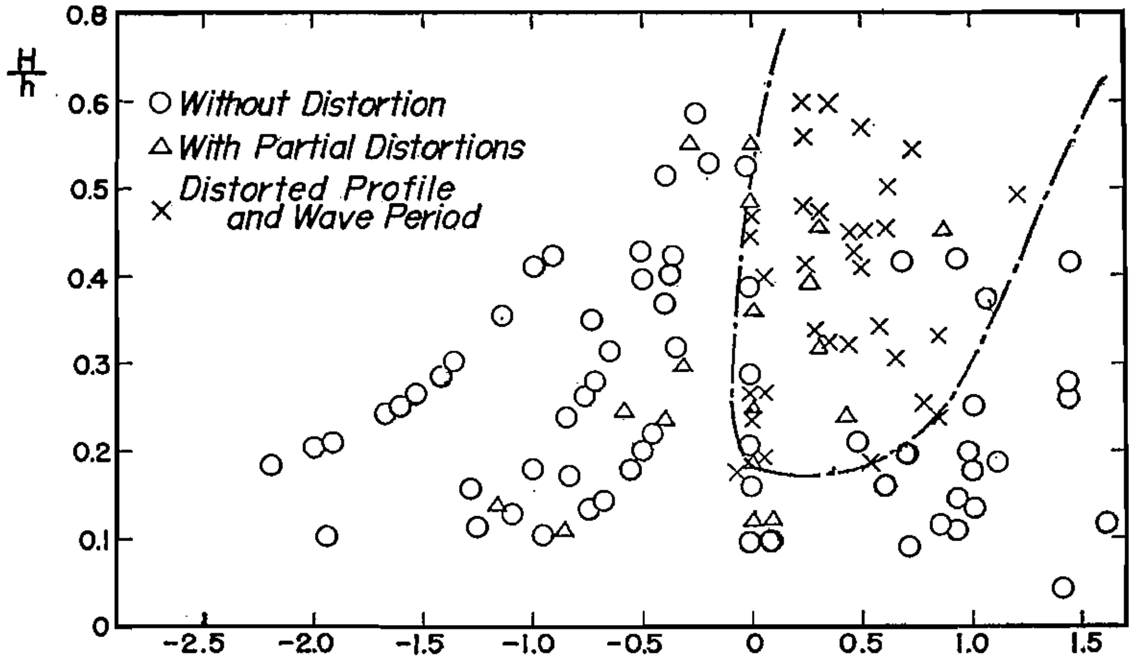

Fig. 7. Classification of Transmitted Wave Porfiles

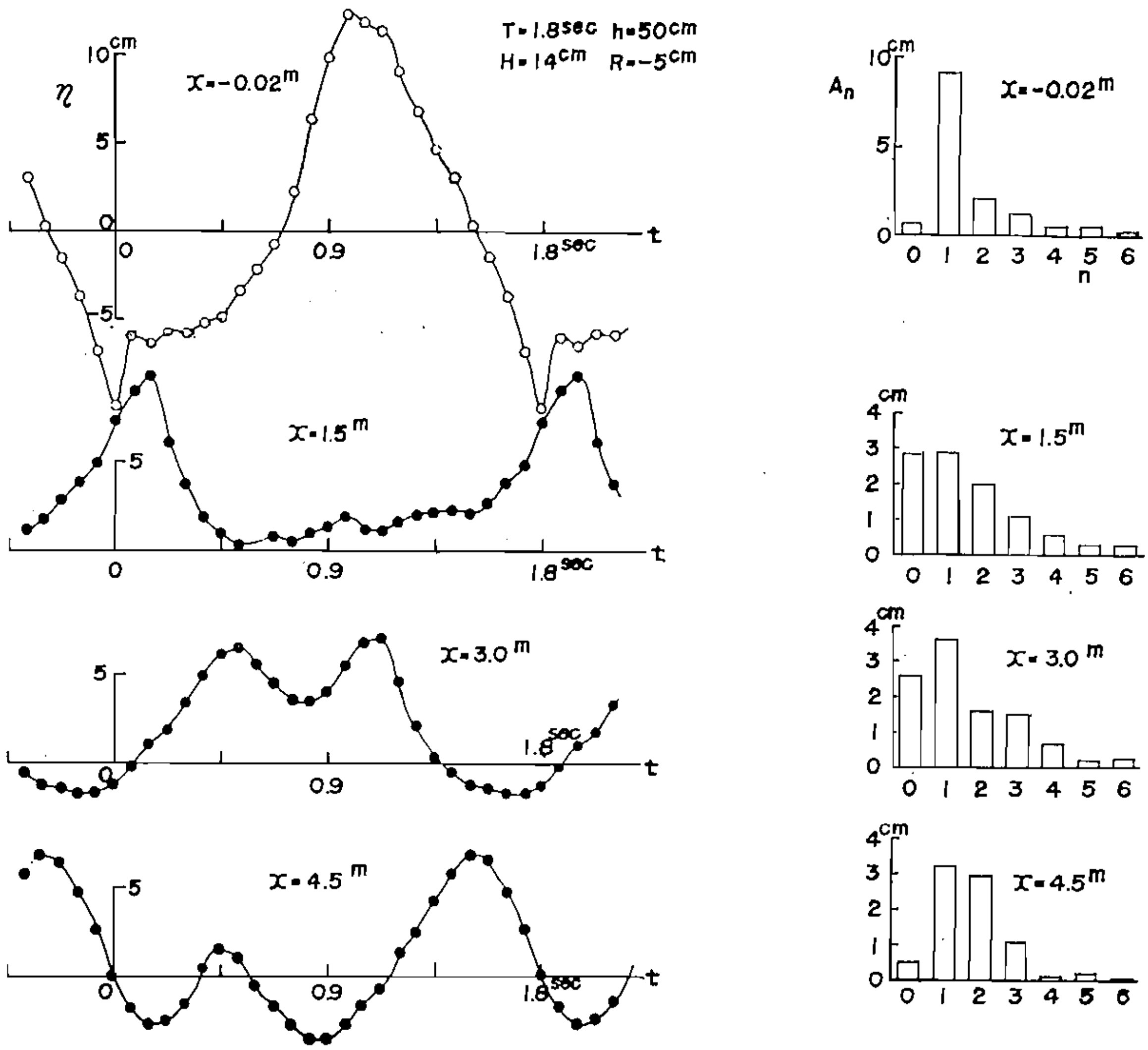

Fig. 8. Transformation of Transmitted Wave Profiles and Amplitudes of Harmonic Components

The simultaneous water level records of Fig. 8 show a propagation of the main wave crest from the left to the right on the time axis. Although the propagation speed can be determined from the time difference between any two stations, the transformation of wave profiles makes the exact determination difficult. Thus, the phase lags of harmonic components of wave profiles were utilized for the determination of the propagation speed of each component wave for the example of Fig. 8.

In the harmonic analysis of wave profiles such as shown in Fig. 8, a wave profile is expressed as:

$$\eta(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n\pi}{T} t - \varphi_n\right) \tag{5}$$

where A_n is the amplitude of the n-th harmonic component and φ_n denotes its phase lag from the common orign of time axis. The values of A_n and φ_n may vary from location to location. If each harmonic component behaves as an independent wave, the wave system can be written as:

$$\eta(x,t) = A_0(x) + \sum_{n=1}^{\infty} A_n(x) \cos \frac{2n\pi}{T} \left(t - \frac{x}{C_n} - \theta_n\right) \tag{6}$$

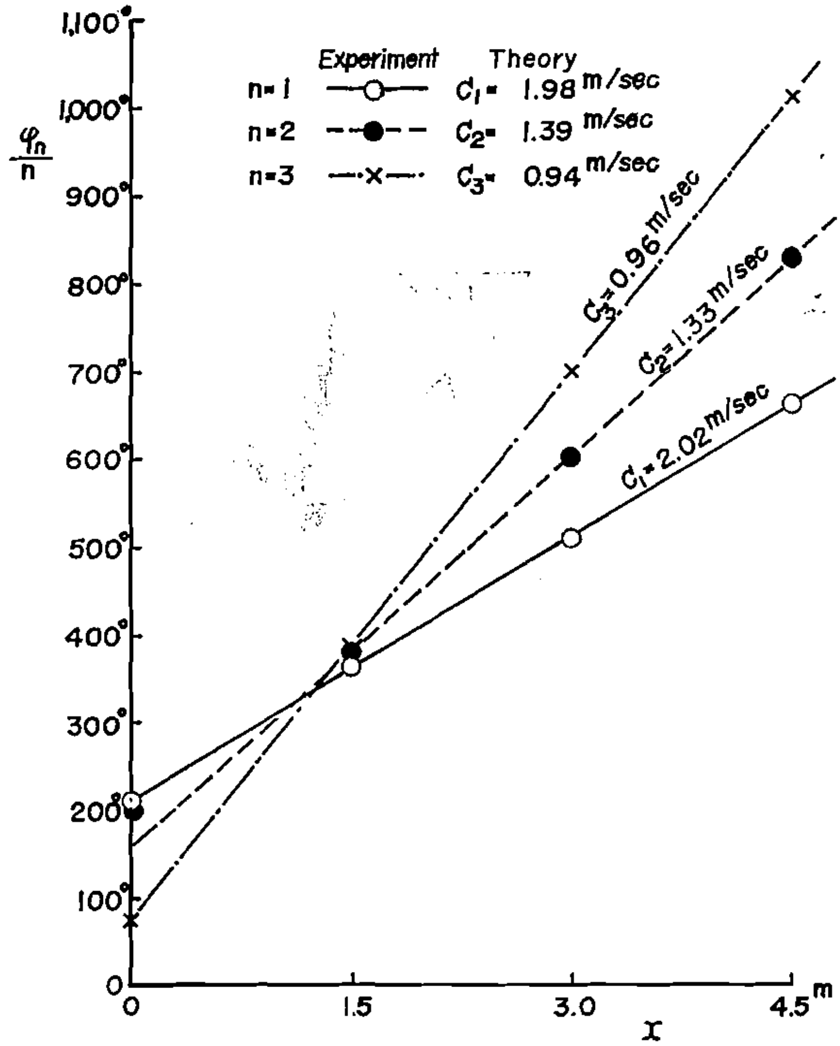

Fig. 9. Propagation Speed of Component Waves

in which C_n denotes the celerity of the n-th harmonic component wave. The phase angle θ_n is constant, depending upon the origins of t and x only. By comparing Eqs. 5 and 6, the following relation between φ_n and x is obtained:

$$\varphi_n = \frac{2n\pi}{T} \left(\frac{x}{C_n} + \theta_n \right) \tag{7}$$

The celerity of the n-th component wave is now calculated from any two measured values of φ_n as:

$$C_n = \frac{2\pi}{T} \cdot \frac{x_2 - x_1}{[\varphi_n(x_2) - \varphi_n(x_1)]/n} \tag{8}$$

The phase lags of the harmonic components of the wave profiles shown in Fig. 8 were examined graphically if they held the relation of Fq. 7. As seen in Fig. 9 in which the values of φ_n/n of the fundamental, second, and third harmonics are plotted against x, the relations between φ_n/n and x are well described with straight lines. The propagation celerities of the component waves are easily obtained from the slopes of these straight lines and with Eq. 8; the celerities obtained are in good agreement with those of small amplitude waves having the periods of $T_n = T/n$.

This agreement is a good indication that a number of wave trains with the periods of T, $T/2$, $T/3$, $T/4$, . . . are generated by the impact of overtopped waves and each wave train propagates independently with its own celerity. Equation 6 therefore is applicable to the system of transmitted waves. The amplitude of each wave train, however, does vary from location to location as seen in the example of Fig. 8. Similar phenomenon has been observed in the case of travelling secondary wave crests in wave channels (see Appendix). The amount of such variations in the amplitudes of component waves is far beyond the errors in measurements and analyses. The trains of component waves are considered to interfere each other and exchange their energies during their propagations.

4. Wave Transmission over Composite Breakwaters

(1) Model Breakwaters

Models of composite breakwaters were composed of a double-walled steel box and three wooden slopes of different sizes as shown in Fig. 10. The height of steel box could be varied by changing the pin holes for bolts and nuts. The width was fixed at B=40 cm (B/h=0.8) as for the models of vertical wall breakwaters. The heights of foundation mounds were selected at the values of d=15, 25, and 35 cm for the water depth of h=50 cm (d/h=0.3, 0.5, and 0.7). The crown of foundation mound was 20 cm wide and the mound slopes were 1 : 3 for the sea side and 1 : 2 for the harbor side.

The mounds were made of wooden boards so as to minimize the frictional loss

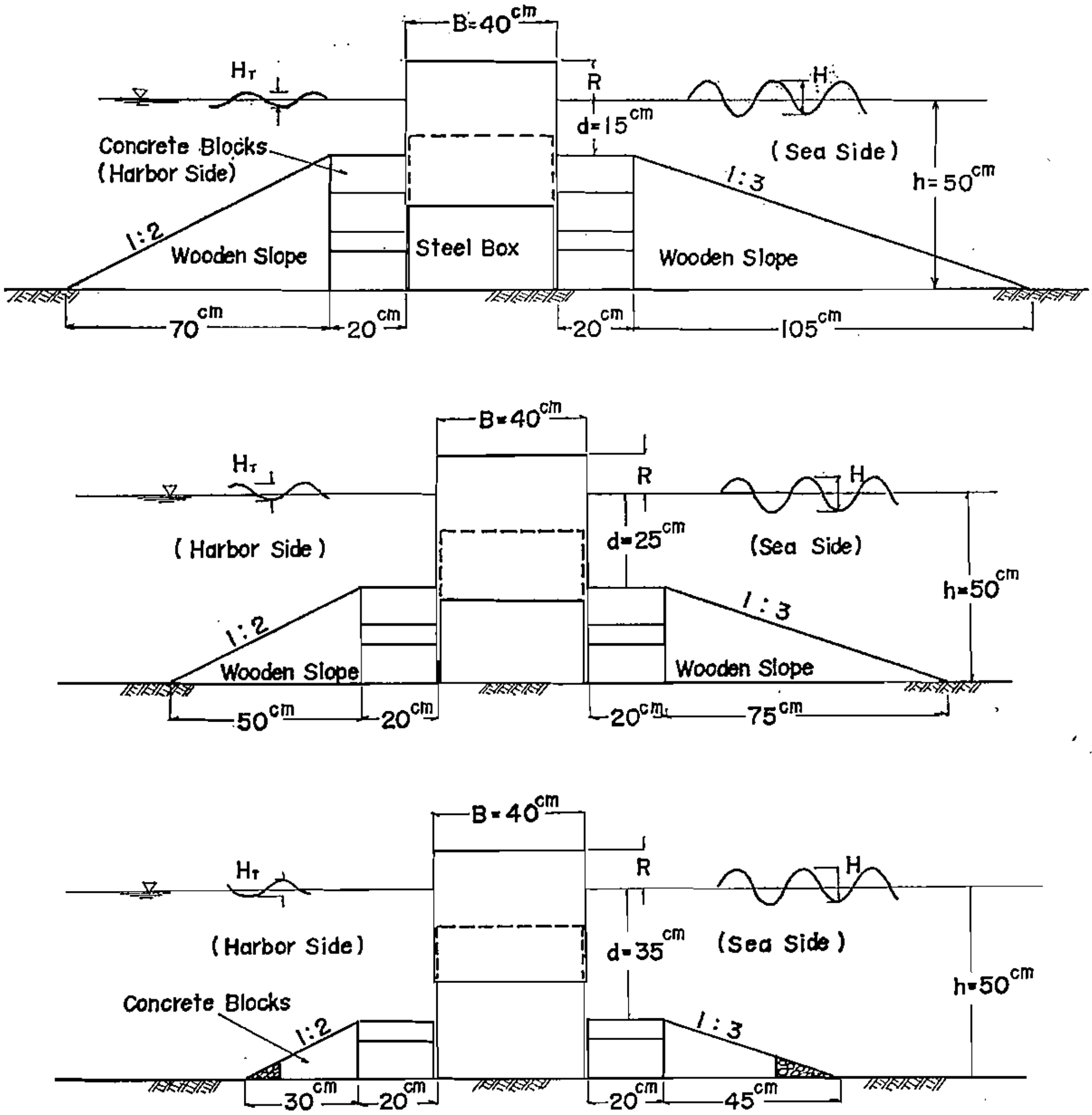

Fig. 10. Models of Composite Breakwaters

in wave energy, because a model mound made of rubble stones would have caused greater energy loss than a prototype mound. The steel box which represented the upper wall part was directly placed upon the channel bottom as seen in Fig. 10, thus preventing any wave transmission through the part of foundation mound. Such the measures were so taken, because the experiment was primarily aimed at investigating the effect of the mound height upon the state of wave overtopping and consequently upon the wave transmission ratio. Any wave transmission through the part of foundation mound made of rubble stones in a model breakwater produces a difficulty in the interpretation of experimental results because of similitude problems for the energy dissipation inside the mound and others.

Experiments were conducted at the constant relative water depth of $h/L=0.14$ with varying wave heights and crown heights for each model of composite breakwater shown in Fig. 10. The selection of relative water depth at $h/L=0.14$ was so made, because the conditions under which actual breakwaters are designed and constructed often yield the relative water depth around the above value. In addition, the little effect of wave characteristics upon the wave transmission ratio over vertical wall breakwaters gave an expectation that the effect would be also little for composite breakwaters.

(2) Effect of Mound Height upon Wave Transmission Ratio

The first experiment of composite breakwaters was made with those of zero crown height so as to investigate the effect of H/h upon the wave transmission ratio (see Table 2 for experimental data). As shown in Fig. 11, the wave transmission ratio

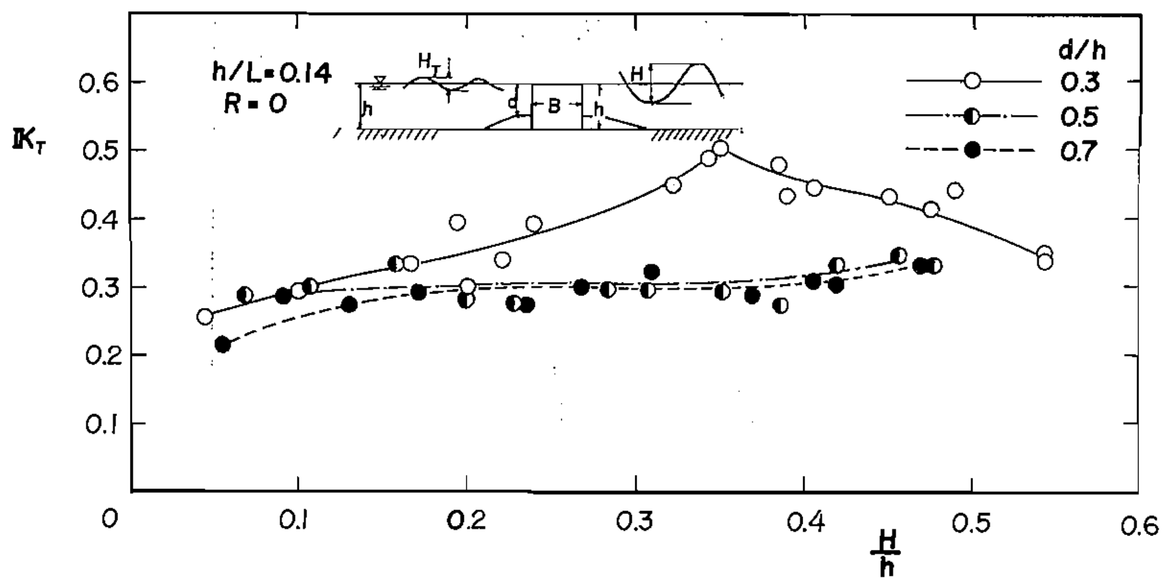

Fig. 11. Effect of Mound Height and Wave Height upon WaveTransmission Ratio over Composite Breakwater

over the high mound breakwater of $d/h=0.3$ is affected by the existence of foundation mound, showing the maximum transmission ratio around $H/h=0.35$ or $H/d=1.2$. At that condition, it was observed that wave broke in the very front of the vertical wall, overtopped the wall very high and splashed down onto the water surface behind the wall.

In the case of breakwaters with medium and low foundation mounds of $d/h=0.5$ and 0.7, the effect of mound upon the wave transmission was not significant; the wave transmission ratio is almost the same as in the case of vertical wall breakwaters shown in Fig. 5. One reason for the insignificance of mound effect is the limitation in the heights of test waves; the waves high enough to break upon the medium and

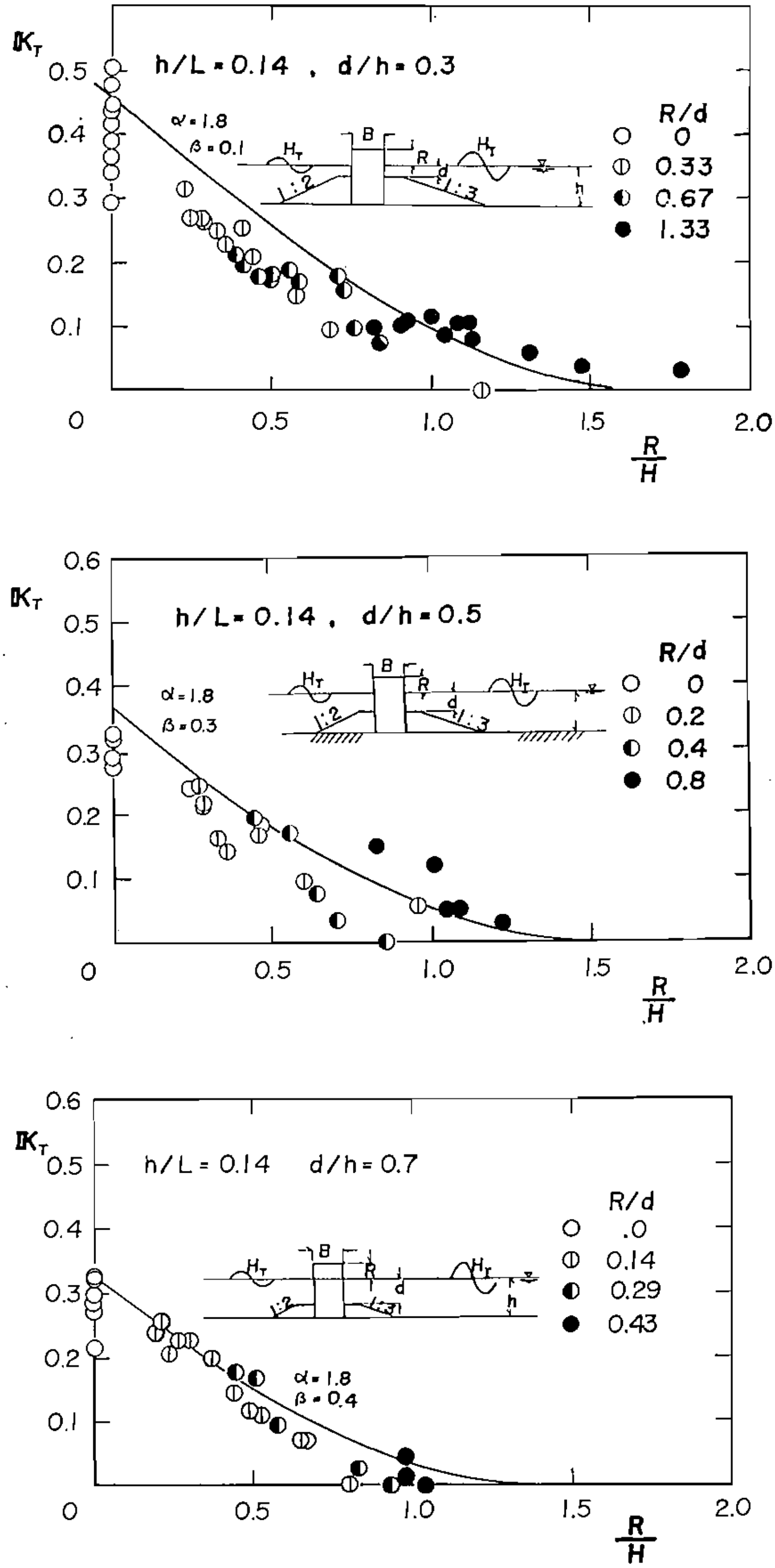

Fig. 12. Wave Transmission Ratio over Composite Breakwaters

low foundation mounds could not be produced. The conditions of constant mound slope and crown width also yielded less possiblities of wave breaking over foundation mound, because the mound length decreased with the lowering of mound height.

The second experiment of composite breakwaters was conducted with the crown height of 5 to 20 cm above the mean water level; the results of the experiments are illustrated in Fig. 12. In the case of the high mound breakwater of $d/h=0.3$, there exist certain wave heights which produce the largest ratio of wave transmission for a given crown height of breakwater, as already shown in Fig. 11. The wave transmission occurs even over a high crown of $R/H>1.5$, possibly because of the forward momentum of rushing waves over the high mound. In the case of the medium and low mound breakwaters the effect of mound becomes less, because the deformation of waves over the mounds is weak. In general, the wave transmission over composite breakwaters is greatly affected by the degree of wave deformation over the foundation mound, especially by the possiblities of wave breaking over there.

(3) Wave Transmission Ratio over Composite Breakwaters

The solid lines shown in Fig. 12 indicate the calculated valued of Eq. 2 fitted to the experimental data. The factors α and β for these curves have the following values for good fitting:

$$\left.\begin{array}{lll} d/h=0.3 & \alpha=1.8 & \beta=0.1 \\ d/h=0.5 & \alpha=1.8 & \beta=0.3 \\ d/h=0.7 & \alpha=1.8 & \beta=0.4 \end{array}\right\} \qquad (9)$$

For vertical wall breakwaters these factors have the values of $\alpha=1.8$ and $\beta=0.4$ for the wave condition of $h/L=0.14$ as seen in Eq. 3 or Fig. 5. These values are the same with those of the low mound breakwater of $d/h=0.7$; that is, the wave transmission ratio is the same with vertical wall breakwaters. With a medium or high foundation mound, however, composite breakwaters produces greater wave transmission than vertical wall breakwaters.

The amount of increase in wave transmission ratio depends upon the degree of wave deformation over the foundation mound, being affected by the relative length and height of the mound to the wavelength and wave height, etc. Because of such various factors related, a minute design diagram or experimental formula inclusive of all these factors concerned is difficult to be established. For practical purposes, however, the following values of α and β in connection with Eq. 2 are hereby proposed:

$$\left.\begin{array}{lll} \alpha=2.0 & \beta=0.1 & \text{for high mound breakwaters} \\ \alpha=2.0 & \beta=0.3 & \text{for medium mound breakwaters} \\ \alpha=2.0 & \beta=0.5 & \text{for low mound breakwaters} \end{array}\right\} \qquad (10)$$

The ratios of wave transmission calculated by Eq. 2 with the values of Eq. 10 are

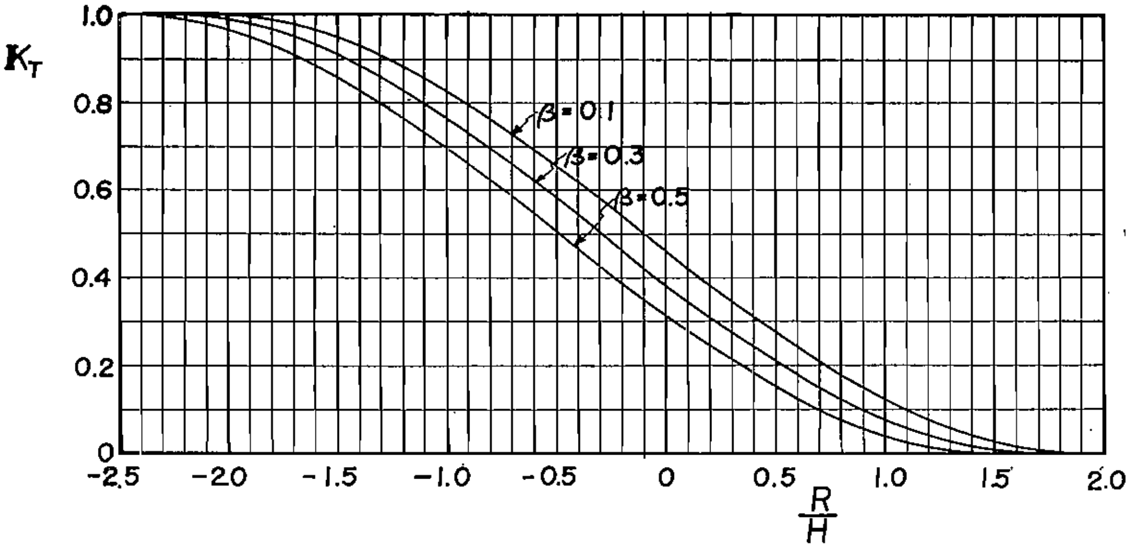

Fig. 13. Calculation Diagram for Wave Transmission Ratio over Breakwaters

shown in Fig. 13 as a design diagram. The value of β should be decreased to some extent when the breakwater crown is very narrow, when the foundation mound is wide enough to be able to deform incident waves greatly, or when the incident waves are already in the state of near breaking in front of breakwaters.

5. Conclusions

The major conclusions of the laboratory tests hereto described are as follows:

(1) The ratio of wave height transmitted behind a breakwater by wave overtopping to the incident wave height, H_T/H_I, is governed almost solely by the ratio of crown height above the mean water level to the incident wave height, R/H_I; the wave characteristics, such as h/L and H/h, do not affect the relation between H_T/H_I versus R/H_I significantly.

(2) An increase in the crown width of a breakwater causes a decreases of the wave transmission ratio.

(3) The heightening of the foundation mound of a composite breakwater generally causes the increase of wave transmission ratio.

(4) The approximate value of transmitted wave height can be calculated with Eqs. 2 and 10, or with Fig. 13.

(5) The transmitted waves produced by wave overtopping are not a single train of waves with a constant period, but they are composed of many wave trains having the periods of T, $T/2$, $T/3$, $T/4$, . . . and travelling with their own celerities.

It should be mentioned here that the above results have been obtained from the experiments with regular trains of long crested waves, tested two-dimensionally in wave channels of uniform sections. The phenomenon of wave transmission over actual breakwaters is more complicated than that in a laboratory. The irregularity in wave heights and period of actual sea waves is one complexity, and the short crestedness of sea waves having a wide range of wave direction is another complexity; the effect of such complex nature of sea waves upon the wave transmission remains to be investigated in the future. The amount of wave energy passing through the rubble mound of a composite breakwater also needs to be clarified. The refinement of the knowledge about the phenomenon of wave transmission over breakwaters will be achieved through these clarifications.

Acknowledgements

The authors sincerely wish to express their thanks to Mr. Shosuke Toki, Assistant Professor of Hokkaido University and formerly member of Hydraulics Division of Port and Harbour Research Institute, and to Mr. Tooru Kikuya, member of Machinery Division, both of whom gave assistance to the execution of laboratory tests at the initial phase of the study.

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(Received by the Institute, March 29, 1967)

Table 1. Experimental Data on Vertical Wall Breakwaters.

Case I $B=40$ cm

$h=50$ cm, $T=0.8$ sec., $h/L=0.5$

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	10	11.78	0.85	0.072	0.462	0.237	0.85
2	5	11.98	1.56	0.130	0.352	0.240	0.42
3	0	12.35	3.41	0.276	0.314	0.247	0
4	−5	10.95	3.99	0.365	0.276	0.219	−0.46
5	−10	11.84	6.72	0.571	0.283	0.236	−0.85
6	−20	12.00	11.4	0.948	0.152	0.240	−1.67
7	30	10.42	0	0	0.538	0.208	2.88
8	20	11.31	0	0	0.543	0.226	1.77
9	10	9.98	0.75	0.075	0.548	0.200	1.00
10	5	9.41	1.45	0.154	0.502	0.188	0.53
11	0	9.48	3.25	0.343	0.433	0.190	0
12	−5	8.88	4.13	0.465	0.301	0.178	−0.56
13	−10	9.99	6.25	0.626	0.201	0.200	−1.00
14	−20	10.04	9.40	0.937	0.165	0.201	−1.99
15	30	5.40	0	0	0.639	0.108	5.56
16	20	5.82	0	0	0.587	0.117	3.44
17	10	6.24	0	0	0.496	0.125	1.60
18	5	5.85	0	0	0.420	0.117	0.86
19	0	5.98	1.46	0.244	0.469	0.120	0
20	−5	5.20	2.48	0.477	0.273	0.104	−0.96
21	−10	5.15	4.13	0.802	0.235	0.103	−1.94
22	−20	5.40	5.10	0.945	0.111	0.108	−3.70

Case II $B=40$ cm

$h=50$ cm, $T=1.05$ sec., $h/L=0.3$

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	30	21.3	0	0	0.409	0.426	1.41
2	20	22.8	1.80	0.079	0.468	0.455	0.88
3	−10	17.50	10.25	0.586	0.321	0.350	−0.57
4	−20	17.65	15.19	0.861	0.247	0.353	−1.13
5	10	15.17	1.55	0.102	0.689	0.304	0.66
6	5	15.90	3.30	0.208	0.636	0.318	0.31
7	0	14.45	4.63	0.321	0.557	0.289	0
8	−5	15.07	5.21	0.345	0.459	0.302	−0.33
9	−10	15.68	7.07	0.451	0.286	0.314	−0.64
10	−20	13.19	11.65	0.883	0.137	0.302	−1.52
11	10	12.67	0.68	0.054	0.609	0.253	0.79

12	5	10·00	2.06	0.206	0.624	0.200	0.50
13	0	9.61	4.33	0.451	0.534	0.192	0
14	− 5	10.35	5.30	0.512	0.451	0.207	−0.48
15	−10	7.82	5.48	0.702	0.258	0.156	−1.28
16	−20	9.10	8.15	0.896	0.098	0.182	−2.20
17	30	7.82	0	0	0.752	0.157	3.84
18	20	8.03	0	0	0.738	0.161	2.49
19	10	8.92	0	0	0.438	0.178	1.21
20	5	6.98	1.00	0.143	0.460	0.140	0.72
21	0	7.95	2.49	0.313	0.499	0.159	0
22	− 5	6.72	3.43	0.510	0.343	0.144	−0.74
23	−10	5.71	4.77	0.836	0.340	0.114	−1.75
24	−20	5.79	5.17	0.892	0.076	0.116	−3.45

Case III B**=40 cm**

h=50 cm, T=1.36 sec., h/L=0.2

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	30	24.6	1.74	0.071	0.566	0.492	1.22
2	20	27.1	4.15	0.153	0.612	0.542	0.74
3	10	29.6	4.82	0.163	0.663	0.592	0.34
4	5	33.4	8.46	0.254	0.661	0.668	0.15
5	0	27.5	9.54	0.342	0.543	0.550	0
6	− 5	26.3	10.21	0.389	0.542	0.526	−0.19
7	−10	25.6	12.11	0.473	0.450	0.512	−0.39
8	−20	20.5	14.3	0.697	0.304	0.410	−0.98
9	10	22.6	4.97	0.220	0.702	0.452	0.442
10	5	19.66	5.62	0.286	0.503	0.393	0.254
11	0	18.09	6.07	0.336	0.567	0.362	0
12	− 5	14.84	6.68	0.450	0.604	0.297	−0.337
13	−10	19.83	9.79	0.494	0.476	0.397	−0.504
14	−20	14.16	10.83	0.767	0.284	0.283	−1.412
15	5	16.06	2.88	0.180	0.586	0.321	0.311
16	0	11.79	4.77	0.405	0.475	0.236	0
17	− 5	9.97	4.81	0.483	0.472	0.199	−0.502
18	−10	13.94	8.14	0.584	0.395	0.270	−0.717
19	−20	10.50	9.42	0.897	0.286	0.210	−1.905
20	0	9.50	2.97	0.313	0.388	0.195	0
21	− 5	7.04	3.47	0.493	0.467	0.141	−0.71
22	−10	6.96	5.68	0.817	0.268	0.140	−1.44

Case IV **B=40 cm**

h=50 cm, T=1.80 sec., h/L=0.14

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	30	20.7	0.88	0.043	0.417	0.414	1.45
2	20	21.1	1.21	0.057	0.483	0.422	0.95
3	10	21.4	2.39	0.112	0.600	0.428	0.47
4	5	20.7	5.25	0.254	0.623	0.414	0.24
5	0	19.24	7.65	0.398	0.569	0.385	0
6	−5	15.32	6.95	0.454	0.568	0.306	0.33
7	−10	20.1	10.32	0.514	0.403	0.401	−0.50
8	−20	21.0	14.75	0.704	0.313	0.419	−0.96
9	30	11.38	0.65	0.057	0.517	0.228	2.64
10	20	13.88	0.65	0.047	0.485	0.278	1.44
11	10	17.34	1.06	0.061	0.634	0.347	0.58
12	5	16.78	2.62	0.156	0.652	0.336	0.30
13	0	13.13	5.05	0.380	0.624	0.263	0
14	−5	11.60	6.36	0.548	0.562	0.232	−0.43
15	−10	13.28	7.14	0.538	0.507	0.266	−0.75
16	−20	14.48	14.35	0.992	0.269	0.298	−1.38
17	30	11.85	0	0	0.638	0.237	2.53
18	20	13.75	0	0	0.562	0.275	1.46
19	10	9.12	0.45	0.049	0.659	0.182	1.10
20	5	8.09	0.54	0.067	0.753	0.162	0.62
21	0	9.04	3.50	0.387	0.716	0.181	0
22	−5	7.33	4.50	0.614	0.524	0.147	−0.68
23	−10	14.03	8.37	0.597	0.371	0.281	−0.71
24	−20	12.79	11.33	0.887	0.279	0.256	−1.56
25	10	5.73	0	0	0.669	0.115	1.75
26	5	5.55	0	0	0.669	0.111	0.90
27	0	5.07	1.46	0.289	0.686	0.101	0
28	−5	5.37	2.97	0.552	0.483	0.107	−0.93

Case V **B=40 cm**

h=35 cm, T=1.5 sec., h/L=0.14

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	10	17.58	2.30	0.131	0.534	0.503	0.57
2	5	16.63	3.19	0.192	0.478	0.476	0.30
3	0	15.60	5.41	0.347	0.494	0.446	0
4	−5	12.90	6.60	0.512	0.494	0.369	−0.39
5	10	11.68	1.12	0.096	0.493	0.333	0.86
6	5	11.35	1.74	0.153	0.572	0.324	0.44

7	0	9.25	2.65	0.287	0.552	0.264	0
8	− 5	8.58	5.10	0.594	0.478	0.246	−0.58
9	10	8.82	0	0	0.303	0.252	1.13
10	5	6.98	0.2	0.029	0.362	0.200	0.72
11	0	5.83	1.69	0.290	0.465	0.167	0
12	− 5	4.83	4.73	0.981	0.357	0.138	−1.14

Case VI B=40 cm

h=35 cm, T=2.0 sec., h/L=0.10

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	10	20.0	4.18	0.208	0.390	0.573	0.50
2	5	21.0	5.48	0.261	0.433	0.600	0.24
3	0	16.74	5.45	0.326	0.441	0.478	0
4	− 5	20.5	9.68	0.472	0.384	0.587	−0.24
5	10	10.67	0.45	0.042	0.479	0.305	0.94
6	5	14.15	1.66	0.117	0.379	0.404	0.35
7	0	11.30	2.64	0.234	0.465	0.323	0
8	− 5	9.98	4.69	0.471	0.455	0.285	−0.50
9	10	4.29	0	0	0.690	0.123	2.33
10	5	4.65	0	0	0.626	0.133	1.08
11	0	3.51	0.97	0.276	0.608	0.100	0
12	− 5	4.45	2.85	0.641	0.494	0.127	−1.12

Case VII B=40 cm

h=35 cm, T=2.76 sec., h/L=0.07

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	10	15.97	3.30	0.206	0.409	0.456	0.63
2	5	16.94	3.84	0.226	0.361	0.485	0.30
3	0	13.91	4.74	0.341	0.393	0.397	0
4	− 5	14.88	8.12	0.546	0.375	0.425	−0.34
5	10	12.60	1.77	0.141	0.381	0.360	0.79
6	5	12.92	2.03	0.157	0.334	0.370	0.39
7	0	9.62	3.08	0.320	0.341	0.275	0
8	− 5	10.57	6.43	0.608	0.247	0.302	−0.47
9	10	5.70	0	0	0.546	0.163	1.75
10	5	5.21	0	0	0.582	0.149	0.96
11	0	3.97	1.19	0.300	0.571	0.113	0
12	− 5	6.11	4.25	0.695	0.357	0.174	−0.82

Case VIII **B=40 cm**

h=50 cm, T=1.8 sec., h/L=0.14

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	0	23.69	7.63	0.322	0.573	0.474	0
2	0	22.19	7.44	0.335	0.544	0.444	0
3	0	20.35	6.33	0.311	0.591	0.407	0
4	0	17.81	5.81	0.326	0.586	0.356	0
5	0	15.63	5.08	0.325	0.624	0.313	0
6	0	14.04	4.87	0.347	0.621	0.281	0
7	0	11.81	3.92	0.331	0.630	0.236	0
8	0	9.81	3.24	0.330	0.656	0.196	0
9	0	8.39	1.92	0.228	0.678	0.168	0
10	0	6.50	1.50	0.231	0.692	0.130	0
11	0	5.08	1.00	0.197	0.774	0.102	0
12	0	3.00	0.50	0.167	0.667	0.060	0

Case IX **B=0.9 cm**

h=40 cm, T=1.6 sec., h/L=0.14

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	0	18.10	9.38	0.517	0.358	0.457	0
2	0	20.85	10.14	0.486	0.486	0.521	0
3	0	20.17	7.96	0.390	0.512	0.503	0
4	0	18.53	7.75	0.418	0.403	0.463	0
5	0	16.85	8.37	0.497	0.371	0.421	0
6	0	16.57	7.00	0.424	0.493	0.414	0
7	0	13.46	6.06	0.450	0.524	0.336	0
8	0	10.30	4.35	0.422	0.607	0.260	0
9	0	7.30	3.10	0.424	0.630	0.183	0

Case X **B= 0.9 cm**

h=50 cm, T=1.8 sec., h/L=0.14

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	0	17.35	7.75	0.447	0.556	0.347	0
2	0	20.35	9.10	0.447	0.518	0.407	0
3	0	22.63	9.38	0.414	0.547	0.453	0
4	0	23.69	10.06	0.425	0.607	0.474	0
5	0	24.94	11.43	0.459	0.569	0.499	0
6	0	14.30	6.69	0.468	0.602	0.286	0
7	0	12.45	6.17	0.496	0.622	0.249	0

8	0	10.68	5.13	0.481	0.645	0.214	0
9	0	9.14	4.01	0.439	0.618	0.183	0
10	0	7.20	2.66	0.369	0.670	0.144	0
11	0	5.00	1.50	0.300	0.709	0.100	0
12	0	3.00	1.00	0.333	0.667	0.060	0

Case XI **B=0.9 cm**

h=50 cm, T=1.8 sec., h/L=0.14

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	12.5	10.40	0	0	0.722	0.208	1.20
2	8.0	10.63	1.0	0.094	0.694	0.213	0.75
3	5.5	10.37	1.88	0.180	0.688	0.207	0.53
4	2.9	10.25	3.17	0.309	0.668	0.205	0.28
5	0.3	10.55	4.87	0.461	0.612	0.211	0.03
6	−2.2	10.62	6.26	0.590	0.530	0.212	−0.21
7	−4.5	10.75	6.71	0.624	0.536	0.215	−0.42
8	−7.0	10.00	7.48	0.748	0.493	0.200	−0.70
9	−9.4	10.67	7.93	0.743	0.390	0.213	−0.88
10	20.0	14.50	0	0	0.742	0.290	1.38
11	17.5	14.30	0	0	0.721	0.286	1.23
12	15.5	14.43	0	0	0.707	0.289	1.08
13	13.0	15.20	0.90	0.059	0.678	0.304	0.84
14	10.0	14.47	1.78	0.123	0.722	0.289	0.69
15	8.0	14.93	3.21	0.215	0.688	0.299	0.54
16	5.0	14.75	5.10	0.346	0.646	0.295	0.34
17	2.5	15.31	6.29	0.411	0.598	0.306	0.16
18	0	15.00	8.33	0.556	0.533	0.300	0
19	−3.5	14.74	9.58	0.608	0.532	0.315	−0.22
20	−5.5	15.64	10.30	0.659	0.496	0.313	−0.35
21	−7.0	16.94	11.76	0.694	0.476	0.339	−0.41
22	−9.5	16.00	10.86	0.674	0.422	0.320	−0.59

Table 2. Experimental Data on Composite Breakwaters

Case I ***d*=15 cm**

h=50 cm, T=1.80 sec., h/L=0.14

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_T}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	0	23.8	9.83	0.359	0.413	0.476	0
2	0	22.5	9.71	0.338	0.431	0.450	0
3	0	20.3	9.03	0.340	0.444	0.406	0
4	0	19.24	9.19	0.292	0.477	0.385	0
5	0	17.56	8.80	0.345	0.502	0.351	0
6	0	16.12	7.21	0.395	0.447	0.323	0
7	0	12.00	4.70	0.417	0.392	0.240	0
8	0	11.06	3.75	0.457	0.339	0.221	0
9	0	8.37	2.80	0.492	0.335	0.167	0
10	0	5.12	1.50	0.537	0.293	0.102	0
11	0	27.1	9.65	0.222	0.356	0.543	0
12	0	24.5	10.77	0.316	0.439	0.490	0
13	0	17.25	8.44	0.319	0.489	0.345	0
14	0	9.76	3.84	0.526	0.393	0.195	0
15	0	2.24	0.57	0.629	0.254	0.045	0
16	5.0	21.8	6.82	0.314	0.407	0.435	0.23
17	5.0	20.5	5.40	0.269	0.383	0.410	0.24
18	5.0	18.00	4.83	0.268	0.388	0.360	0.28
19	5.0	17.50	4.60	0.262	0.411	0.350	0.29
20	5.0	15.20	3.74	0.246	0.441	0.304	0.33
21	5.0	13.87	3.15	0.226	0.478	0.278	0.36
22	5.0	12.14	3.01	0.247	0.513	0.243	0.41
23	5.0	11.30	2.37	0.210	0.514	0.226	0.44
24	5.0	8.60	1.50	0.174	0.615	0.172	0.58
25	5.0	7.30	0.70	0.096	0.782	0.146	0.69
26	5.0	4.25	0	0	0.693	0.086	1.16
27	10.0	21.6	3.86	0.178	0.534	0.432	0.46
28	10.0	20.1	3.65	0.182	0.555	0.402	0.50
29	10.0	20.0	3.38	0.168	0.624	0.400	0.50
30	10.0	18.10	3.48	0.192	0.678	0.362	0.55
31	10.0	17.17	2.96	0.172	0.666	0.343	0.58
32	10.0	14.03	2.51	0.178	0.662	0.281	0.71
33	10.0	13.65	2.13	0.156	0.640	0.273	0.73
34	10.0	13.15	1.31	0.099	0.674	0.263	0.76
35	10.0	11.95	0.90	0.075	0.622	0.239	0.84
36	10.0	6.26	0.74	0.118	0.588	0.125	1.60

37	10.0	26.3	5.55	0.211	0.436	0.526	0.38
38	10.0	24.2	4.66	0.192	0.448	0.484	0.41
39	20.0	24.3	2.38	0.098	0.412	0.486	0.82
40	20.0	24.6	2.41	0.098	0.512	0.492	0.81
41	20.0	21.8	2.41	0.111	0.479	0.436	0.92
42	20.0	22.3	2.20	0.099	0.519	0.446	0.90
43	20.0	20.3	2.31	0.114	0.546	0.406	0.99
44	20.0	19.3	1.74	0.090	0.516	0.386	1.04
45	20.0	17.5	1.40	0.080	0.560	0.350	1.14
46	20.0	15.25	0.99	0.065	0.587	0.312	1.31
47	20.0	15.28	0.83	0.054	0.600	0.312	1.31
48	20.0	13.60	0.50	0.037	0.517	0.272	1.47
49	20.0	11.28	0.35	0.031	0.526	0.226	1.78
50	20.0	20.14	2.44	0.120	0.521	0.403	1.00
51	20.0	18.65	2.08	0.111	0.550	0.373	1.07
52	20.0	18.02	1.90	0.105	0.628	0.360	1.11

Case II $d=25$ **cm**

$h=50$ cm, $T=1.80$ sec., $h/L=0.14$

No.	R (cm)	H_I (cm)	H_T (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	0	23.9	7.88	0.330	0.440	0.478	0
2	0	22.8	7.84	0.344	0.443	0.457	0
3	0	21.0	6.94	0.330	0.501	0.420	0
4	0	19.31	6.22	0.322	0.516	0.387	0
5	0	17.63	5.14	0.292	0.510	0.352	0
6	0	15.44	4.53	0.294	0.595	0.309	0
7	0	14.19	4.20	0.296	0.560	0.284	0
8	0	11.88	3.29	0.277	0.580	0.238	0
9	0	9.94	2.79	0.281	0.573	0.199	0
10	0	7.88	2.60	0.330	0.619	0.158	0
11	0	5.38	1.60	0.298	0.674	0.108	0
12	0	3.50	1.00	0.286	0.714	0.069	0
13	5.0	5.25	0.30	0.057	0.773	0.105	0.95
14	5.0	8.44	0.80	0.095	0.734	0.169	0.59
15	5.0	10.79	1.80	0.168	0.705	0.216	0.46
16	5.0	13.73	2.00	0.145	0.694	0.275	0.36
17	5.0	15.25	2.50	0.164	0.689	0.305	0.33
18	5.0	17.56	3.86	0.220	0.623	0.351	0.28
19	5.0	18.10	4.40	0.243	0.579	0.362	0.27
20	5.0	20.1	4.97	0.247	0.497	0.402	0.25

21	5.0	20.9	5.27	0.252	0.535	0.418	0.24
22	5.0	23.0	6.75	0.293	0.481	0.460	0.21
23	10.0	22.6	4.44	0.197	0.599	0.452	0.44
24	10.0	22.1	4.33	0.196	0.601	0.442	0.45
25	10.0	21.4	3.90	0.182	0.647	0.428	0.47
26	10.0	18.35	3.15	0.172	0.684	0.367	0.55
27	10.0	16.85	1.58	0.094	0.704	0.337	0.60
28	10.0	15.86	1.26	0.079	0.677	0.317	0.63
29	10.0	14.40	0.53	0.036	0.683	0.288	0.70
30	10.0	11.80	0	0	0.710	0.236	0.85
31	20.0	24.7	3.76	0.152	0.740	0.494	0.81
32	20.0	19.95	2.40	0.121	0.768	0.399	1.00
33	20.0	19.30	1.02	0.053	0.830	0.386	1.04
34	20.0	18.55	1.02	0.055	0.822	0.371	1.08
35	20.0	16.45	0.30	0.030	0.788	0.329	1.22

Case III $d=35$ **cm**

$h=50$ cm, $T=1.80$ sec., $h/L=0.14$

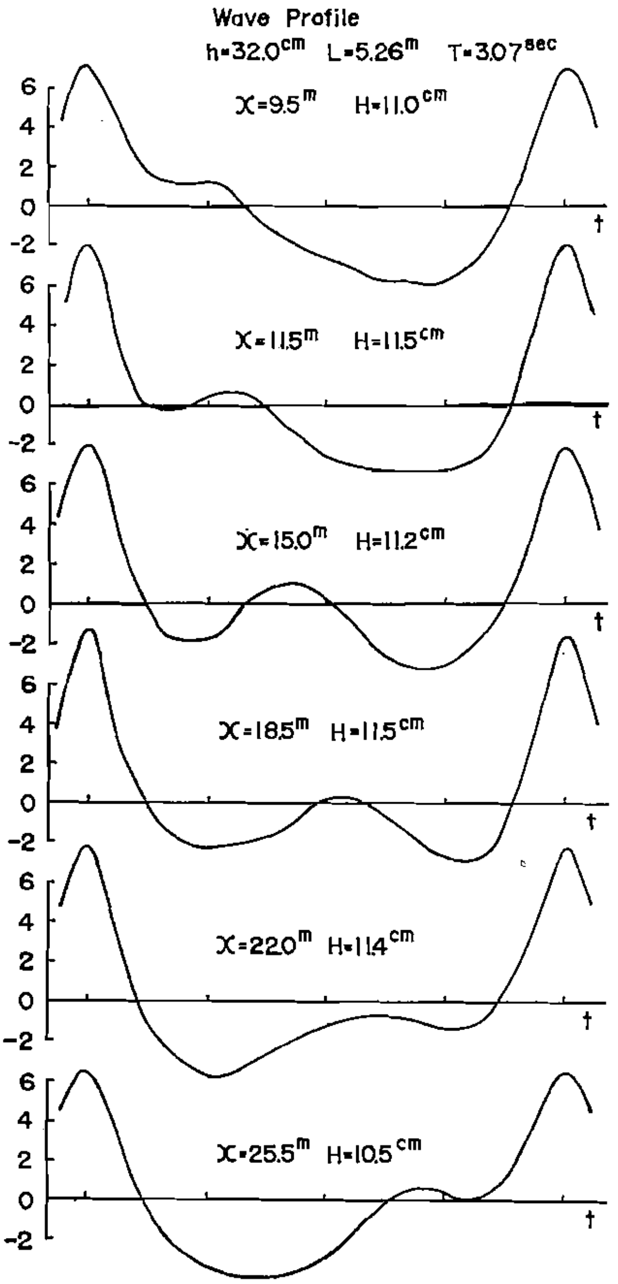
No.	R (cm)	H_I (cm)	H_I (cm)	$\frac{H_T}{H_I}$	$\frac{H_R}{H_I}$	$\frac{H_I}{h}$	$\frac{R}{H_I}$
1	0	23.5	7.70	0.328	0.548	0.471	0
2	0	21.0	6.34	0.302	0.559	0.420	0
3	0	20.3	6.25	0.308	0.562	0.406	0
4	0	18.50	5.26	0.285	0.598	0.370	0
5	0	15.50	4.98	0.322	0.645	0.310	0
6	0	13.38	3.98	0.298	0.627	0.268	0
7	0	11.78	3.23	0.275	0.627	0.236	0
8	0	10.08	3.00	0.298	0.663	0.201	0
9	0	8.57	2.50	0.292	0.650	0.171	0
10	0	6.57	1.80	0.274	0.695	0.131	0
11	0	4.56	1.30	0.285	0.781	0.091	0
12	0	2.81	0.60	0.213	0.822	0.056	0
13	5.0	6.20	0	0	0.822	0.124	0.81
14	5.0	7.70	0.55	0.071	0.822	0.154	0.65
15	5.0	10.22	1.21	0.119	0.705	0.204	0.49
16	5.0	11.47	1.64	0.143	0.739	0.230	0.44
17	5.0	13.80	2.75	0.199	0.688	0.276	0.36
18	5.0	16.50	3.75	0.227	0.629	0.330	0.30
19	5.0	18.64	4.19	0.225	0.630	0.373	0.27
20	5.0	20.9	4.30	0.206	0.600	0.418	0.24
21	5.0	23.6	6.05	0.256	0.605	0.472	0.21
22	5.0	25.6	6.14	0.240	0.588	0.513	0.20

23	10.0	10.75	0	0	0.744	0.215	0.93
24	10.0	12.05	0.33	0.027	0.695	0.241	0.83
25	10.0	15.05	1.09	0.072	0.721	0.301	0.67
26	10.0	17.43	1.64	0.094	0.682	0.349	0.57
27	10.0	19.27	2.18	0.113	0.710	0.385	0.52
28	10.0	19.38	3.27	0.169	0.607	0.388	0.52
29	10.0	22.5	4.00	0.178	0.615	0.440	0.44
30	20.0	19.35	0	0	0.643	0.387	1.03
31	20.0	21.0	0.3	0.014	0.671	0.420	0.98
32	20.0	21.5	1.0	0.046	0.619	0.430	0.96

Appendix

Travelling Secondary Wave Crests in Wave Channels*

by Yoshimi GODA



1. The phenomenon of secondary waves is well known among experienced experimenters of water waves[1,3]. The word of secondary waves, or secondary wave crests, refers to the irregural swells on normal wave profiles such as shown in Fig. A-1. Examples of wave profiles with secondary waves were first presented by Morison and Crooke[1]. They noticed that secondary waves moved with a less speed than the celerity of main crests. The waves with secondary wave crests, therefore, are not of permanent type; this property makes it a very troublesome one for experimenters. The position of a secondary wave crest on a wave profile receeds gradually as the wave advances in a test channel. The

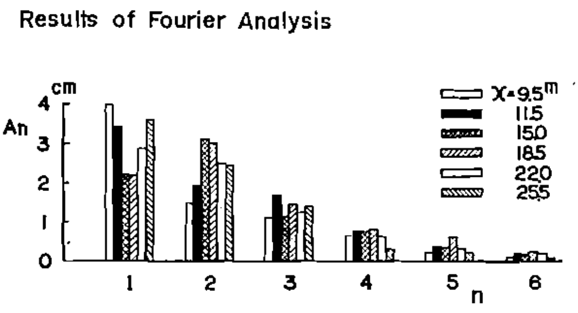


Fig. A-1. An Example of Travelling Secondary Wave Crest

*) This note was first prepared in Japanese as Note No. 8 of the Breakwater Laboratory of the Hydraulics Division of the Port and Harbour Research Institute in August 1961. It was later presented at the Seminar at the Hydrodynamics Laboratory of Massachusetts Institute of Technology in U.S.A. in January 1962, when the author attended there as a graduate student. The present text is based on the memorandum for that Seminar, several copies of which have been distributed to the parties interested.

wave profile varies with respect to location and the wave height also shows a variation along a wave channel; the wave envelope shows a kind of beat as if there exist some reflected waves, although the beat length is several times the wavelength.

2. The appearance of secondary waves is limited, however, to the relatively shallow water waves with large wave heights. If a test is carried out in a region where the relative water depth h/L is greater than 0.15, no secondary wave will be observed. Figure A-2 shows experimental data on the appearance of secondary wave crests

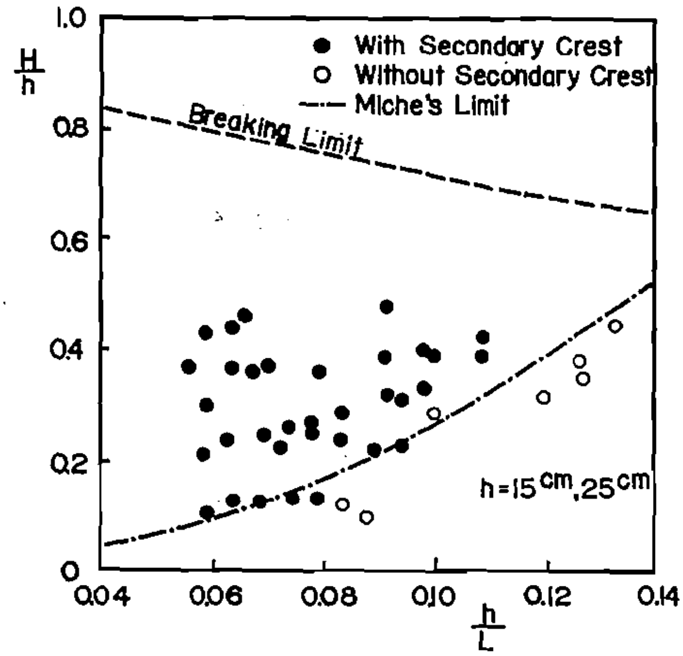


Fig. A-2. Appearance Limit of Secondary Wave Crests in a Test Channel

during tests in a wave channel, 33 m long 0.5 m wide, and 1.0 m deep. The theoretical curve in the figure was originally given by Miche [2] expressed in the following form:

$$(H/L)_{\text{critical}} = \frac{1}{3\pi} \sinh^2(2\pi h/L) \tanh(2\pi h/L) \qquad (1)$$

The formation of a secondary wave crest is somewhat theoretically expected as shown by Miche. The profile of finite amplitude waves of permanent type is expressed in the form of Fourier's series:

$$\eta = A_1 \cos\theta + A_2 \cos 2\theta + A_3 \cos 3\theta + \cdots \qquad (2)$$

where: $\theta = 2\pi\left(\frac{t}{T} - \frac{x}{L}\right)$

$A_1, A_2, A_3, \ldots$ =function of H, L and h.

The condition of secondary wave crest formation is zero or negative curvature at $\theta=\pi$. Hence,

$$\left(\frac{d^2\eta}{d\theta^2}\right)_{\theta=\pi}=A_1-4A_2+9A_3-16A_4+\cdot\cdot\cdot\leq 0 \qquad (3)$$

When a wave height increases, the coefficients A_2, A_3, ... increase more rapidly than A_1. Hence the condition (3) can be satisfied for waves with the wave height larger than a critical value. Miche obtained his equation (1) by taking the first two terms of his equation for the wave profile.

3. One of problems associated with secondary waves is the travelling of secondary wave crests. Since it moves with a less speed than the main crest, it is detached from the main wave crest, appeared at the middle of wave trough, caugth up by the next wave crest, absorbed in the profile of main wave, and appears again on the next wave trough. Such a transformation of wave profile is well illustrated in Fig. A-1. These appearances and disappearances of secondary wave crests can be observed repeatedly as far as the channel extends. Such travelling secondary wave crests are beyond the scope of finite amplitude wave theories, because they are constructed on the assumption of a permanent wave profile.

The phenomenon of secondary waves is better understood by assuming that the wave in a wave channel consists of infinite number of independent progressive waves, the frequencies of which are n-times the fundamental one, or that of wave paddle motion, and that the secondary wave crest is due to the heigher frequency waves. By this assumption, the wave profile is expressed as follows:

$$\eta=A_o+\sum_{n=1}^{\infty}\eta_n=A_o+\sum_{n=1}^{\infty}A_n\cos\left(n\sigma t-k_nx+\omega_n\right) \qquad (4)$$

where: $\sigma=2\pi/T$
$n^2\sigma^2=k_ng\tanh kh$

and each independent wave has its own celerity of:

$$C_n=\sqrt{\frac{g}{k_n}\tanh k_nh} \qquad (5)$$

A test has been made for the actual velocities of secondary wave crests. In order to measure the velocity, two resistance type wave gages were set up at successive appearance points of secondary wave crests. As shown in Fig. A-3, the measured velocities of secondary wave crests drop between the celerity of twice frequency waves C_2 and that of thrice frequency waves C_3. There is a tendency that the velocity of a secondary wave crest decreases as the wave height increases. Another tendency is that the celerity of a secondary wave crest approaches C_3 from C_2 as the relative

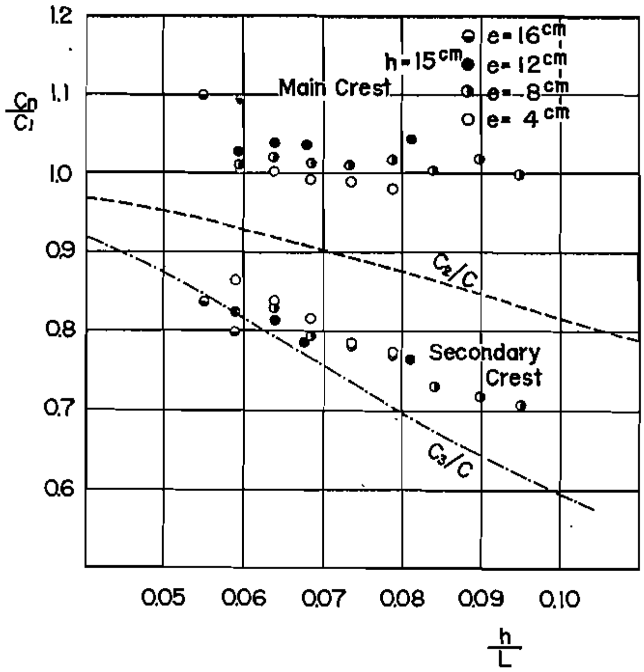


Fig. A-3. Celerities of Main and Secondary Wave Crests

water depth decreases. These tendencies support the assumption that the phenomenon of secondary waves is due to independent waves of higher frequencies, because higher harmonic components of a wave profile increases with an increase of wave steepness and with decrease of relative water depth.

It should be mentioned here that the amplitudes of component waves are not constant during their propagation, however. As seen in the results of the Fourier analysis shown in Fig. A-1, the amplitudes of harmonic components vary with respect to the distance x. This suggests the existence of interaction among component waves while travelling with their own celerities.

4. Another point of interest in the test was the appearance length of secondary wave crests, or the distance between two successive appearance points of secondary wave crests. This length is a characteristic one for waves with secondary wave crests, because the wave properties vary regularly with this length. Wave heights, wave profiles, and other properties should be investigated at least over one appearance length of secondary wave crests.

The appearance length of secondary waves is constant along a wave channel for given waves. This length is related to the wavelength as a function of relative water depth and wave steepness. When the ratio of the appearance length to the wavelength ℓ/L obtained in the experiment was plotted against the relative water depth h/L with the relative wave height H/h as a parameter, a very clear relationship was observed. Figure A-4 shows the experimental diagram for the ratio of appearance

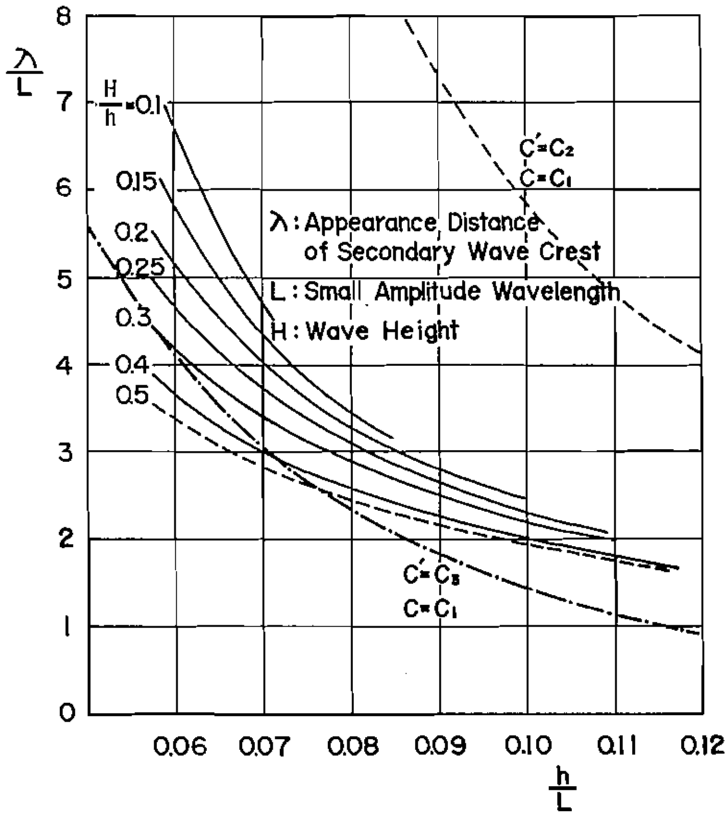


Fig. A-4. Experimental Diagram for Appearance Distance of Secondary Wave Crest

distance to wavelength. Generally it decreases with an increase of wave height or with an increase of relative water depth.

The appearance distance of secondary wave crests can be derived from the velocity of secondary waves as follows. Suppose there are two waves, a main wave and a secondary wave, and they have their own celerities C and C' respectively. The time t which is required for the main wave to travel the distance of appearance length is $t=\ell/C$. Since the secondary wave is just one wave behind by the main wave in travelling the distance ℓ, the time t' which is required for the secondary wave to travell the distance is: $t'=\ell/C'=t+T$. Hence, $\ell/C'=\ell/C+L/C$

$$\therefore\ \ell/L=C'/(C-C') \qquad (6)$$

If the wave celerity based on the small amplitude theory C_1 is taken as C and C_2 as C', the ratio of appearance length to wavelength ℓ/L is calculated with Eq. 6 as the dashed line in Fig. A-4. If C_3 is taken as C', the calculated ratio becomes like the dash-dot line in Fig. A-4. Since the actual wave celerity is greater than C_1, this approximation gives a greater value of ℓ/L.

The data in Fig. 4 are applied only when secondary waves appear. It has been observed in wave channels that the wave envelopes taken along the channels show the beat-like variations with repetition length longer than the wavelength. Such beat-like phenomena may be explained in a similar way with the assumption of infinite number of independent waves. The only difference between two phenomena will be the fact that higher frequency waves are not large enough to produce secondary wave crests in the case of beat-like phenomenon.

5. The above analysis is dealing with secondary waves actually observed. The reason why secondary waves are brought forth in a wave channel is not understood. But the phenomenon of secondary waves is not a peculiar one in a particular wave channel. It has been observed in many wave channels and wave basins where waves were generated by sinusoidal movements of wave paddles; the most of paddle motions contained little harmonic components. This suggests that the sinusoidal movement of wave paddle itself might be a cause of secondary wave appearance. Water particles near a wave paddle are forced to move sinusoidally by it, while proper movements of water particles under finite amplitude waves in relatively shallow water are like those given by cnoidal wave theory: i.e., large displacements in short duration under a wave crest and small displacements in long duration under a wave trough. The discrepancy between forced movements and proper ones of water particles seems to bring forth secondary waves in a wave channel. If a cnoidal-wave-like-motion can be given to a wave paddle, the formation of secondary wave crests may be suppressed.

6. Summing up the above results, the following conclusions are made on the phenomenon of secondary waves:

1) A secondary wave crest appears in a wave channel under certain wave condition.
2) A phenomenon of secondary waves can be explained by the assumption that the wave in a wave channel consists of infinite number of independent progressive waves, the frequencies of which are n-times the fundamental one, or the frequency of wave paddle motion.
3) The actual velocities of secondary waves have the values between the celerity of twice frequency waves and that of thrice frequency waves. This result supports the above assumption.
4) A diagram for the appearance length of secondary wave crests has been given experimentally.

References

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2. Miche, M.: "Mouvements Ondualatories de la Mer en Profondeur Constante ou Decroissante," *Ann. des Ponts et Chausees, Tome 114,* 1944 (referred by Horikawa (3)).

3. Horikawa, K.: "Secondary Wave Crest Formation," *Trans. of Japan Soc. Civil Eng., No. 66,* Jan. 1960.