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by

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ON THE f^{-5} LAW OF WIND-GENERATED WAVES

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Abstracts: Using the first perturbed vorticity equation, it is shown that the infinitesimal vorticies in water are rapidly amplified when the surface progressive wave has the density distribution of frequency spectrum of surface profile in the form of $E(f)\sim f^{-n}$ $(n>5)$. This amplification of vortices does not exist if $n<5$. At $n=5$, the logarithmic infinity appears and may be physically considered as the equilibrium condition. This result may be interpreted that, if $n>5$, some of the wave energy in the concerned part of the spectrum is transfered to the induced vortex motion.

An experiment was added to clarify this property of progressive random waves. Wind waves are generated in a uni-directional air-water tunnel, and the wind is excluded from the water surface in a midway. After some passage in a calm condition, the effect of breaking disappears and the capillary waves become weak. At this condition of stationary state, the decay of waves is examined. The result is agreeable with the above-mentioned analytical consideration.

1. Introduction

An equilibrium condition

$$E(f)=\text{Const.}\quad g^2 f^{-n}(n\simeq 5) \qquad (1)$$

$E(f)$: frequency spectrum of surface profile
g: acceleration of gravity
f: frequency

was pointed out by O. M. Phillips (1958, 1963) for the high frequency part of frequency spectrum of wind-generated waves. It is understood physically as the condition that the energy given by wind to waves is balanced by the dissipation caused by the surface instabilities of waves and g^2f^{-5} is dimensionally correct. In field observations, R. W. Burling (1959) and B. Kinsman (1961) showed that high frequency parts of actually observed spectrum are very near to this epuilibrium condition, and in their cases n in (1) usually situates between 4.5 and 5.5.

The observation of ocean waves by floating buoy published by M. S. Longuet-Higgins, D. E. Cartwright and N. D. Smith (1963) also shows that the relation (1) is considerably agreeable with the observed data. Our previous experiment (1963) of an air-water tunnel shows that, in cases of short fetch and strong wind, n is greater rather than 5, but even in these cases n becomes small and approaches to 5 at the part of very high frequency. Accordingly, the relation (1) may be generally applicable to wind-generated waves in cases when the wind is uni-directional and the generated waves has the directional spreading but has not so much reverse progressive components.

2. The physical interpretation

We consider progressive waves in deep water. x axis is taken along the principal direction of the progress of waves at the water surface. y axis is taken vertically to x axis at the water surface, and z axis is taken to vertical to X (x, y) plane, upwards positive. To simplify the problem the motion is assumed to be a stationary random (real) process for X (x, y), and wave spectrum to spread symmetrically for x axis. The reverse progressive components of waves are neglected.

We consider the Fourier-Stieltjes type expression of surface profile, and make to correspond $-K$ to wave number vector K. Therefore the surface profile may be expressed

$$\eta=\frac{1}{g}\left(\frac{\partial\varphi}{\partial t}\right)_{z=0}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{k_1, k_2} e^{iK.X} e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}\,t+\varepsilon(K)]} \qquad (2)$$

k_1 k_2

$$\varepsilon(-K)=-\varepsilon(K),\ C_{k_1,k_2}=C_{-k_1,-k_2}$$

Here φ ; velocity potential of wave motion, k_1 : wave number for x direction ($k_1= |K|\cos\theta$), k_2 : wave number for y direction ($k_2=|K|\sin\theta$), C : phase velocity vector of actual components of progressive waves. In the present case, $\mathrm{sgn}(K.C)=+1$ for $k_1>0$, $\mathrm{sgn}(K.C)=-1$ for $k_1<0$.

The wave number vector K is related to the period of component wave as

$$\frac{2\pi}{T}=\sqrt{g|K|} \qquad (3)$$

The two-dimensional spectrum of wave profile is represented by

$$<\eta^2>=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(K)dK \qquad (4)$$

and $\phi(K)$ is related to C_{k_1,k_2} by $C_{k_1,k_2}=\sqrt{\phi(K)\delta K}$

Frequency spectrum $E(f)$ is given by

$$<\eta^2>=\int_0^{\infty}E(f)df\left(=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(K)dK\right) \qquad (5)$$

and, at the present problem, the following relation is easily introduced.

$$\left.\begin{array}{lllll} E(f) & f^{-3} & f^{-5} & f^{-7} & f^{-9} \\ \phi(K) & |K|^{-3} & |K|^{-4} & |K|^{-5} & |K|^{-6} \\ \sqrt{\phi(K)} & |K|^{-1.5} & |K|^{-2} & |K|^{-2.5} & |K|^{-3} \end{array}\right\} \quad (6)$$

The velocity potential φ of this wave motion is given by

$$\varphi=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\phi(K)\delta K}\frac{i\sqrt{g}}{\mathrm{sgn}(K.C)\sqrt{|K|}}e^{iK.X}e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}t+\varepsilon(K)]}e^{|K|z} \quad (7)$$

$k_1 \; k_2$

Accordingly, particle velocity $U_0(u_0, v_0, w_0)$ by this wave motion is given by

$$u_0=-\frac{\partial\varphi}{\partial x}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\phi(K)\delta K}\frac{\sqrt{g}}{\mathrm{sgn}(K.C)\sqrt{|K|}}k_1e^{iK.X}e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}t+\varepsilon(K)]}e^{|K|z} \quad (8)$$

$$v_0=-\frac{\partial\varphi}{\partial y}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\phi(K)\delta K}\frac{\sqrt{g}}{\mathrm{sgn}(K.C)\sqrt{|K|}}k_2e^{iK.X}e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}t+\varepsilon(K)]}e^{|K|z} \quad (9)$$

$$w_0=-\frac{\partial\varphi}{\partial z}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}-\sqrt{\phi(K)\delta K}\; i\sqrt{g}\,\mathrm{sgn}(K.C)\sqrt{|K|}\,e^{iK.X}e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}t+\varepsilon(K)]}e^{|K|z} \quad (10)$$

Then we consider the vorticity equation in water, expressing the vorticity ω (ξ, η, ζ) and velocity U.

$$\frac{\partial\omega}{\partial t}+U.\nabla(\omega)=\omega.\nabla(U)+\nu\nabla^2\omega \quad (11)$$

$$\nabla=i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}, \qquad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

Inserting $\omega=\omega_0+\varepsilon\omega_1$, $U=U_0+\varepsilon U_1$ into (11), we obtain the first order perturbed vorticity epuation

$$\frac{\partial\omega_1}{\partial t}+U_0.\nabla(\omega_1)=\omega_1.\nabla(U_0)+\nu\nabla^2\omega_1 \quad (12)$$

$$\nabla.U_0=0, \qquad \nabla.\omega_1=0$$

Here, as the initial condition, $\omega_0(\xi_0, \eta_0, \zeta_0)=0$ is naturally assumed. In (12) the left hand side of equation means the change of the first perturbed vorticity ω_1 along the particle motion of wave, and the first term of the right hand side means the generation of vorticity and the second term shows the dissipation of vorticity by molecular viscosity. At the initial time $|\omega_1|$ is considered to be very small. By this way, to examine the development of ω_1, we must investigate the behavior of $\omega_1.\nabla(U_0)$. That is : concerned to ξ_1

$$\xi_1\frac{\partial u_0}{\partial x}+\eta_1\frac{\partial u_0}{\partial y}+\zeta_1\frac{\partial u_0}{\partial z} \tag{13}$$

concerned to η_1

$$\xi_1\frac{\partial v_0}{\partial x}+\eta_1\frac{\partial v_0}{\partial y}+\zeta_1\frac{\partial v_0}{\partial z} \tag{14}$$

concerned to ζ_1

$$\xi_1\frac{\partial w_0}{\partial x}+\eta_1\frac{\partial w_0}{\partial y}+\zeta_1\frac{\partial w_0}{\partial z} \tag{15}$$

We put the Fourier transform of u_0, v_0, w_0, ξ_1, η_1, ζ_1 in $X\ (x, y)$ plane as $F_i(K, z)$ ($i=1, 2, \cdots\cdots, 6$) respectively. From the condition of real stationary process, $F_i(K,z) =\overline{F_i(-K, z)}$ (bar means the complex conjugate, and z is only combined to $|K|$.) is consistent.

The Fourier transform of (13) is

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_4(K',z)i(k_1-k_1')F_1(K-K',z)dK'$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_5(K',z)i(k_2-k_2')F_1(K-K',z)dK'$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_6(K',z)|K-K'|F_1(K-K',z)dK' \tag{16}$$

(14) and (15) are also transformed similarly. $F_1(K, z)$ is expressed by

$$u_0(X, z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_1(K, z)e^{iK.X}dK \tag{17}$$

and so from (8)

$$F_1(K, z)=\sqrt{\frac{\phi(K)}{\delta K}}\frac{\sqrt{g}}{\mathrm{sgn}(K.C)\sqrt{|K|}}k_1e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}t+\varepsilon(K)]}e^{|K|z} \tag{18}$$

In a similar way,

$$F_2(K, z)=\sqrt{\frac{\phi(K)}{\delta K}}\frac{\sqrt{g}}{\mathrm{sgn}(K.C)\sqrt{|K|}}k_2e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}t+\varepsilon(K)]}e^{|K|z} \tag{19}$$

$$F_3(K, z)=-i\sqrt{\frac{(\phi K)}{\delta K}}\mathrm{sgn}(K.C)\sqrt{g|K|}e^{i[-\mathrm{sgn}(K.C)\sqrt{g|K|}t+\varepsilon(K)]}e^{|K|z} \tag{20}$$

Using (18), the first term of (16) is

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_4(K',z)i(k_1-k'_1)F_1(K-K', z)dK'$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_4(K',z)\sqrt{\frac{\phi(K-K')}{\delta(K-K')}}\frac{\sqrt{g}i(k_1-k_1')^2}{\mathrm{sgn}((K-K').C)\sqrt{|K-K'|}}$$

$$\cdot\ e^{i[-\mathrm{sgn}((K-K').C)\sqrt{g|K-K'|}t+\varepsilon(K-K')]}e^{|K-K'|z}dK' \tag{21}$$

Now we use the relation $dK'=dk_1'dk_2'=d(k_1-k_1')d(k_2-k_2')=d(K-K')$, and (21) is transformed to

$$\iint_{(K-K')} F_4(K',z)\sqrt{\frac{\phi(K-K')}{\delta(K-K')}}\frac{\sqrt{g}\,i(k_1-k_1')^2}{\mathrm{sgn}((K-K').C)\sqrt{|K-K'|}}$$

$$\cdot\, e^{i[-\mathrm{sgn}((K-K').C)\sqrt{g|K-K'|}t+\varepsilon(K-K')]}\, e^{|K-K'|z}\, d(K-K')$$

$$=F_4(K,z)\iint_{(K-K')\to 0}\sqrt{\frac{\phi(K-K')}{\delta(K-K')}}\frac{\sqrt{g}\,i(k_1-k_1')^2}{\mathrm{sgn}((K-K').C)\sqrt{|K-K'|}}$$

$$\cdot\, e^{i[-\mathrm{sgn}((K-K').C)\sqrt{g|K-K'|}t+\varepsilon(K-K')]}\, e^{|K-K'|z} d(K-K')$$

$$+\iint_{\substack{(K-K')\\ \text{other part}}} F_4(K',z)\sqrt{\frac{\phi(K-K')}{\delta(K-K')}}\frac{\sqrt{g}\,i(k_1-k_1')^2}{\mathrm{sgn}((K-K').C)\sqrt{|K-K'|}}$$

$$\cdot\, e^{i[-\mathrm{sgn}((K-K').C)\sqrt{g|K-K'|}t+\varepsilon(K-K')]} e^{|K-K'|z} d(K-K') \qquad (22)$$

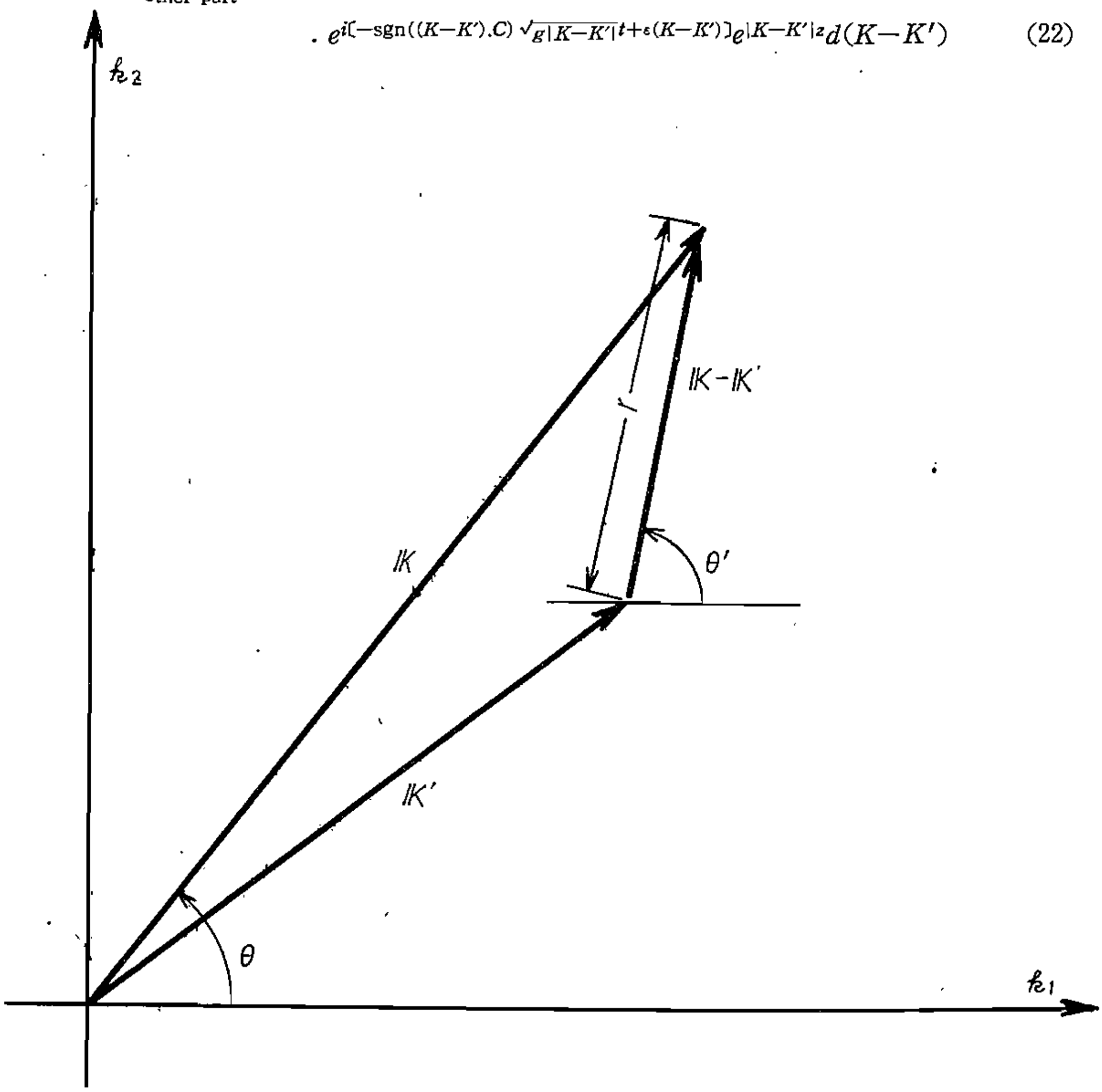


Fig-1. Relation of K, K' and $K-K'$

Fig-1 shows the relation of three vectors (K, K', $K-K'$) of wave number. We consider the cas when the frequency spectrum of wave is expressed by the f^{-5} law, and so from (6)

$$\phi(K)=A^2|K|^{-4}f(\theta) \qquad (f(\theta)\geqq 0) \tag{23}$$

At the present problem, we set

$$f(\theta)=f(-\theta)=f(\theta+\pi) \tag{24}$$

From (23)

$$\sqrt{\phi(K-K')}=A\,|K-K'|^{-2}\sqrt{f(\theta')} \tag{25}$$

So the first integral of (22) becomes

$$F_4(K,z)\int\!\!\int_{(K-K')\to 0}\frac{A\,|K-K'|^{-2}\sqrt{f(\theta')}}{\sqrt{\delta(K-K')}}\,\frac{\sqrt{g}\,i\,|K-K'|^2\cos^2\theta'}{\mathrm{sgn}((K-K').C)\sqrt{|K-K'|}}$$

$$\cdot\, e^{i[-\mathrm{sgn}((K-K').C)\sqrt{g|K-K'|}t+\varepsilon(K-K')]}e^{|K-K'|z}d(K-K') \tag{26}$$

putting $K-K'=\gamma e^{i\theta'}$, $\gamma=|K-K'|$, it becomes

$$F_4(K,z)\int\!\!\int_{\substack{\gamma\to 0\\ \pi\geqq\theta'\geqq-\pi}}\frac{A\sqrt{f(\theta')}}{\sqrt{\gamma\delta\gamma\delta\theta'}}\,\frac{\sqrt{g}\,i\cos^2\theta'}{\mathrm{sgn}((K-K').C)\sqrt{\gamma}}$$

$$\cdot\, e^{i[-\mathrm{sgn}((K-K').C)\sqrt{g\gamma}t+\varepsilon(\gamma e^{i\theta'})]}e^{\gamma z}\gamma\, d\gamma\, d\theta' \tag{27}$$

Integration of (27) concerned to θ' does not become zero when the sign of $\mathrm{sgn}((K-K').C)$, the relation $\varepsilon(K-K')=-\varepsilon(-(K-K'))$ and the relation of (24) are considered. Integration of (27) concerned to γ simply contains

$$F_4(K,z)\int_{0(+1)}^{0(+2)}\frac{A\sqrt{g}\,i}{\sqrt{\delta\gamma}}d\gamma \tag{28}$$

When γ becomes zero, $\delta\gamma$ becomes $0(\gamma^2)$, and so $\sqrt{\delta\gamma}$ becomes $0(\gamma)$.

Therefore (28) becomes

$$F_4(K,z)\int_{0(+1)}^{0(+2)}\frac{A\sqrt{g}\,i}{\sqrt{\delta\gamma}}d\gamma\sim F_4(K,z)A\sqrt{g}\,i\,\Big|\log_e\gamma\Big|_{0(+1)}^{0(+2)} \tag{29}$$

It means that (27) contains the logarithmic infinity.

When (23) is presented by

$$\phi(K)=A^2|K|^{-\alpha}f(\theta) \tag{30}$$

Then (i) if $\alpha>4$, the above-mentioned integration contains $\left|\frac{1}{\gamma^\beta}\right|_{0(+1)}^{0(+2)}(\beta>0)$, and the order of infinity increases with the increase of α. (ii) if $\alpha<4$, the integration contains $\left|\frac{1}{\gamma^\beta}\right|_{0(+1)}^{0(+2)}(\beta<0)$, and so the integration becomes zero. We can also obtain the similar result from the second and the third terms of (16).

If we take the Fourier transforms of (14) and (15), we can make the same computation by making use of (19) and (20) respectively, and the result is same. By this way we can find that the Fourier transforms of (13) (14) and (15) become divergent with the limiting boundary of the f^{-5} representation of frequncy specctrum of wave profile, and the order of divergence increases with the increase of n in the representation $E(f) \sim f^{-n} (n \geqq 5)$. On the contrary, if $n < 5$, this divergence does not occur.

The existence of this divergence means physically that the first perturbed vorticity $\omega_1(\xi_1, \eta_1, \zeta_1)$ grows up with the wave motion when the no perturbed vorticity $\omega_0(\xi_0, \eta_0, \zeta_0)$ is considered to be zero, and the wave motion is irrotational originally. Stillmore, as the no perturbed vorticity ω_0 is assumed to be zero, the energy of this first perturbed vorticity, which increases with time, should be supplied from the wave motion.

3. The experimental result

If the computational result in 2. is right and the effect is not so small, the progressive waves, which are originated by wind and have the spectrum representation of $E(f) \sim f^{-n} (n > 5)$ in the high frequency part, will show the rapid decay of spectrum intensity at the concerned part of spectrum. The results of our experiment clearly support this expectation. To take up only this effect, we must exclude the wind influence on water surface and the effect of breaking as the surface instability. When we can succeed to exclude the above two effects from the wave, the effect of capillary wave also becomes small. The influence of viscosity is not eliminated from the experiment, and so this effect is estimated by computation.

The experimental waterway is the same as the one mentioned in the reference of T. Hamada (1963). The width is 150cm, and depth of water to bottom in this case is taken to 15cm. Air flow touches the water surface at the air inlet and blows on water for the length of 1,360cm and generates water waves. Then the air flow is excluded from the water surface strictly. Section C-2, which is used to the measurement of frequency spectrum of waves is distant 965cm from the section of the exclusion of air flow in the direction of wave progress. Section D, which is also used to the measurement is distant 450.5cm from the Section C-2 in the same direction.

We measure the frequency spectrums of wave profile at both C-2 and D at the stationary wave condition, and examine the variation of the spectrums. The distance 965cm between the section of exclusion of air flow and the section C-2 is taken to exclude the effect of surface breaking of waves. The number of revolution of blower is taken to r.p.m. 300 and r.p.m. 400. The wind velocities by these rotation numbers are shown in the reference of T. Hamada (1963). Though the air flow is excluded from the water surface at the mid-way in the present case, and so the dynamical condition is somewhat different from the case of the reference, the wind velocity may

Fig 2-1

Fig. 2-2

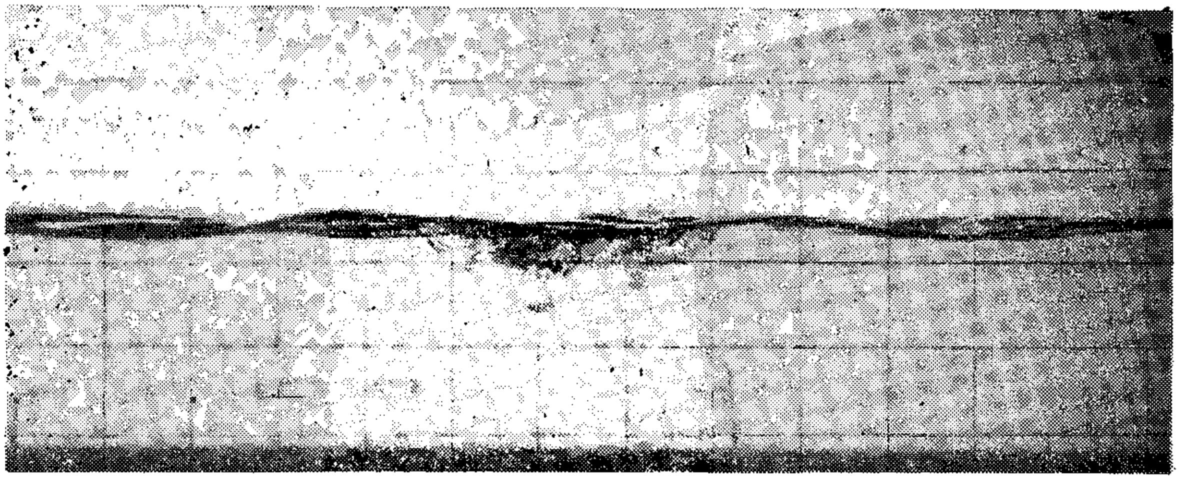


Fig. 3-1

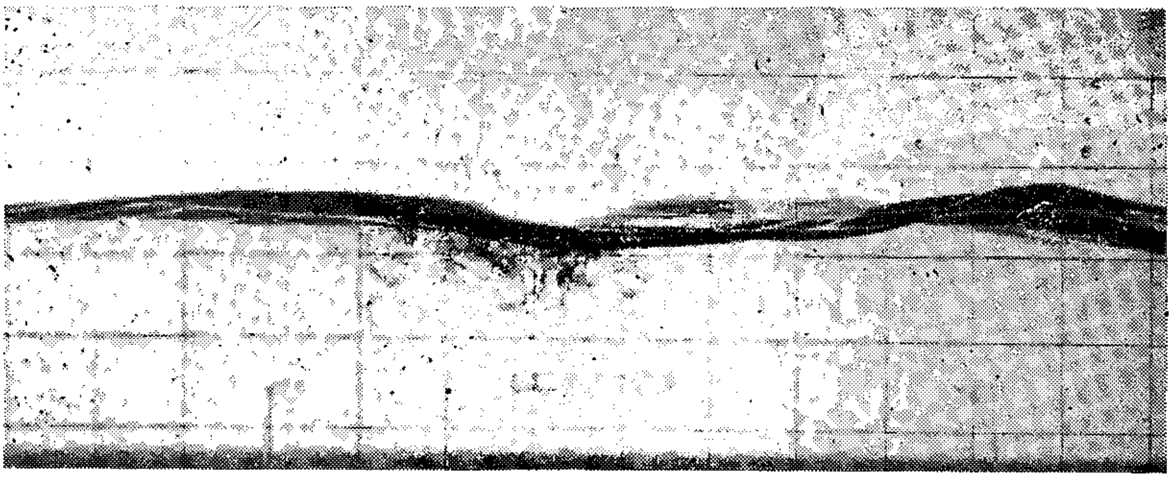


Fig. 3-2

not be so different. Fig-2 and Fig-3 are photographic examples of waves at C-2 and D, respectively Fig-2-1, Fig-3-1 correspond to r.p.m. 300 of blower, and Fig-2-2, Fig-3-2 are r.p.m. 400 of blower. Dark streaks in water are the colouring matter.

In such experiment of air-water tunnel, the homogenity of wave properties in the lateral direction is usually a little broken (T. Hamada (1963)). So the measurement of frequency spectrum was done at the centre and its both sides distant 50cm from the centre. Three measurements of frequency spectrum at a point in a stationary condition were made. Each measurement of wave profile needs two minutes, and the χ^2- freedom of spectrum by each measurement is $N=36$. Therefore the representative spectrum, which is the average of three measured spectrums for the same point, has the freedom of 36×3. The spectrums which we used for the analysis were the average of three representative spectrums taken from the centre and its both sides in C-2 and D sections, respectively.

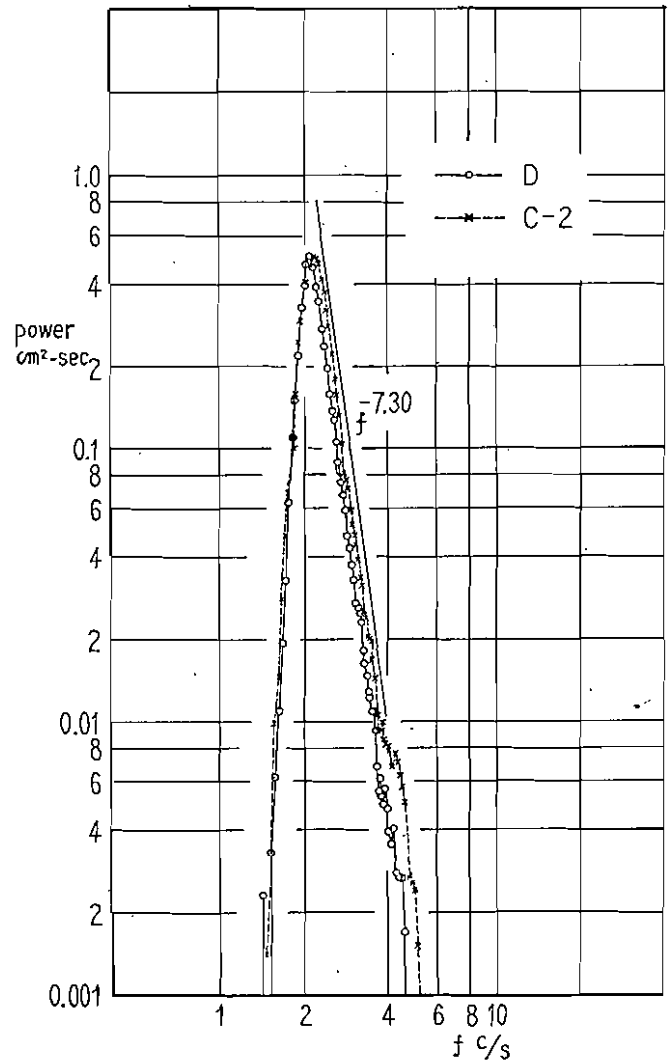


Fig-4. Spectrum intensity at C-2 and D (r.p.m. 300 of blower)

Fig-4 and Fig-5 show the spectrums obtained in this way. Abscissa is taken to the logarithm of wave frequncy f c/s, and ordinate is taken to the logarithm of the power of frequency spectrum $E(f)$cm²-sec.. Fig-4 corresponds to the case of r.p.m. 300 of blower. From this figure we can easily recognize the decay of power between C-2 and D. Fig-5 corresponds to the case of r.p.m. 400 of blower, and the value of power is greater than that of Fig-4. But the tendency of decay between C-2 and D is same. In Fig-4 a straight line drawn along the high frequency part of spectrum indicates $E(f) \sim f^{-7.30}$, and in Fig-5 it means $E(f) \sim f^{-8.94}$. Accordingly each spectums nave steeper gradient than $E(f) \sim f^{-5}$ in the high frequency part, and the rapid decay of spectrum intensity is expected in the same part.

Then we estimate the effect of viscosity on wave motion. The wave is caused by the uni-directional, steady continuous wind. Both sides of waterway are limited to a constant width. So at the stationary condition the effect of visous damping

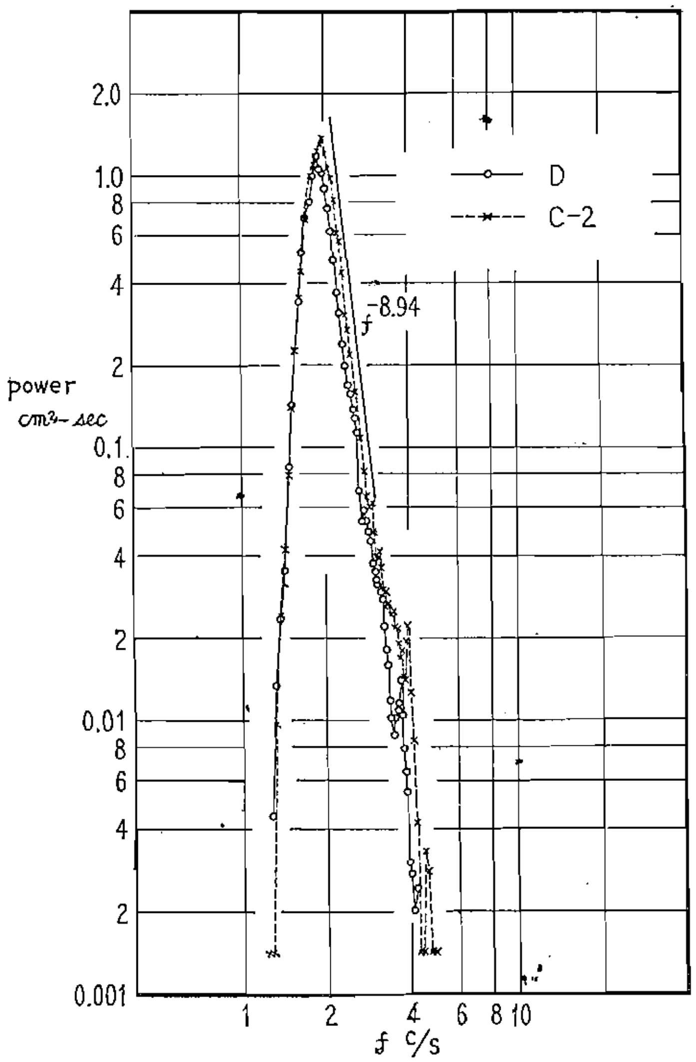


Fig.-5 Spectrum intensity at C-2 and D (r.p.m. 400 of blower) of frequency spectrum of waves can be estimated as follows;

$$E(f)=\int_{-\pi/2}^{\pi/2}\exp.\left\{-k^2\sqrt{\frac{\nu}{2kc_0}}\frac{2F_e\sec\alpha}{\sinh kh\cosh kh+kh}-\frac{8\nu k^2}{c_0}\frac{F_e\sec\alpha}{1+\dfrac{kh}{\sinh kh\cosh kh}}\right\}$$

$$\times 4\phi(f,\alpha)_{F_e=0}\frac{2\pi kf}{g\tanh kh+khg\,\mathrm{sech}^2 kh}d\alpha \qquad (31)$$

$$E(f)_{F_e=0}=\int_{-\pi/2}^{\pi/2}4\phi(f,\alpha)_{F_e=0}\frac{2\pi kf}{g\tanh kh+khg\,\mathrm{sech}^2 kh}d\alpha \qquad (32)$$

ν : viscous coefficient (usually taken molecular viscosity), c_0 : phase velocity of component wave, F_e : straight distance of the two considered sections, α : directional angle of component wave (coincides with θ in 2.)

In (31) the first term in exponent indicates the effect of bottom friction, and the second term means the inner friction (concentrates near surface). Besides these two terms, there is some viscous loss of side friction.

To compute accurately (31) and (32), the directional function $\phi(f,\alpha)_{Fe=0}$ should be determined. But in our experiment this is not measured and the determination is made by the following presumption. As stated in the reference of T. Hamada (1963), when the depth of water to bottom is taken 50cm, the phase velocity of the wave, which has the assumed frequency $\overline{f}_{zero\ up\text{-}cross}$ in the Gaussian assumption of random waves, is agreeable with the average value of translation celerities of appeared wave crests along the side wall at she same section. But in the present case of the water depth 15cm, these two celerities show the obvious difference and the average value of translation celerities of appeared wave crests along the side wall is always larger.

Table-1 .(all measurement was done under the direct wind action)

water depth cm	station	distance from air inlet cm	r.p.m. of blower	$\overline{f}$ zero up-cross	$c^{(1)}$ cm/sec	$c^{(2)}$ cm/sec	$c^{(2)}/c^{(1)}$	$\alpha_1 = \sec^{-1}\frac{c^{(2)}}{c^{(1)}}$
15	B	975	400	2.70	57.7	84.7	1.46	47°
15	C	1875	400	2.12	72.9	100.1	1.37	43°
15	D	2775	400	1.83	82.6	112.0	1.36	42°
50	C	1875	400	1.95	79.9	81.6	1.02	12°

note $c^{(1)}$: phase velocity of wave correspond to $\overline{f}$ zero up-cross.
$c^{(2)}$: the average value of translation celerity of appeared wave crest along the side wall.

Table-1 shows these relations. We can consider that the remarkable difference of $c^{(1)}$ and $c^{(2)}$ in Table-1 in the case of depth of water 15cm is caused by the angular spreading of random waves, and, in the reference of the value of $\sec^{-1}c^{(2)}/c^{(1)}$, we assume the intensity of $\phi(f,\alpha)$ as

Fig-6 $[\log_e E(f)]_D - [\log_e E(f)]_{C-2}$

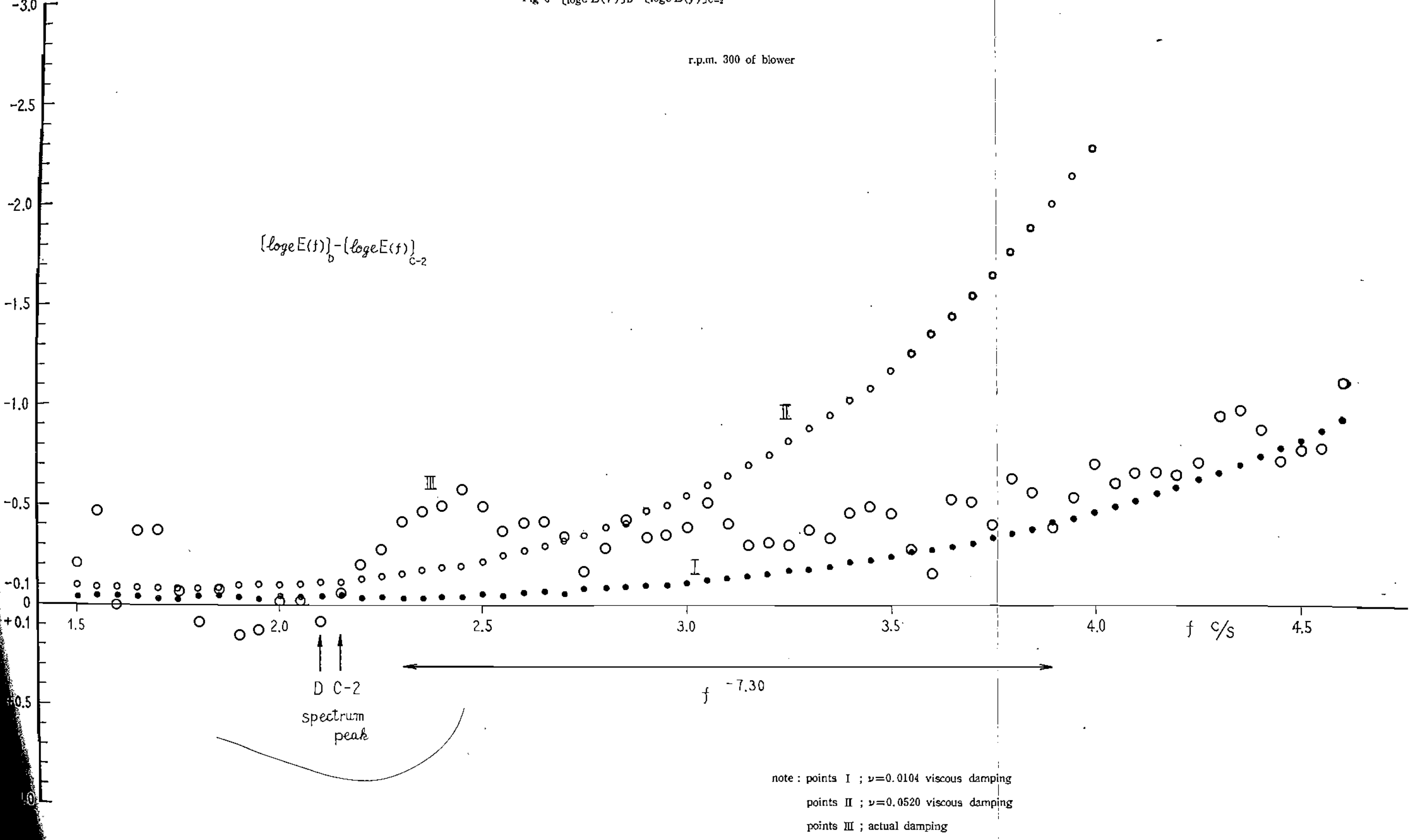
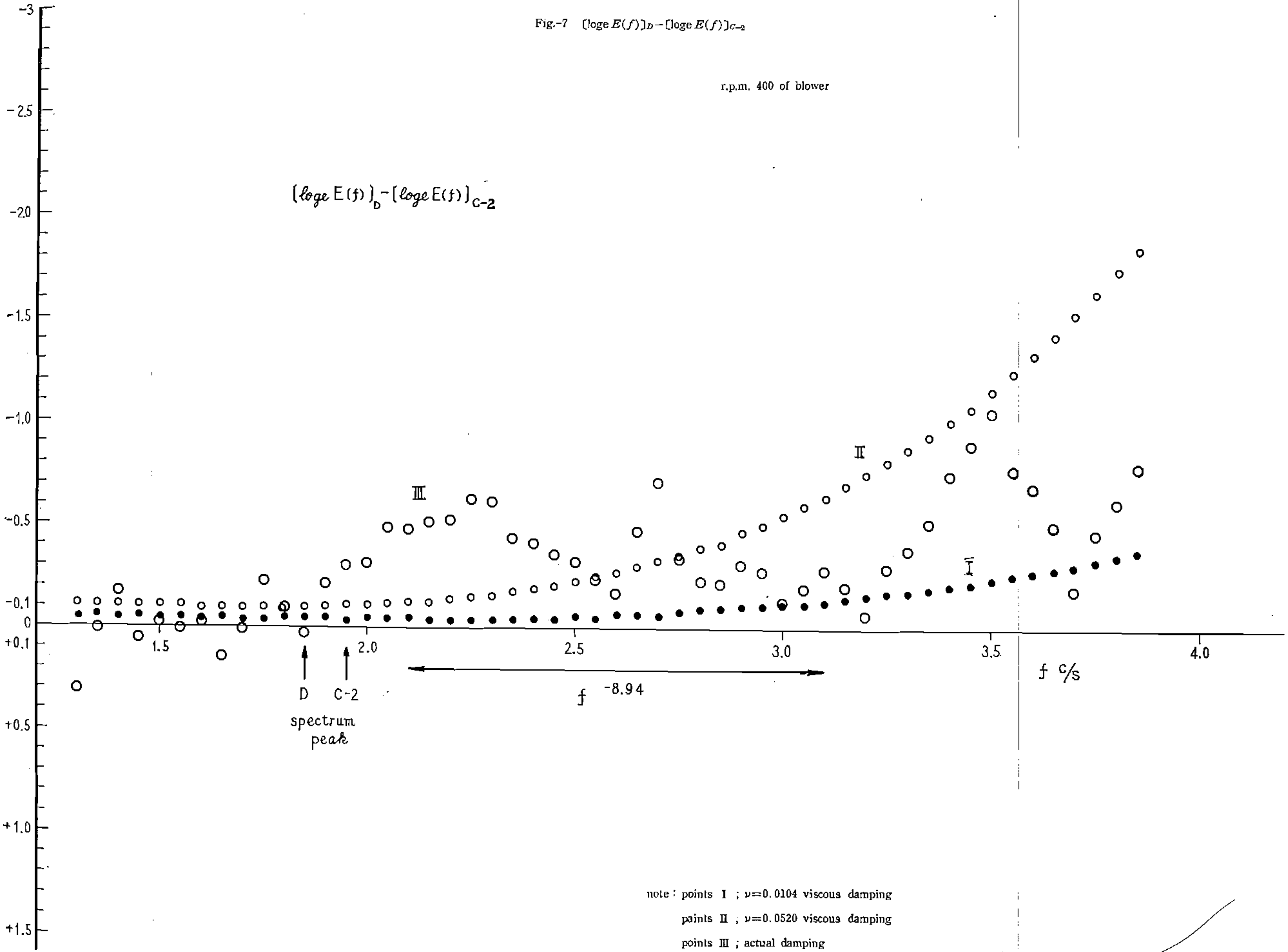


Fig.-7 $[\log_e E(f)]_D - [\log_e E(f)]_{C-2}$

r.p.m. 400 of blower



note : points I ; ν=0.0104 viscous damping

paints II ; ν=0.0520 viscous damping

points III ; actual damping

$$\phi(f,\alpha)=\phi_1(f) \qquad |\alpha|\leqq 50^\circ$$
$$=0 \qquad |\alpha|> 50^\circ \tag{33}$$

(33)shows that the intensity of $\phi(f,\alpha)$ is uniform for α in $|\alpha|\leqq 50^\circ$, and in the outside of $|\alpha|=50^\circ$ it becomes zero. Following to this assumption and putting $\sec\alpha \fallingdotseq 1+\alpha^2/2$, (31) and (32) are easily computed.

Fig-6 and Fig-7 show $[\log_e E(f)]_D-[\log_e E(f)]_{C\text{-}2}$ computed in this way and the actually measured $[\log_e E(f)]_D-[\log_e E(f)]_{C\text{-}2}$. Fig-6 corresponds to r.p.m. 300 of blower and Fig-7 is for r.p.m. 400 of blower. In the computation viscosity ν is taken 0.0104 (which corresponds to the molecular viscosity at the mean temperature of water through experiment) and 5×0.0104. The ratio of the numerical value of the first and the second term in exponent of (31) has the following tendency. The ratio becomes nearly to 1 at f=2c/s, and increases about 10 at f=1.5c/s, and falls into about 1/18 at f=2.5c/s, when we take the value of ν as 0.0104.

Many experiments which have been done up to the present day show that the bottom friction is greater than the estimation given by the effect of molecular viscosity, and so near f=1.5c/s, the viscous damping should be supposed to approach points II in Fig-6 and Fig-7. But actual decay of $E(f)$ in the both figures indicates that in the high frequncy part, where the bottom friction is very small, the viscous damping due to molecular viscosity should be predominant. Points II is far away from the measured values (points III). On the contrary, points, I are agreeable with the observed data in the part of 4.0~4.5c/s of f in Fig-6. If the effect of friction of side walls is taken into the computation, the agreement may be more satisfactory. In Fig-7, in the part of 3.4~3.6c/s of f, the influence of the nonlinear effect (L. J. Tick (1959)) of measured spectrums disturbs the agreement, but as the whole the same tendency as that of Fig-6 may be perceived.

In the both figures the positions of peak of spectrums and the range of frequency in which $E(f)\sim f^{-7.30}$ (Fig-6) and $E(f)\sim f^{-8.94}$ (Fig-7) are marked. In the marked range values of points III are far greater than those of points I. The maximum of of the difference amounts to more than ten times of the values of points I, and it means that the actual decay of $E(f)$ is remarkably greater than that given by (31) in the concerned part of frequency spectrum. The difference of values of points III and points I is shown in Fig-8. In this figure the difference decreases with the increase of frequency. This accords with the experimental fact that n approaches to 5 with the increase of frequency in the expression of $E(f)\sim f^{-n}$.

By this way we can find experimentally that the energy of progressive waves in the part of spectrum expression $E(f)\sim f^{-n}$ $(n>5)$ decreases conspicuously, and this phenomenon supports the analysis in 2.

In the computation in 2. we used the relation in deep water $k=4\pi^2 f^2/g$. The

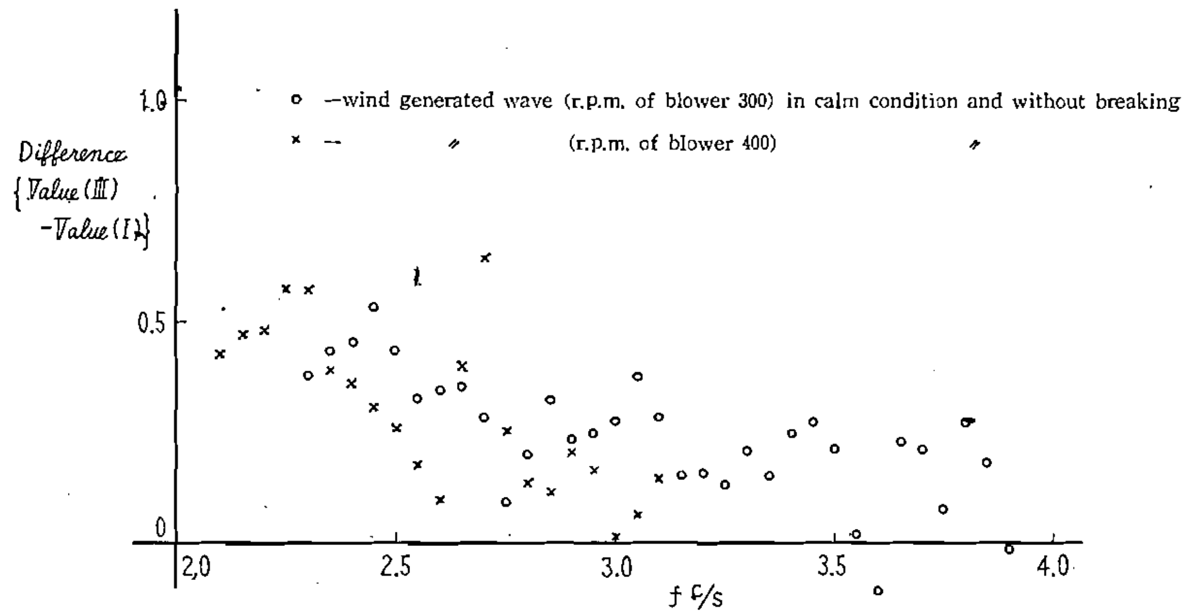


Fig-8 The difference of values of points III to values of points I

experiment was done in water of finite depth, and in (31) (32) we used the shallow water theory. The ratio of k given by $4\pi^2 f^2/g$ to k by shallow water theory for the same f is estimated 0.905 for $f=1.50$c/s, 0.985 for $f=2.00$c/s and 0.999 for $f=2.50$ c/s, and so the difference induced by this approximation is not so effective in the present consideration.

4. Conclusion

In the two-dimensional stationary random process of progressive waves, if n is greater than 5 in the frequency spectrum $(E(f) \sim f^{-n})$ of surface profile, the perturbed vortex motion is induced by the wave motion of the concerned part of the spectrum, and some of the wave energy in the same part of the spectrum is transfered to the induced vortex motion.

If n is smaller than 5, this phenomenon does not occur. The f^{-5} law gives the limiting boundary of this instability.

The experiment to support this theoretical estimation was added and the result is agreeable with the theory, The instability by this theoretical estimation may have an important influence on the development of wind waves and the decay of wind generated swells.

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Appendix

In the case when the water depth to the bottom is finite, the relation of wave number and its period is generally expressed by

$$\frac{2\pi}{T}=\sqrt{g|K|\tanh|K|h} \qquad (1)$$

Accordingly, in the case of very shallow water,

$$\frac{2\pi}{T}\sim|K|\sqrt{gh} \qquad (2)$$

We treat the problem in very shallow water in this appendix. Using (2), (2) in the text is modified to

$$\left.\begin{aligned}&\eta=\frac{1}{g}\left(\frac{\partial\varphi}{\partial t}\right)_{z=0}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{k_1,k_2}e^{iK.X}e^{i[-\mathrm{sgn}(K.C)|K|\sqrt{gh}\,t+\varepsilon(K)]}\\ &\varepsilon(-K)=-\varepsilon(K),\qquad C_{k_1,k_2}=C_{-k_1,-k_2}\end{aligned}\right\} \qquad (3)$$

and

$$|K|=\frac{2\pi f}{\sqrt{gh}},\qquad df=\frac{\sqrt{gh}}{2\pi}dk\qquad (k=|K|) \qquad (4)$$

Accordingly the expression which corresponds to (6) in the text is

$$\left.\begin{array}{llllll} E(f) & f^{-1} & f^{-2} & f^{-3} & f^{-4} & f^{-5}\\ \phi(K) & |K|^{-2} & |K|^{-3} & |K|^{-4} & |K|^{-5} & |K|^{-6}\\ \sqrt{\phi(K)} & |K|^{-1} & |K|^{-1.5} & |K|^{-2} & |K|^{-2.5} & |K|^{-3}\end{array}\right\} \qquad (5)$$

Using the same method of computation as in the text, $\phi(K)=A^2|K|^{-3}f(\theta)$ gives the logarithmic infinity as same as $\phi(K)\sim|K|^{-4}$ in deep water. So the equilibrium condition in the case of very shallow water is

$$E(f)\sim g^{1/2}h^{3/2}f^{-2} \tag{6}$$

But it must be noticed that, in the computations of the terms of $\zeta_1\frac{\partial u_0}{\partial z}$, $\zeta_1\frac{\partial v_0}{\partial z}$, $\xi_1\frac{\partial w_0}{\partial x}+\eta_1\frac{\partial w_0}{\partial y}$ $\phi(K)\sim|K|^{-5}$ gives the logarithmic infinity, and so ζ_1 seems weak in the present case.

In the case of very shallow water, the bottom friction is effective to the decrease of wave motion, and so (6) will be modified by this effect. But (6) indicates that, even in the case of very shallow water, if n is greater than 2 in the spectrum expression $E(f)\sim f^{-n}$, the induced vortex motion is generated by the wave motion concerned, and that some of the wave energy is transfered to the vortex motion. This phenomenon is clearly discriminated from the generation of vortices from the bottom boundary layer, and may be influential to the development of wind waves in very shallow water.