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Evaluation of Resisting Moment Against Sliding as Rotating Mass

Two Dimensional Influence Charts of Stresses in a Semi-Infinite Elastic Body

by

Masatoshi Sawaguchi

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PORT AND HARBOUR RESEARCH INSTITUTE
MINISTRY OF TRANSPORT

1-1, 3 Chome, Nagase Yokosuka-City, Kanagawa-Prefecture, Japan

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概　要

粘土地盤がある荷重によってスベリ破壊を起こす時，荷重条件によっては局部的にスリベが発生することもあるとして，その際に考慮すべき側面抵抗を公式化し，種々の場合の形のスベリについて具体化し，使い易すいようにグラフを作成した。その際地盤の強度は深さと共に直線的に変化し，又可働率は１と仮定した。この結果を使って円筒体スベリ面抵抗の影響と，正方形フーチングの抵抗モーメントの計算を例題として述べてある。

Evaluation of Resisting Moment Against Sliding as Rotating Mass

MASATOSHI SAWAGUCHI*

SYNOPSIS

As one of calculation method in stability analysis of low embankments on clay was the evaluation of resisting moment proposed, assuming that the sliding soil mass was a rotating body by such a reason why the range of sliding area was limited, and moreover, the shape of the cross-section of the sliding mass might be a circular arc. As the ground is composed of clay, the $\phi=0$ analysis could be applied though the undrained strength of clay was assumed to be in proportion with the depth. This evaluation was applied to several cases where the sliding most occures. Finally the formula was demonstrated to the possibility of calculating of the resisting moment of a footing against sliding.

§ 1. Introductio

In the case that the range within which failure occurs is limited in its length and, strictly speaking, the length of side areas is relatively in a short distance in comparison with its width, the resisting moment acting on the slip surface has better to be estimated not only in two dimensional problem but also on both sides of the sliding mass; otherwise, the design will be apt to become uneconomical. In this report is a sliding mass considered to be a part of rotating mass and the resisting moment against sliding is evaluated, assuming that the shearing strength of soil is empirically in proportion of the depth of the ground. In this case the shearing strength of the soil is assumed to be given by undrained compression strength, and also its mobilization is assumed to be at the peak value everywhere over the slip surface.

§ 2. Method of Analysis

2—1 Evaluation

Supposing that the sliding of mass with its length within limited range occurs around a horizontal axis which is situated at some height, H, above the ground, the resisting moment against sliding can be obtained as the following general equation,

* Chief Researcher, Soil and Structure Division

in which the shape of the slip surface on the vertical section is expressed as $f(x)$.

The infinitesimal area of the slip surface is shown in Fig. 1, where the direction of the resisting force acting on this area, dF, is perpendicular to the axis, xx'. As the infinitisimal area is at a distance, $f(x)+H$, from the axis, the resisting moment against sliding on this area, dM_r, is expressed as

$$dM_r=\{f(x)+H\}df \qquad (1)$$

If this moment is integrated all over the slip surface, the resisting moment acting on there is given in the form

$$M_r=\int_0^L\int_{-\alpha}^{\alpha}\{f(x)+H\}df \qquad (2)$$

where α is the angle of circular arc of the slip line as shown in Fig. 2, and L is the maximum length of the sliding mass parallel to the axis as shown Fig. 3.

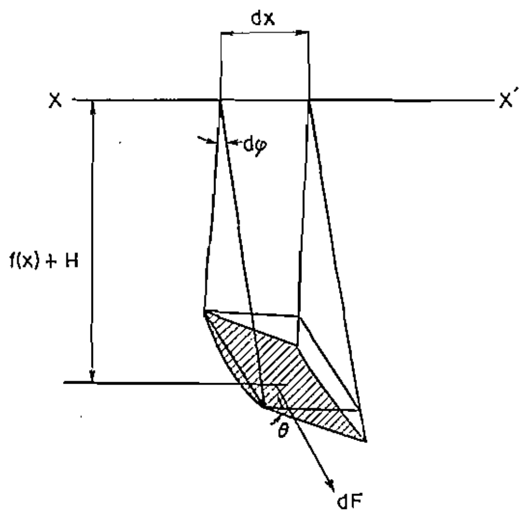

Fig. 1

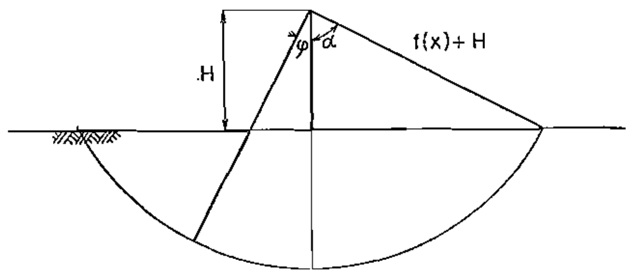

Fig. 2

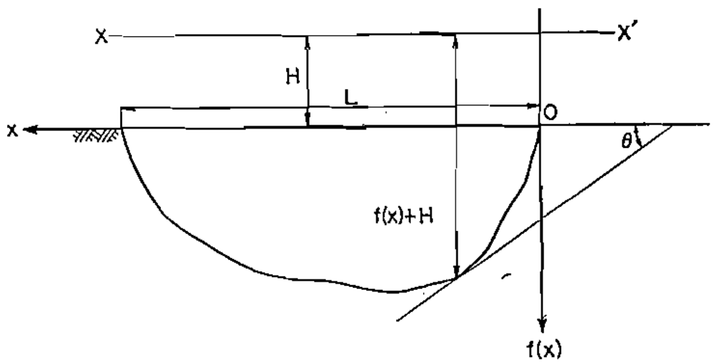

Fig. 3

Let us scrutinize the area, dF, in the equation above. Supposing that the undrained shearing strength, C_u, at a given depth, z, is in proportion of the depth, that is,

$$C_u = C_0 + k(z-H) \qquad (3)$$

where C_0 and k are the constants of the undrained shearing strength with respect to the depth, the resisting force acting on the infinitesimal area is given in the form

$$dF = \{C_0 + k(z-H)dA \qquad (4)$$

where dA is the magnitude of this area and can be obtained as follows; supposing that the angle between the tangent on the slip surface and the axis is θ, $\tan\theta = f'(x)$, and so

$$\begin{aligned} dA &= \{f(x)+H\}\sec\theta\, d\varphi\, dx \\ &= \{f(x)+H\}[\{f'(x)\}^2+1]^{1/2} d\varphi\, dx \end{aligned} \qquad (5)$$

where φ is the angle between the direction of a given infinitesimal area and the horizontal line as shown in Fig. 3.

Substituting eq. (3)(4) and (5) into eq. (2), we have the resisting moment

$$M_r = \int_0^L \int_{-\alpha}^{\alpha} \{f(x)+H\}^2 [\{f'(x)\}^2+1]^{1/2} \{C_0 + k(z-H)\} d\varphi\, dx \qquad (6)$$

As z in this equation is

$$\{f(x)+H\}\cos\varphi$$

so, finally, the general equation in the resisting moment is expressed in the form

$$M_r = \int_0^L \int_{-\alpha}^{\alpha} \{f(x)+H\}^2 [\{f'(x)\}^2+1]^{1/2} [C_0 + k\{(f(x)+H)\cos\varphi - H\}] d\varphi dx \qquad (7)$$

Furthermore, if M_{cr} and M_{kr} denote the terms of the equation above corresponding to C_0 and k respectively,

$$M_{cr} = C_0 \int_0^L \int_{-\alpha}^{\alpha} \{f(x)+H\}^2 [\{f'(x)\}^2+1]^{1/2} d\varphi\, dx \qquad (8)$$

$$M_{kr} = k \int_0^L \int_{-\alpha}^{\alpha} \{f(x)+H\}^2 [\{f'(x)\}^2+1]^{1/2} [\{f(x)+H\}\cos\varphi - H] d\varphi \cdot dx \qquad (9)$$

As $\qquad \alpha = \cos^{-1}\dfrac{H}{f(x)+H}$

$$M_{cr} = 2C_0 \int_0^L \{f(x)+H\}^2 [\{f'(x)\}^2+1]^{1/2} \cos^{-1}\frac{H}{f(x)+H} dx$$

$$\begin{aligned} M_{kr} &= k \int_0^L \int_{-\alpha}^{\alpha} \{f(x)+H\}^3 [\{f'(x)\}^2+1]^{1/2} \cos\varphi \cdot d\varphi \cdot dx \\ &\quad - 2_k H \int_0^L \{f(x)+H\}^2 [\{f'(x)\}^2+1]^{1/2} \cos^{-1}\frac{H}{f(x)+H} dx \end{aligned}$$

where the first term of M_{kr} is

$$k\int_0^L\int_{-\alpha}^{\alpha}\{f(x)+H\}^3[\{f'(x)\}^2+1]^{1/2}\cos\varphi\cdot d\varphi\cdot dx$$

$$=2_k\int_0^{\alpha}\{f(x)+H\}^3[\{f'(x)\}^2+1]^{1/2}\sin\alpha\cdot dx$$

Here, we setting

$$F=\int_0^L\{f(x)+H\}^2[\{f'(x)\}^2+1]^{1/2}\cos^{-1}\frac{H}{f(x)+H}dx \tag{10}$$

$$G=\int_0^L\{f(x)+H\}^2[\{f'(x)\}^2+1]^{1/2}[\{f(x)+H\}^2-H]^{1/2}dx \tag{11}$$

the resisting moment against sliding is, as a result, expressed briefly in the form

$$M_r=2C_0F+2k(G-HF) \tag{12}$$

In what follows, a few specific forms of $f(x)$ will be given and evaluated to practical use.

2—2 Case 1 Cylinder

In this case, as $f(x)=D$ and $f'(x)=0$, so

$$F=\int_0^L(D+H)^2\cos^{-1}\frac{H}{D+H}dx$$

$$=L(D+H)^2\cos^{-1}\frac{H}{D+H} \tag{13}$$

$$G=\int_0^L(D+H)^2\{(D+H)^2-H^2\}^{1/2}dx$$

$$=L(D+H)^2\{(D+H)^2-H\}^{1/2} \tag{14}$$

Then, if dimensionless functions F_1 and G_1 are used instead of F and G for convenience,

$$F=LH^2F_1$$

$$G=LH^3G_1$$

where

$$F_1=\left(1+\frac{D}{H}\right)^2\cos^{-1}\frac{1}{1+D/H} \tag{15}$$

$$G_1=\left(1+\frac{D}{H}\right)^2\left\{\left(1+\frac{D}{H}\right)^2-1\right\}^{1/2} \tag{16}$$

As a result, the resisting moment against sliding in this case is

$$M_r=2C_0LH^2F_1+2kLH^3(G_1-F_1) \tag{17}$$

The results of calculation with F_1 and G_1 are shown in Fig. 4.

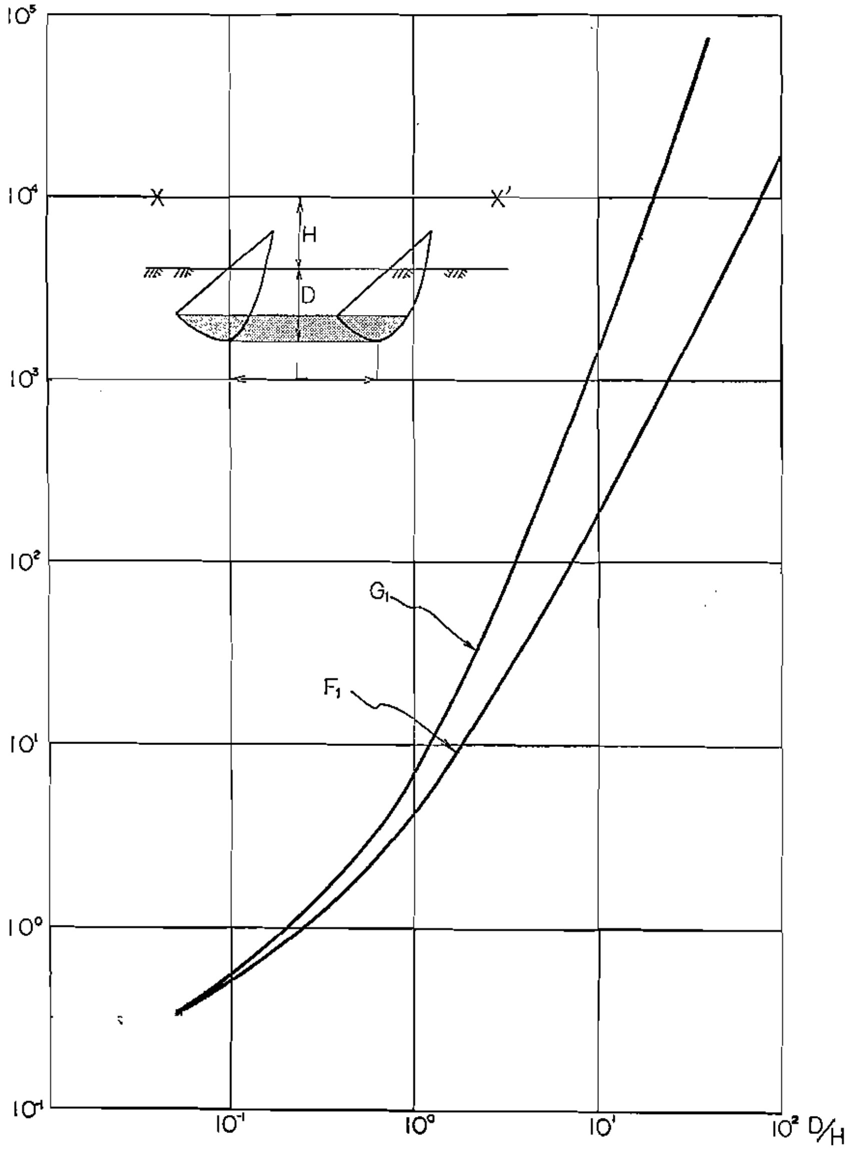

Fig. 4

2—3 Case 2 Cone

This case is not practicable but useful for the general equation in case 3.

$$f(x)=\frac{D}{L}x \qquad\qquad f'(x)=\frac{D}{L}$$

If these equations are substituted into eq. (10) and (11) in the same way as in case 1,

$$F=\int_0^L\left(H+\frac{D}{L}x\right)^2\left\{1+\left(\frac{D}{L}\right)^2\right\}^{1/2}\cos^{-1}\frac{H}{H+\frac{D}{L}x}dx \tag{18}$$

Though the process of integration of this equation is omitted here, as a result,

$$F=\frac{H^3}{3}\left\{1+\left(\frac{L}{D}\right)^2\right\}^{1/2}F_2\left(\frac{D}{H}\right) \tag{19}$$

where

$$F_2\left(\frac{1}{1+D/H}=t_0\right)=\frac{\cos^{-1}t_0}{t_0^3}-\frac{1}{2}\left(\frac{\sqrt{1-t_0^2}}{t_0^2}+\frac{1}{2}\ln\frac{1+\sqrt{1-t_0^2}}{1-\sqrt{1-t_0^2}}\right) \tag{20}$$

Likewise,

$$G=\int_0^L\left(H+\frac{D}{L}x\right)^2\left\{1+\left(\frac{D}{L}\right)^2\right\}^{1/2}\left\{\left(H+\frac{D}{L}x\right)^2-H^2\right\}^{1/2}\cdot dx \tag{21}$$

This becomes

$$G=\frac{H^4}{4}\left\{1+\left(\frac{L}{D}\right)^2\right\}^{1/2}G_2\left(\frac{D}{H}\right) \tag{22}$$

where

$$G_2\left(\frac{D}{H}\right)=\left(1+\frac{D}{H}\right)\left\{\left(1+\frac{D}{H}\right)^2-1\right\}^{3/2}+\frac{1}{2}\left(1+\frac{D}{H}\right)\left\{\left(+1\frac{D}{H}\right)^2-1\right\}^{1/2} \tag{23}$$
$$-\frac{1}{2}\cos^{-1}\left(1+\frac{D}{H}\right)$$

So that, the resisting moment against sliding is

$$M_r=\frac{2}{3}C_0H^3\left\{1+\left(\frac{L}{D}\right)^2\right\}^{1/2}F_2+\frac{1}{2}kH^4\left\{1+\left(\frac{L}{D}\right)^2\right\}\left(G_2-\frac{4}{3}F_2\right) \tag{24}$$

G_2 and F_2 will be given in Fig. 5.

2—4 Case 3 Disk

This is a special one of case 2 as shown in Fig. 5, and the equation of moment can be obtained by making L approach to zero.

Thus, the resulting moment against sliding is expressed as

$$M_r=\frac{2}{3}C_0H^3F_2+\frac{1}{2}kH^4\left(G_2-\frac{4}{3}F_2\right) \tag{25}$$

where F_2 and G_2 are all the same as in case 2 because these equations include only the maximum depth of the slip surface, D, and the height up to the axis from the ground surface, H.

2—5 Case 4 Ellipsoid of revolution

As $f(x)$ in this case is in general expressed as

$$f(x)=R_1\sqrt{1-\left(\frac{x}{R_2}\right)^2}+H \tag{26}$$

so

$$f'(x)=-\frac{R_1}{R_2}\frac{x/R_2}{\sqrt{1-(x/R_2)^2}}$$

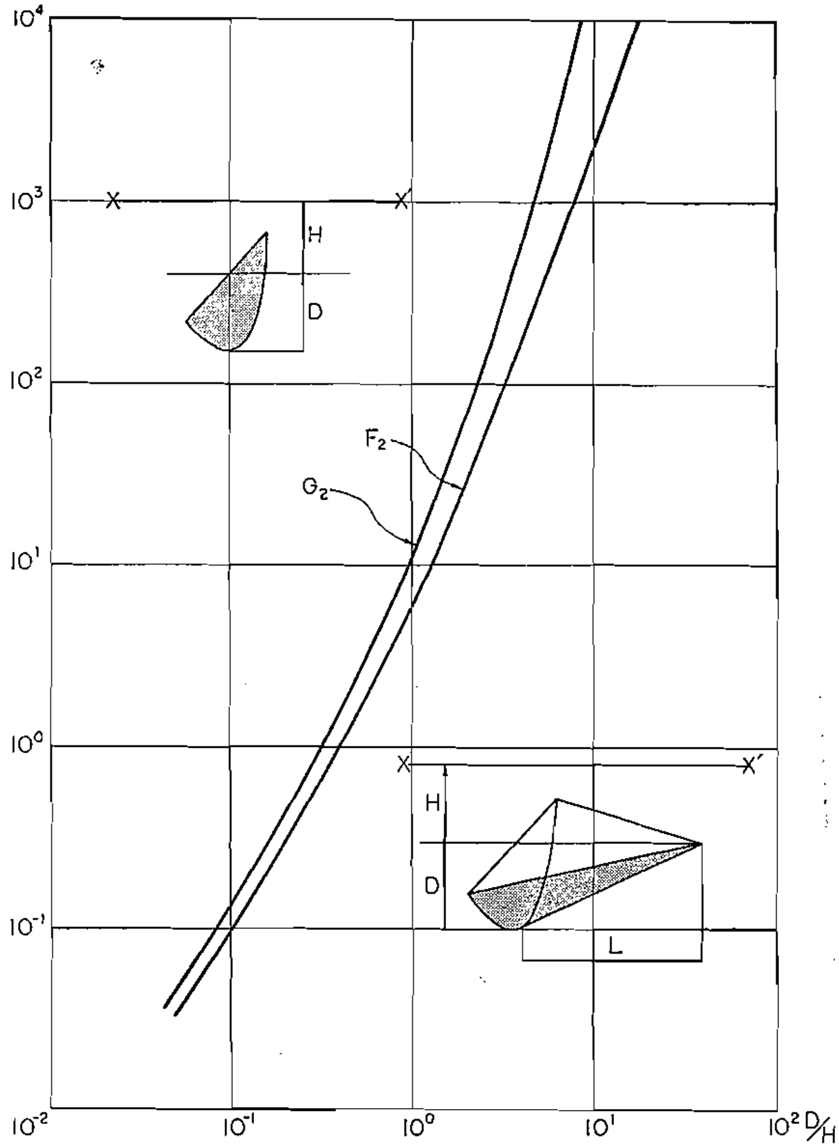

Fig. 5

Substituting these equation into eq. (10) and (11), and expressing them with dimensionless functions, F_3 and G_3, the resisting moment against sliding is given in the form

$$M_r = 4C_0R_1^2R_2F_3 + 4kR_1^3R_2\left(G_3 - \frac{H}{R_1}F_3\right) \tag{27}$$

where

$$F_3\left(\frac{H}{R_1}\,\frac{R_1}{R_2}\right) = \int_0^{L/R_2}(1-u^2)^{1/2}\left[1+\left\{\left(\frac{R_1}{R_2}\right)^2-1\right\}u^2\right]^{1/2}\cos^{-1}\frac{H/R_1}{(1-u^2)^{1/2}}du \tag{28}$$

$$G_3\left(\frac{H}{R_1}\,\frac{R_1}{R_2}\right) = \int_0^{L/R_2}(1-u^2)^{1/2}\left[1+\left\{\left(\frac{R_1}{R_2}\right)^2-1\right\}u^2\right]^{1/2}\left\{1-\left(\frac{H}{R_1}\right)^2-u^2\right\}^{1/2}du \tag{29}$$

$$\frac{L}{R_2}=\left\{1-\left(\frac{H}{R_1}\right)^2\right\}^{1/2}$$

As it is impossible for us to solve the equation above, the numerical solution was calculated by the digital computer and the result is given in Fig. 6.

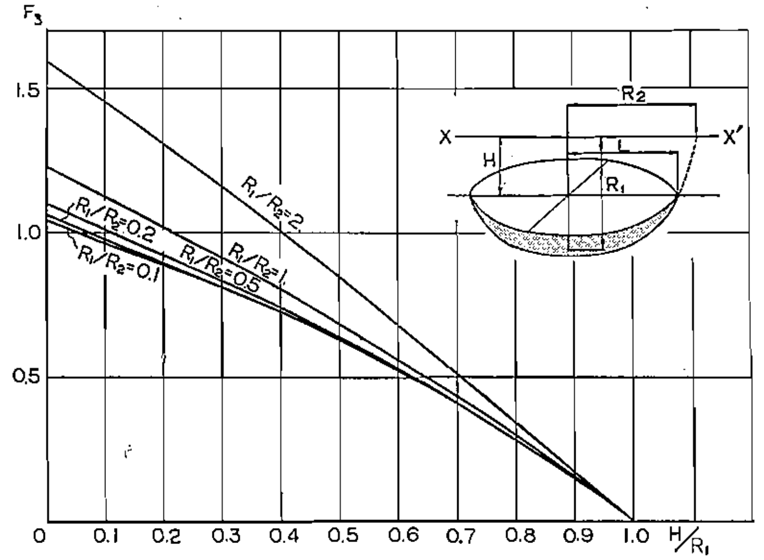

Fig. 6 (1)

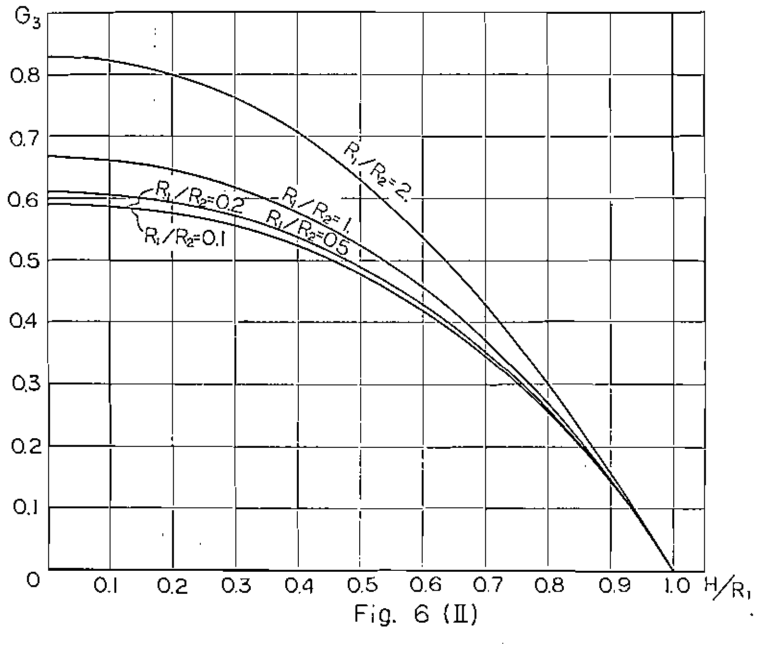

Fig. 6 (2)

2—6 Case 5 Sphere

This is the case of $R=R_1=R_2$ in case 4.
Accordingly, the resisting moment against sliding is written as follows;

$$M_r=4C_0R^3F_3'+4kR^4\left(G_3'-\frac{H}{R}F_3'\right) \tag{30}$$

where

$$F_3'=\int_0^{L/R}(1-u^2)^{1/2}\cos^{-1}\frac{H/R}{(1-u^2)^{1/2}}du \tag{31}$$

$$G'=\int^{L/R}(1-u^2)^{1/2}\left\{1-\left(\frac{H}{R}\right)^2-u^2\right\}^{1/2}du \tag{32}$$

$$\frac{L}{R}=\left\{1-\left(\frac{H}{R}\right)^2\right\}^{1/2}$$

The solution is given in the same figure as in case 4.

2—7 Example 1

If we suppose that a failure occurs within a limited area, it is interesting to investigate what difference will be made between the case of including the effect of the resisting moment acting on the side areas on the total resisting moment against sliding and the case where this effect on its sides is neglected. For this purpose, let us assume $D=H=6$ m, $C_0=1$ t/m² and $k=0.125$ t/m³ and make the value of L variable. With the values of dimensionless functions in the figure of case 1, which correspond to the condition above,

$$F_1=4.2 \qquad G_1=7.0$$

$$M_r=2\times1\times6^2\times4.2\times L+2\times0.125\times6^3\times L\times(7.0-4.2)$$

$$=453.6L \text{ t-m}$$

In case 3 under the same condition, from Fig. 5,

$$F_2=6.2 \qquad G_2=11$$

and then

$$M_r=\frac{2}{3}\times1\times6^3\times6.2+\frac{11}{2}\times6^4\times\left(11-\frac{4}{3}\times6.6\right)$$

$$=16573 \text{ t-m}$$

Thus, the ratio of the combined resisting moment against sliding to that one without the effect of the side areas,

$$\frac{453.6L}{453.6L+16573}$$

varies by the length, L, as shown in Fig. 7.

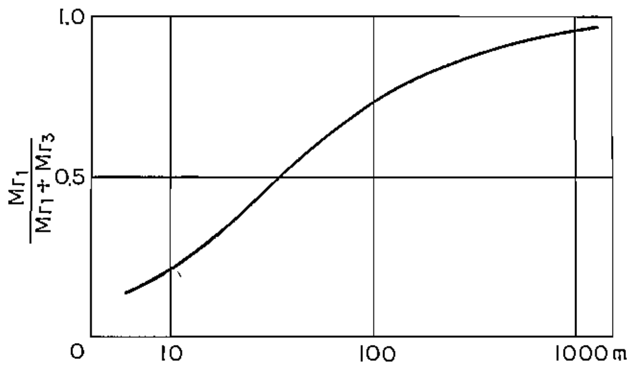

Fig. 7

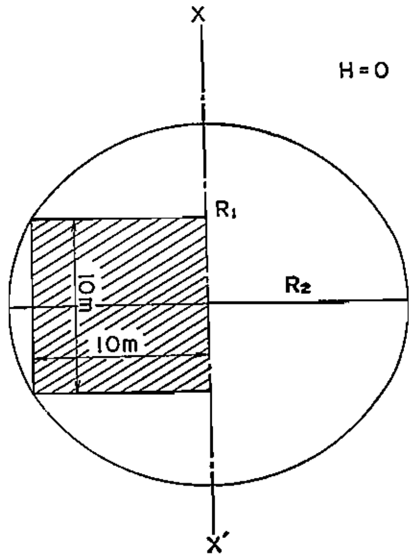

Fig. 8

This figure shows that, when the effect of side areas on the total resisting moment is neglected, more than two in factor of safety is still included in the estimation under $L=35$ m, and it seems very uneconomical.

2—8 Example 2

In the case that an isolated footing fails, it can be empirically supposed that the sliding mass becomes an ellipsoid of revolution.

Supposing, for example, that such a square isolated footing as shown in Fig. 8 rotates around one of its sides as an axis; that is, $H=0$, let us calculate the resisting moment against sliding as a shape of ellipsoid of revolution. As it is resonable to assume that whole of a square footing lies on the surface of the ground of the sliding mass, and the elliptic curve on the surface passes through the corners of the square footing, so, let us examine whether we can obtain the magnitude of R_1 and R_2 in the least factor of safety by varying the ratio, R_1/R_2. In this case also, the soil properties are assumed to be quite the same as in example 1.

As $H=0$, R_1 and R_2 have the relation as follows;

$$\left(\frac{10}{R_2}\right)^2+\left(\frac{5}{R_1}\right)^2=1$$

F_3 and G_3 can be found in Fig. 6 for each value of R_1/R_2.

R_1/R_2	2	1	0.5	0.2	0.1
F_3	1.60	1.23	1.10	1.06	1.05
G_3	0.83	0.67	0.61	0.59	0.59

From the value above and eq. (33), the resisting moment can be calculated;

R_1/R_3	2	1	0.5	0.2	0.1	
M_r	65300	12240	4610	4555	7570	t-m

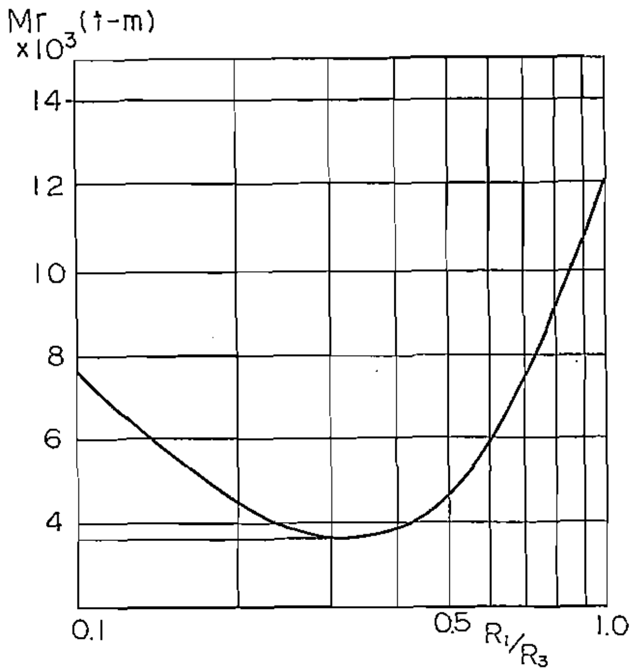

Fig. 9

Fig. 9 is the plotting of these values. This shows that M_r is at the minimum at $R_1/R_2=0.3$, that is, from eq. (33) together,

$$R_1=6\,\mathrm{m}$$

$$R_2=20\,\mathrm{m}$$

Thus, if the condition of the ground and the dimension of a footing are given, the sliding mass as an ellipsoid of revolution can be determined for a given height.

§ 3. Conclusion

If the undrained shearing strength of soil under a structure is assumed to be linear with respect to the depth, the resisting moment against sliding can be obtained by assuming that the shape of the sliding mass is a rotating one with a horizontal axis above the ground at a given height. When the strength of soil is uniform, the solution can be obtained by setting $k=0$ as a special case.

Finally, the auther wishes to express his gratitude to Mr. Nakase who had given much suggestion to this report.

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