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Contribution to the Bearing Capacity of Soil Stratum

by

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Contribution to the Bearing Capacity of Soil Stratum

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SYNOPSIS

In the stability analysis of low embankments on rather impervious soil stratum, the $\phi=0$ analysis can be used unless the construction is slow. A design graph for low embankments is compiled using circular slip surface by the $\phi=0$ analysis for the case when the undrained strength of soil increases linearly with depth. The same analysis is applied to the problem of the ultimate bearing capacity of long footing. A brief description of Odenstad's method for designing low embankments is given in the appendix.

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CONTENTS

§ 1. Introduction

In most practical problems, the stability of soil masses against failure under their own weight or under the action of applied loads are examined by the method based on the principle of limit design. The most general definition of factor of safety against complete failure, which can be applied irrespective of the failure surface, is expressed in terms of the proportion of the measured shear strength that must be mobilized to just maintain limiting equilibrium. The shear strength parameters to which the factor of safety is applied in setting up the equations expressing the condition of limiting equilibrium depend of whether the analysis is carried out in terms of effective stress or total stress.

The problem of bearing capacity of soil stratum may be most simply illustrated in terms of the construction of a low embankment on a saturated soft clay stratum with a horizontal surface. In this case the excess pore pressure set up in an element of clay beneath the embankment is given in terms of the pore pressure coefficients and the increments of principal stresses. The pore pressure will have its greatest value at the end of construction. Unless construction is slow or the clay contains permeable layers, little dissipation of pore pressure will occur during the construction period. After construction is completed the pore pressure will decrease by dissipation, until finally the pore pressure corresponds ground water level.

The factor of safety given by the effective stress analysis will thus show a minimum value at or near end of construction, after which it will rise to the long term equilibrium value.

The use of the effective stress method for the end of construction case means that the pore pressure must be predicted or measured in the field. As this estimate involves an assumption about the stress distribution and the determination of pore pressure coefficients, it is usually avoided by going directly to the $\phi=0$ analysis which is applicable to the end of construction case with zero drainage.

It has been shown[1] that when a saturated soil is loaded to failure, allowing no drainage to occur, it will show a strength c_u, $\phi=0$.

This value, c_u*, will depend on the mode of consolidation of soil before shearing (isotropic or anisotropic), and the rate of change of c_u with the magnitude of

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* $$c_u=\frac{p \sin \phi'[K-(1-K)A_f]+c' \cos \phi'}{1+(2A_f-1) \sin \phi'}$$

where p : consolidation pressure
c' : cohesion intercept in terms of effective stress
ϕ' : angle of shearing resistance in terms of effective stress
K : principal strees ratio
A_f : pore pressure coefficient at failure

the consolidation pressure p, will depend on the stress history of the soil (normally-consolidated or over-consolidated).

The construction of an embankment on a staturated soil doposit may lead to the full mobilization of the undrained strength c_u, depending, of course, on the rate of construction as well as on the consolidation characteristics of the foundation itself. For comparatively impervious foundations unless construction is slow enough to allow excess pore pressure to dissipate, the shear strength to be mobilized in the foundation may be taken as that of the undrained case c_u, $\phi=0$.

In what follows we shall concern ourselves with the stability analysis of foundation of low embankments assuming the $\phi=0$ analysis applicable.

In the $\phi=0$ analysis we need not have any information about the pore pressure. We have further simplification by using a slip circle analysis since the geometry of the problem is quite simple. However, the practical calculation work involved can be tedious because we have to draw many trial slip circles to find the minimum factor of safety.

So far some works have been done to reduce the amount of calculation work in the $\phi=0$ analysis. Fig. 1. shows diagrammatically the various cases considered by D.W. Taylor(2), B. Jakobson(3), S. Odensted*(4), R.E. Gibson and N. Morgenstern(5), together with that treated in the present paper.

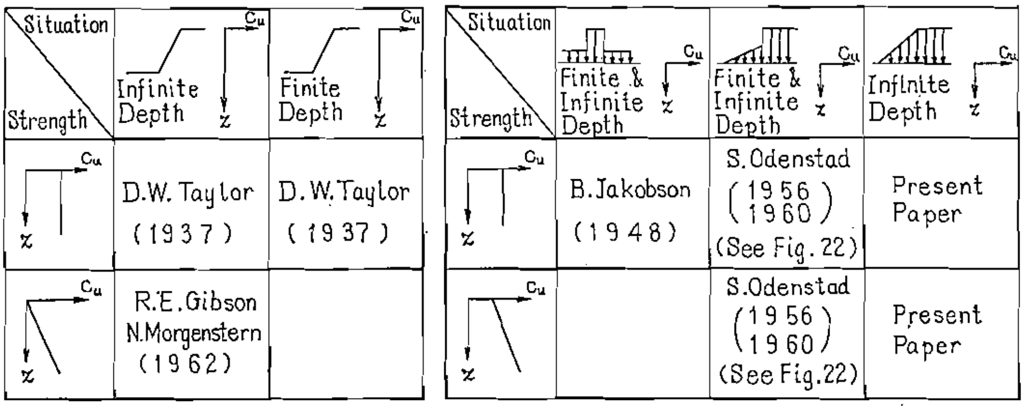


Fig.-1. Cases in which convenient design charts are available.

§ 2. Stability analysis of foundations of low embankments

2-1. Outline

Let us consider a low embankment on a saturated soil stratum with a horizontal surface, as shown on Fig. 2.

* A brief description of the Odenstad's method is given in the appendix.

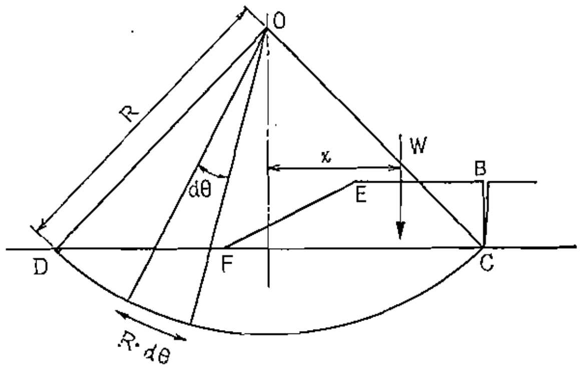


Fig.-2. Situation considered in the present paper.

The factor of safety against failure on the slip circle is expressed as

$$F = M_r / M_a \qquad (1)$$

where M_r is the restoring moment due to the shearing resistance along the arc, and M_a is the disturbing moment due to weight of the embankment.

The restoring moment is expressed as

$$M_r = R \sum_{C}^{D} (R d\theta) c_u \qquad (2)$$

The value of M_r depends on a shape of the sector OCD and on the shear strength. Therefore the value of M_r does not change by shifting the centre position 0 horizontally.

The disturbing moment M_a is written as

$$M_a = W \cdot x \qquad (3)$$

where W is the weight of the embankment BEFC, and x is the distance between 0 and centre of gravity of mass BEFC. For simplicity of analysis it is assumed that a vertical tension crack would develop in the fill above the point at which the failure surface emerges from the foundation, as shown in Fig. 2. As seen in the figure, if the chord length CD and the abscissae of 0 are fixed, M_a is not effected by change of slip circle.

Referring to Fig. 3, for three slip circles the restoring moments on the slip circles Nos, 1 and 2 are equal, and the disturbing moments on the slip circles Nos. 1 and 3 are equal. Then if we find the relationship between M_r and the shape of the sector, and also the relationship between M_a and geometry of the sector and the embankment, it will be possible to examine the stability by combining these two moments.

In the following sections, these two moments shall be studied separately. Finally they shall be combined to obtain the general solution of the stability analysis.

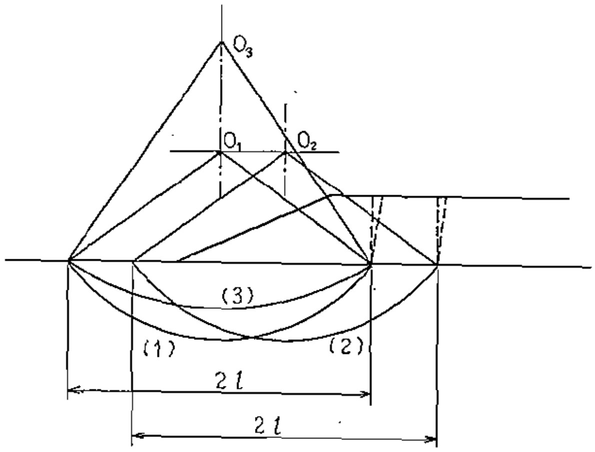


Fig.-3. Comparison of disturbing and restoring moments.

2-2. Restoring moment

In the simplest case it is assumed that the undrained shear strength increases linearly with depth, and can be expressed as

$$c_u = c_0 + kz \qquad (4)$$

where k is a constant for particular soil. The constant k has the same dimensions as density.

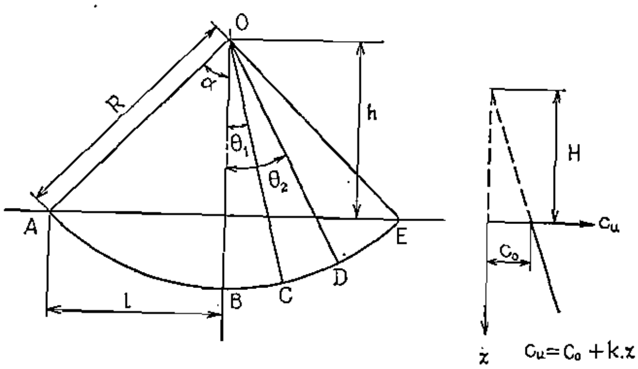


Fig.-4. Caluculation of restoring moment.

Now let us consider the restoring moment about 0 (See Fig. 4), produced by the shearing resistance along the circular arc CD, unit width of soil mass being considered.

$$M_r\Big)_{\theta_1}^{\theta_2}=\int_{\theta_1}^{\theta_2}(c_0+kz)R^2d\theta$$

$$=\int_{\theta_1}^{\theta_2}\{c_0+kR(\cos\theta-\cos\alpha)\}R^2d\theta$$

$$=R^2\Big[c_0\theta+kR(\sin\theta-\theta\cos\alpha)\Big]_{\theta_1}^{\theta_2} \qquad (5)$$

where the angle θ is measured from the vertical line through the centre 0.

This expression for the restoring moment is useful, in particular, in the case where the foundation soil has different shear strength distributions, *e. g.* in the case of the vertical sand drains. The second term of equation (5) represents the part of the moment due to the increment of strength kz, and the table of values of $(\sin\theta-\theta\cos\alpha)$ has been published.[6]

The restoring moment for the whole length of arc ABCDE (Fig. 4.) is obtained simply by substituting $\theta_1=-\alpha$ and $\theta_2=+\alpha$ into equation (5). Then we obtain the following expression for the restoring moment,

$$M_r=2R^2[c_0\alpha+kR(\sin\alpha-\alpha\cos\alpha)]$$

$$=2\frac{l^2}{\sin^2\alpha}[c_0\alpha+k(l-h\alpha)]$$

$$\because\quad R\sin\alpha=l,\quad R\cos\alpha=h$$

$$=2kl^3\frac{\alpha}{\sin^2\alpha}\Big[\frac{H}{l}+\frac{1}{\alpha}-\cot\alpha\Big]$$

$$=2kl^3\cdot f(\alpha, H/l) \qquad (6)$$

where $H=c_0/k$ (See Fig. 4.), and

$$f(\alpha, H/l)=\frac{\alpha}{\sin^2\alpha}\Big[\frac{H}{l}+\frac{1}{\alpha}-\cot\alpha\Big] \qquad (7)$$

By calculating the value of $f(\alpha, H/l)$, we can examine the relationship between the restoring moment and the sector angle for a given chord length. Values of $f(\alpha, H/l)$ are plotted against the angle α for values of H/l on Fig. 5. In this figure the dashed line is the locus of the minimum values of $f(\alpha, H/l)$ for each value of H/l.

The minimum value of $f(\alpha, H/l)$ is obtained from the condition $\frac{\partial}{\partial\alpha}\{f(\alpha, H/l)\}=0$, which gives the relationship between the parameter H/l and angle α

$$\frac{H}{l}=\frac{3\cot\alpha-2\alpha\cot^2\alpha-\alpha\cdot\operatorname{cosec}^2\alpha}{1-2\alpha\cot\alpha}$$

By the above expression the H/l value which gives the minimum value of $f(\alpha, Hl)$ for any value of α can be calculated. Results of the calculation are listed in Table 1.

Table-1 $f(\alpha, H/l)_{min.}$ and corresponding values of H/l and α

α	H/l	$f(\alpha, H/l)_{min.}$
0.	0	0.33333
0.1	0.00036	0.33823
0.15	0.00086	0.34216
0.2	0.00220	0.34989
0.25	0.00438	0.35967
0.3	0.00773	0.37213
0.4	0.01955	0.40708
0.5	0.04161	0.45926
0.6	0.08050	0.53723
0.7	0.14847	0.65747
0.8	0.27111	0.85486
0.85	0.36979	1.00565
0.9	0.51264	1.21770
0.95	0.73070	1.53376
1.0	1.09290	2.04896
1.05	1.78695	3.02223
1.10	3.57421	5.50421
1.125	6.14263	9.05661
1.15	16.99761	24.04452
1.16	48.67488	67.76431
1.163	106.61316	147.72300
1.164	272.45791	376.59932

H/l	$f(\alpha, H/l)_{min.}$*	α**
0.	0.333	0
0.05	0.473	0.515
0.1	0.570	0.635
0.15	0.663	0.700
0.2	0.742	0.750
0.3	0.900	0.817
0.4	1.055	0.863
0.5	1.205	0.897
0.6	1.350	0.924
0.7	1.500	0.945
0.8	1.635	0.962
0.9	1.795	0.978
1.0	1.925	0.990
2.0	3.310	1.059
3.0	4.700	1.089
4.0	6.080	1.106
5.0	7.475	1.118
10.0	14.5	1.138
20.0	28.5	1.153
50.0	69.8	1.160
100.0	139	1.163

* : Readings from the $f(\alpha, H/l)_{min.}$ vs H/l plot
**: Readings from the H/l vs α plot

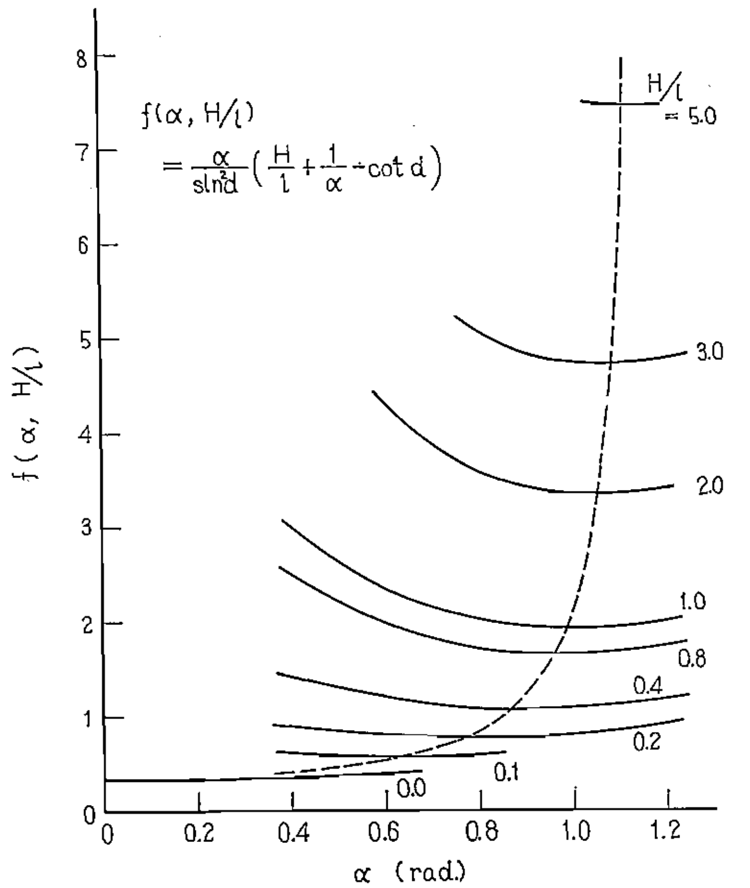


Fig.-5. Relationship between $f(\alpha, H/l)$ and angle α.

If c_u is constant with depth, *i. e.* $k=0$, equation (6) can be written

$$M_r=2l^2c_0\frac{\alpha}{\sin^2\alpha}=2R^2\alpha c_0$$

Also the angle α at which the restoring moment becomes minimum for a given chord length $2l$ is obtained by putting

$$\frac{\partial M_r}{\partial\alpha}=2c_0l^2\left(\frac{\sin\alpha-2\alpha\cos\alpha}{\sin^3\alpha}\right)=0$$

giving $\alpha=1.165$ (rad.), which is well known in the $\phi=0$ analysis. Also the minimum restoring moment in this case is expressed as

$$M_{r.min}=2.76\,l^2c_0$$

This expression is particularly useful since it immediately determines the minimum restoring moment obtainable from any chosen chord length of slip circle for a soil of constant c_u with depth.

If $c_0=0$, *i.e.* $H=0$, equation (7) becomes

$$f(\alpha,0)=\frac{\alpha}{\sin^2\alpha}\left(\frac{1}{\alpha}-\cot\alpha\right)$$

The minimum value of $f(\alpha, 0)$ is 1/3 at $\alpha=0$.

The dashed line in Fig. 5. implies that if l is given for a known combination of c_0 and k, the angle α which gives the minimum restoring moment can be readily obtained. Fig. 6. shows the relationship between the parameter H/l and the angle α for the minimum restoring moment.

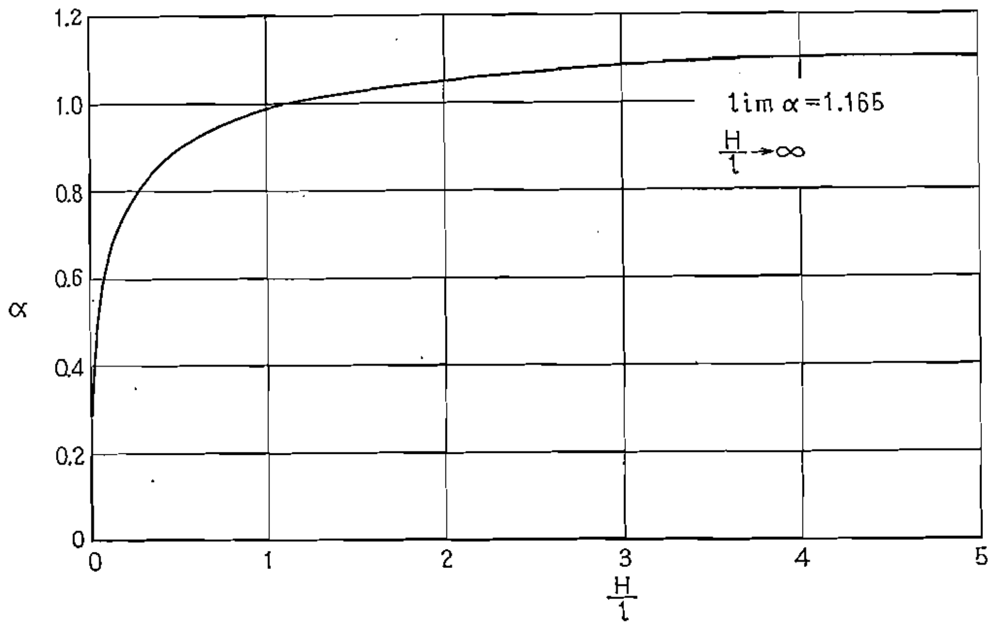
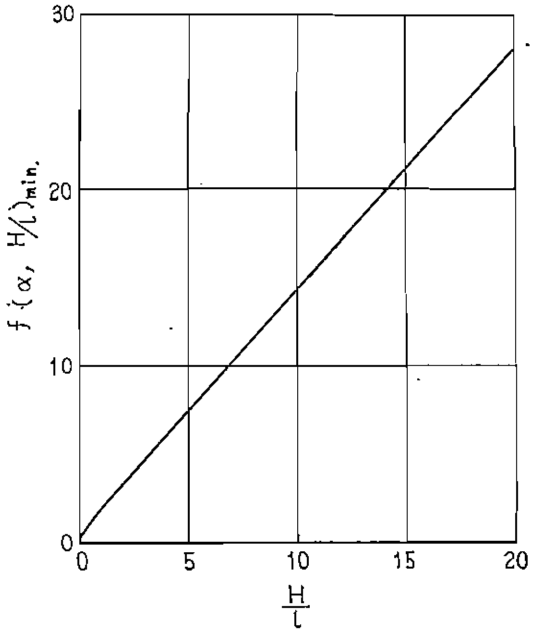


Fig.-6. Relationship between parameter H/l and angle α which gives the minimum value of f (α, H/l).

Fig. 7. shows the relationship between the parameter H/l and the corresponding minimum value of $f(\alpha, H/l)$. As shown in the figure, the minimum value of $f(\alpha, H/l)$ is an approximately linear function of the parameter H/l for the range of $H/l \geq 0.8$. Then $f(\alpha, H/l)_{min.}$ may be expressed for the range of $H/l \geq 0.8$, as

$$f(\alpha, H/l)_{min.} = 0.526 + 1.380(H/l) \tag{8}$$



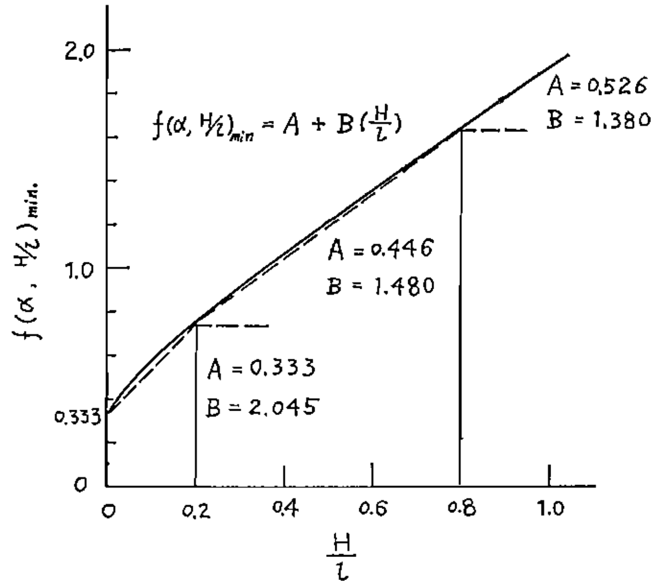


Fig.-7. Relationship between parameter H/l and the minimum value of $f(\alpha, H/l)$.

Fig.-8. Sectional approximation of $f(\alpha, H/l)_{min.}$ vs H/l relationship.

Fig. 8. shows the $f(\alpha, H/l)_{min}$ versus H/l relationship for the H/l values less than 1.0. The non-linear part of $f(\alpha, H/l)_{min}$ may be approximately defined by two linear equations:

$$f(\alpha, H/l)_{min.} = 0.446 + 1.480(H/l), \quad 0.8 \geq H/l \geq 0.2 \tag{9}$$

$$= 0.333 + 2.045(H/l), \quad 0.2 \geq H/l \geq 0 \tag{10}$$

These approximate expressions for $f(\alpha, H/l)_{min.}$ are a little conservative as shown in Fig. 8.

Then the minimum restoring moment is generally given as

$$M_r = 2kl^3\left(A + B\frac{H}{l}\right) \tag{11}$$

where A and B are constants depending on the range of H/l values considered.

As an example let us consider the slip circle having a chord length of $2l = 20m$ in the clay stratum where $c_0 = 1.0\, t/m^2$ and $k = 0.1\, t/m^3$, *i.e.* $H = 10m$. From Fig. 7, $f(\alpha, H/l)_{min}$ value corresponding to $H/l = 1$ is 1.9. So the minimum restoring moment is

$$M_r = 2kl^3 \cdot f(\alpha, H/l)_{min.} = 2 \times 0.1 \times 10^3 \times 1.9 = 380\ ^{t\text{-}m}/_{m}$$

And from Fig. 6. sector angle α of the circle which gives the minimum restoring moment is $\alpha=0.98$ rad. $=56.2°$. These situations are illustrated in Fig. 9.

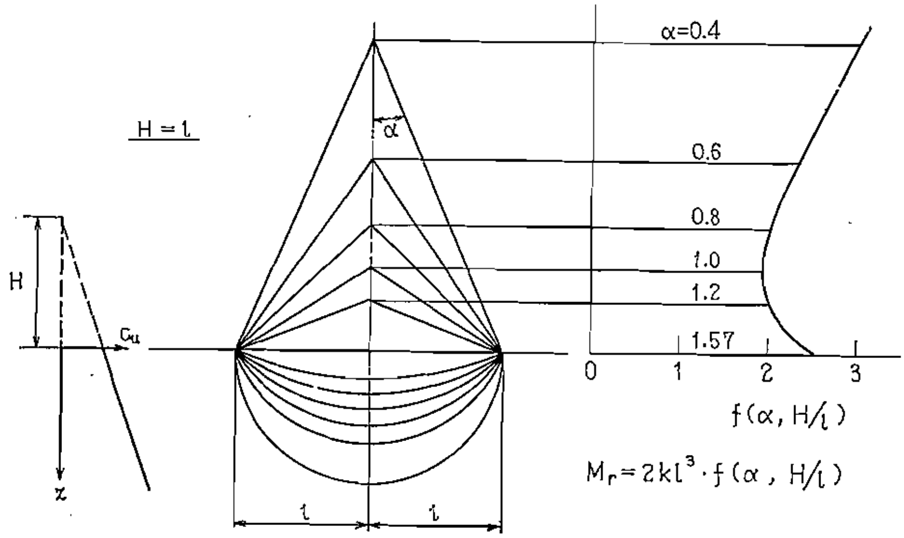


Fig.-9. Change of restoring moment with sector angle for a given chord length.

2-3. Disturbing moment

As mentioned in **2-1** the disturbing moment depends on the chord length and the centre position of the slip circle.

Let us consider the embankment where the load intensity is q and length of the side slope is x. The problem is to find the centre position of the slip circle which gives the maximum disturbing moment for a given chord length $2l$. For simplicity of analysis it is assumed that a vertical tension crack would develop in the fill above the point at which the failure surface emerges from the foundation.

It is convenient to consider two cases with respect to the geometry of the embankment and the slip circle.

Case 1.—This is the situation where the chord length is smaller than the side slope length, i. e. $2l \leq x$.

Referring to Fig. 10 (a) it is evident that the maximum disturbing moment occurs when the both ends of slip circle are inside the side slope. And the resulting maximum disturbing moment is therefore,

$$\dot{M}_a=\frac{2}{3}\frac{q}{x}l^3 \qquad (12)$$

for the range of $x-l \geq y \geq l$, as shown in Fig. 10 (b).

Case 2.—The case where the chord length is larger than the side slope length,

i. e. $2l \geq x$ is considered. It is easily appreciated from Fig. 11 (a) that the maximum disturbing moment occurs when both ends of the slip circle are outside the side slope. In this case the moment is expressed as;

$$M_a = q(l-x+y)\left(x-y+\frac{y+l-x}{2}\right)+\frac{qx}{2}\left(x-y-\frac{x}{3}\right)$$

$$= \frac{q}{2}(l-x+y)(l+x-y)+\frac{qx}{6}(2x-3y) \tag{13}$$

The maximum value of the disturbing moment is obtained from the condition that

$$\frac{\partial}{\partial y}M_a = \frac{q}{2}(x-2y)=0\,,$$

giving $y=x/2$. And therefore the maximum disturbing moment is obtained by substituting $y=x/2$ into equation (13), with the result,

$$M_a = \frac{q}{2}\left(l^2 - \frac{x^2}{12}\right) \tag{14}$$

The variation of M_a as expressed by equation (13) and obtained by shifting the centre position of the slip circle is illustrated in Fig. 11 (b).

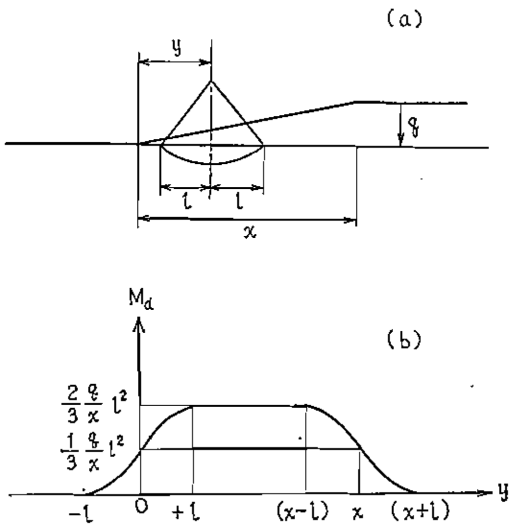


Fig.-10. Change of disturbing moment with abscissae of centre of slip circle ($2l<x$)

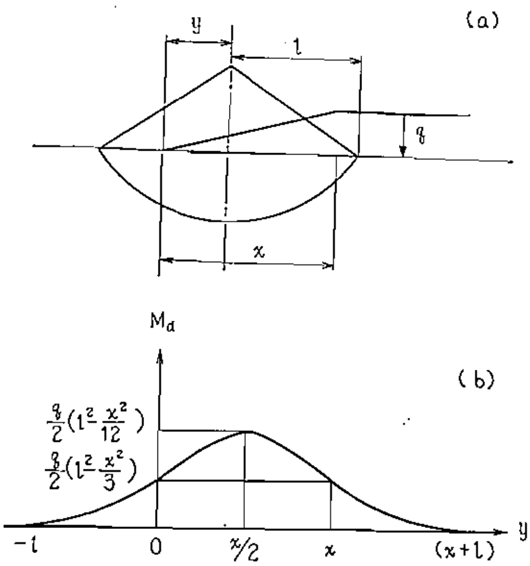


Fig.-11. Change of disturbing moment with abscissae of centre of slip circle ($2l>x$)

2-4. General solution

In the preceding sections we have obtained the expression for the minimum restoring moment and the maximum disturbing moment for a given chord length of the slip circle. Therefore it will be possible to find the general solution for the minimum safety factor by comparing these two moments.

The general expression for the minimum restoring moment is written from equation (11) by using a new parameter $n=l/x$,

$$M_{r.min.}=2kn^3x^3\left(A+\frac{BH}{nx}\right) \tag{15}$$

As mentioned previously, A and B are dependent on the range of H/l value, i. e. H/nx considered. And the maximum disturbing moment is expressed from equations (12) and (14), as

$$M_{d.max.}=\frac{2}{3}qn^3x^2 \qquad \frac{1}{2}\geq n>0 \tag{16}$$

$$=\frac{q}{2}x^2\left(n^2-\frac{1}{12}\right) \qquad n\geq\frac{1}{2} \tag{17}$$

The general expression for the factor of safety, therefore, becomes;

$$F=\frac{2\cdot k\cdot n^3\cdot x^3\cdot\left(A+\frac{BH}{nx}\right)}{\frac{2}{3}\cdot q\cdot n^3\cdot x^2}$$

$$=\frac{3k}{q}\cdot x\cdot\left(A+\frac{BH}{nx}\right), \qquad \frac{1}{2}\geq n>0 \tag{18}$$

$$F=\frac{2\cdot k\cdot n^3\cdot x^3\cdot\left(A+\frac{BH}{nx}\right)}{\frac{q}{2}\cdot x^2\cdot\left(n^2-\frac{1}{12}\right)}$$

$$=\frac{4k}{q}\cdot x\cdot n^3\cdot\frac{A+\frac{BH}{nx}}{n^2-\frac{1}{12}}, \qquad n\geq\frac{1}{2} \tag{19}$$

From equation (18) it is obvious that the factor of safety becomes minimum when n is equal to $1/2$. The same value of factor of safety is also obtained from equation (19) by substituting $n=1/2$. Therefore the required answer will be obtained by investigating equation (19) for n values larger than $1/2$.

By differentiating equation (19) with respect to n we get

$$\frac{\partial F}{\partial n}=\frac{4k}{q}\cdot x\cdot\frac{n}{\left(n^2-\frac{1}{12}\right)^2}\cdot\frac{1}{12}\left(12An^3-3An-2B\frac{H}{x}\right) \tag{20}$$

Thus the minimum value of factor of safety can be obtained by substituting into equation (19) the value $n=n_0$ which satisfies the condition that,

$$12An^3-3An-2B\frac{H}{x}=0 \tag{21}$$

It must be remembered that the constants A and B depend on the value of the parameter $H/l=H/nx$. Therefore after solving equation (21) for various values of H/x, it is necessary to examine the range of parameter H/n_0x.

The procedures of calculation are as follows;

(a) Obtain the n_0 values from equation (21) for each combination of parameters A and B, by substituting values of $\beta=H/x$.

(b) Find the range where the H/n_0x values are appropriate to constants A and B.

(c) Substitute the n_0 values into equation (19).

Thus we get the minimum value of factor of safety F,

$$F=\frac{4kx}{q}\cdot n_0^3\cdot\frac{A+\frac{BH}{n_0x}}{n_0^2-\frac{1}{12}}$$

$$=\frac{4k}{q}\cdot\frac{H}{\beta}\cdot n_0^3\cdot\frac{A+B\frac{\beta}{n_0}}{n_0^2-\frac{1}{12}}$$

$$=\frac{kH}{q}\cdot f(A,\ B,\ n_0,\ \beta)$$

$$=\frac{c_0}{q}\cdot f(A,\ B,\ n_0,\ \beta) \tag{22}$$

where the numerical values of A, B, n_0 and β are known. These calculations are listed in Table 2. Fig. 12 shows the relation between n_0 and H/x. As seen

Table-2 (a) Values of n_0 and parameter Fq/c_0

$\frac{H}{l}=\frac{H}{n_0x}=0\sim0.2$ $A=0.333$ $B=2.045$

$4.09\cdot\frac{H}{x}$	n_0	$\frac{Fq}{kx}$	$\frac{Fq}{c_0}$	$\frac{H}{n_0x}$	$\frac{x}{H}$	$\frac{H}{x}$
0	0.500	1.0	∞	0	∞	0
0.01	0.505	1.030	429.167	0.005	416.667	0.0024
0.02	0.510	1.061	216.531	0.010	204.082	0.0049
0.1	0.544	1.287	52.746	0.045	40.984	0.0244
0.2	0.580	1.557	31.841	0.084	20.450	0.0489
0.3	0.611	1.820	24.830	0.120	13.643	0.0733
0.4	0.638	2.073	21.196	0.153	10.225	0.0978
0.5	0.663	2.321	18.993	0.184	8.183	0.1222

$$\frac{Fq}{kx}=\frac{4n_0^3}{n_0^2-\frac{1}{12}}\left(0.333+\frac{2.045}{n_0}\frac{H}{x}\right) \qquad \frac{Fq}{c_0}=\frac{Fq}{kx}\cdot\left(\frac{x}{H}\right)$$

$$\frac{\partial F}{\partial n}=\frac{4k}{q}\cdot x\cdot\frac{n}{\left(n^2-\frac{1}{12}\right)^2}\cdot\frac{1}{12}\left(4n^3-n-4.09\frac{H}{x}\right)$$

Table-2 (b)

$$\frac{H}{l}=\frac{H}{n_0 x}=0.2\sim0.8 \qquad A=0.446 \qquad B=1.480$$

$2.96\frac{H}{x}$	n_0	$\frac{Fq}{kx}$	$\frac{Fq}{c_0}$	$\frac{H}{n_0 x}$	$\frac{x}{H}$	$\frac{H}{x}$
0.4	0.611	2.432	18.002	0.221	7.402	0.1351
0.6	0.650	2.940	14.503	0.312	4.933	0.2027
0.8	0.684	3.431	12.695	0.395	3.700	0.2703
1.0	0.715	3.911	11.577	0.472	2.960	0.3378
1.2	0.743	4.390	10.830	0.546	2.467	0.4054
1.4	0.768	4.857	10.268	0.616	2.114	0.4730
1.6	0.792	5.317	9.836	0.682	1.850	0.5405
1.8	0.814	5.779	9.500	0.747	1.644	0.6081

$$\frac{Fq}{kx}=\frac{4n_0^3}{n_0^2-\frac{1}{12}}\left(0.446+\frac{1.480}{n_0}\cdot\frac{H}{x}\right)$$

$$\frac{\partial F}{\partial n}=\frac{4k}{q}\cdot x\cdot\frac{n}{\left(n^2-\frac{1}{12}\right)^2}\cdot\frac{1}{12}\left(5.352\,n^3-1.338\,n-2.96\frac{H}{x}\right)$$

$$\frac{Fq}{c_0}=\frac{Fq}{kx}\cdot\left(\frac{x}{H}\right)$$

Table-2 (c)

$$\frac{H}{l}=\frac{H}{n_0 x}\geq 0.8 \qquad A=0.526 \qquad B=1.380$$

$2.76\frac{H}{x}$	n_0	$\frac{Fq}{kx}$	$\frac{Fq}{c_0}$	$\frac{H}{n_0 x}$	$\frac{x}{H}$	$\frac{H}{x}$
2 0	0.804	6.532	9.014	0.901	1.380	0.7246
3.0	0.887	8.796	8.092	1.225	0.920	1.0870
4.0	0.955	11.018	7.607	1.518	0.690	1.4493
5.0	1.016	13.203	7.288	1.783	0.552	1.8116
6.0	1.068	15.367	7.069	2.035	0.460	2.1739
8.0	1.157	19.656	6.781	2.505	0.345	2.8986
10	1.237	23.904	6.598	2.929	0.276	3.6232
15	1.396	34.407	6.331	3.893	0.184	5.4348

$$\frac{Fq}{kx}=\frac{4n_0^3}{n_0^2-\frac{1}{12}}\left(0.526+\frac{1.38}{n_0}\cdot\frac{H}{x}\right)$$

$$\frac{\partial F}{\partial n}=\frac{4k}{q}\cdot x\cdot\frac{n}{\left(n^2-\frac{1}{12}\right)}\cdot\frac{1}{12}\left(6.312\,n^3-1.578\,n-2.76\frac{H}{x}\right)$$

$$\frac{Fq}{c_0}=\frac{Fq}{kx}\cdot\left(\frac{x}{H}\right)$$

in the figure there is scattering of plotted points. It is caused by the fact that three straight lines are used to represent the $f(\alpha, H/l)_{min}$ *vs* H/l relationship.

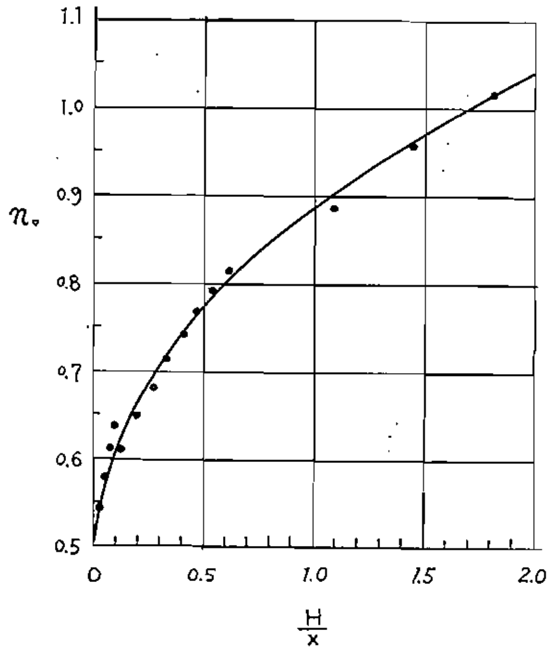


Fig.-12. Relationship between n_0 and parameter H/x.

Parameter Fq/c_0 is plotted against x/H on Fig. 13. The figure implies that for a proposed embankment the length of side slope corresponding to an assigned factor of safety is readily obtained for given values of c_0, k and q.

When a foundation has the uniform strength c_0, i. e. $x/H=0$, Fq/c_0 becomes 5.52. This is the well known value for the factor of bearing capacity. In such a case the embankment may stand even with the vertical face, provided that it is supported by some kind of wall against collapse.

If c_0 is zero, the parameter Fq/c_0 becomes infinite. Therefore we have to use the other expression for the side slope length x.

Substituting $H=0$ and $A=1/3$ into equation (21) we get

$$4n^3-n=0 \qquad \therefore \quad n=\frac{1}{2}$$

From equation (22), using $A=1/3$ for $H/nx=0$, we obtain

$$F=\frac{4kx}{q}\cdot\left(\frac{1}{2}\right)^3\cdot\frac{\frac{1}{3}}{\left(\frac{1}{2}\right)^2-\frac{1}{12}}=\frac{kx}{q}$$

$$\therefore \quad x=\frac{q}{k}\cdot F \tag{23}$$

Note, however, the sector angle α of the most critical slip circle becomes zero as shown in Fig. 6. This implies that this case is the extreme limit of the analysis of this kind.

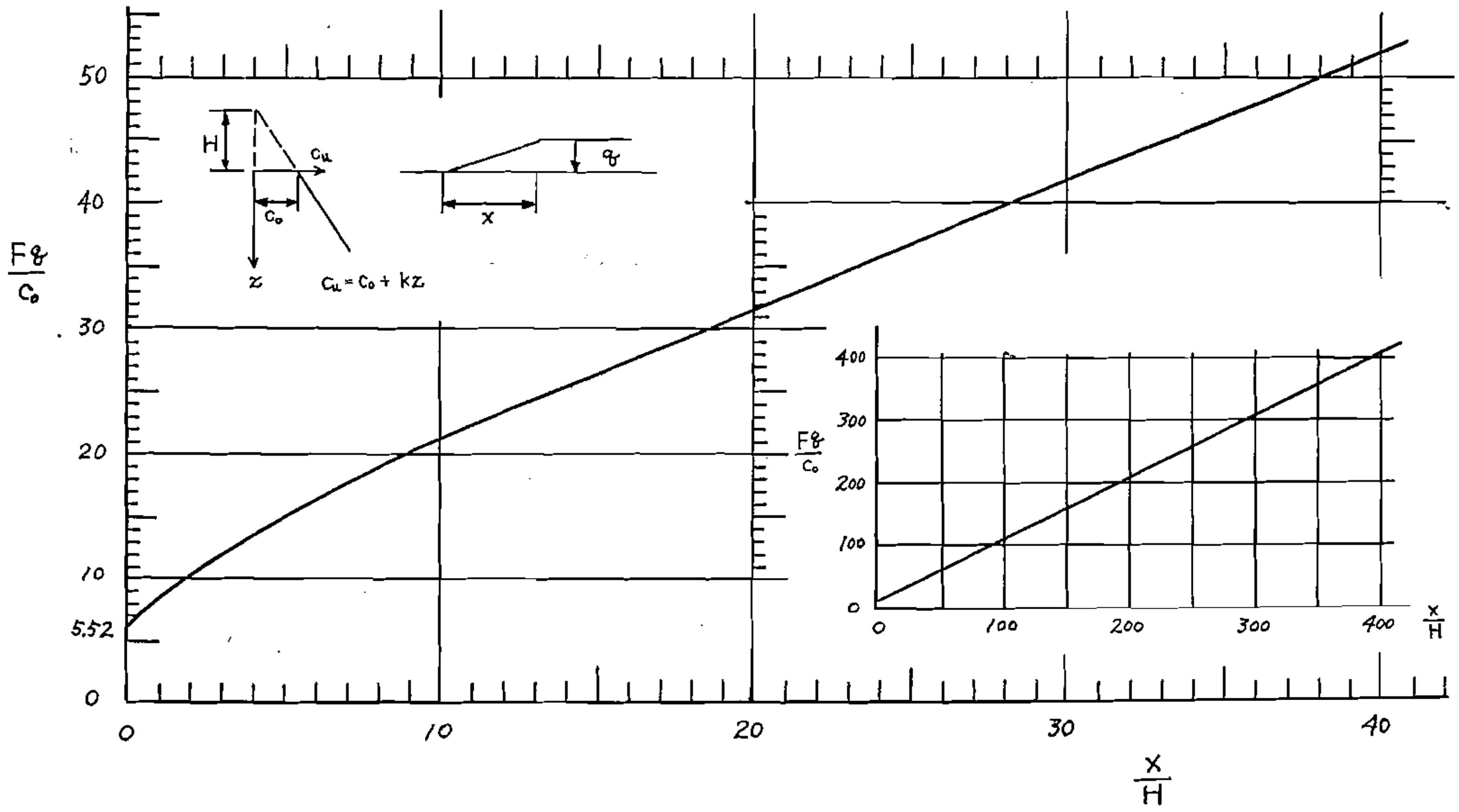


Fig.-13. Design graph of low embankment—relationship between parameters Fq/c_0 and x/H.

2-5. Position of the most critical circle

Position of the slip circle is determined by the chord length and the sector angle. In section 2-2 it was found that the sector angle 2α of the most critical circle was readily obtained for a given chord length $2l$ and the shear strength parameters c_0 and k. In Fig. 6 the angle α is plotted against the parameter H/l which is equal to H/n_0x.

When we obtain the x value for a given combination of c_0, k, q and F, we know at the same time the n_0 value from Fig. 12. Therefore we can obtain the value of H/n_0x, hence the sector angle of the circle.

As for a geometry of the most critical slip circle, the depth of the circle may be most important in practical problems. Because if soil stratum is shallow relative to the geometry of the embankment, we can not apply the slip circle analysis.

Depth D of the most critical slip circle is given as;

$$D=R(1-\cos\alpha)$$

$$=n\cdot x\cdot\frac{1-\cos\alpha}{\sin\alpha} \tag{24}$$

Then

$$D/H=\frac{nx}{H}\cdot\frac{1-\cos\alpha}{\sin\alpha} \tag{25}$$

For a given value of Fq/c_0, values of n_0 and x/H can be obtained from Fig. 12 and Fig. 13 respectively. With these two values the angle can be found corresponding to the value of H/n_0x from Fig. 6. Then we can calculate the value of D/H by equation (25).

Table-3 Calculation of depth of the critical slip circle

$\frac{Fq}{c_0}$	$\frac{x}{H}$ (1)	n_0 (2)	$\frac{H}{n_0x}$	α (3)	$\frac{D}{H}$ (4)
10	2.0	0.76	0.658	0.93	0.76
20	9.4	0.62	0.171	0.72	2.20
30	18.8	0.58	0.092	0.63	3.54
40	28.6	0.57	0.061	0.57	4.80
50	38.8	0.56	0.046	0.51	5.67

(1) from Fig. 13
(2) from Fig. 12
(3) from Fig. 6
(4) $D/H=\frac{n_0x}{H}\,\frac{1-\cos\alpha}{\sin\alpha}$

These calculations are tabulated in Table 3. And the value of D/H is plotted against the parameter Fq/c_0 on Fig. 14. The figure implies that for known values of c_0, k, q and F the depth of the most critical slip circle can be obtained as a proportion of $H=c_0/k$.

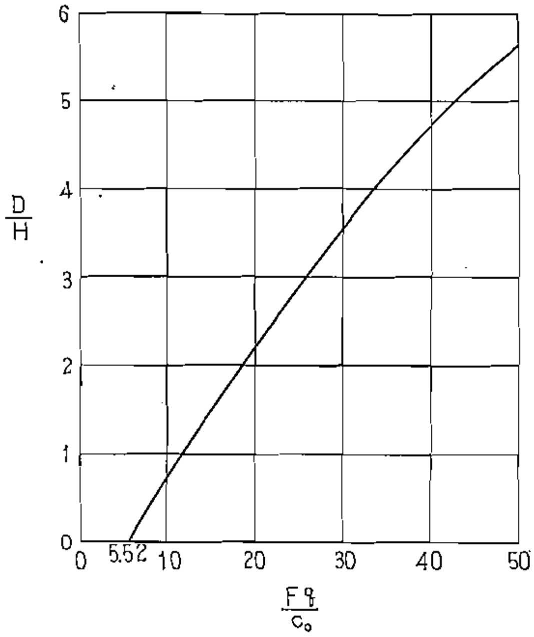


Fig.-14. Relationship between parameters Fq/c_0 and D/H for the critical slip circle.

Now we can see that if values of c_0, k, q and F are given, the geometry of the side slope and the most critical slip circle can be obtained as follows;

(a) the side slope length x from Fig. 13.

(b) the chord length $2l=2n_0x$ of the circle from Fig. 12.

(c) the depth of the circle from Fig. 14.

If depth of the soil stratum considered is less than D which is given by Fig. 14, above mentioned calculation does not hold any longer. In such a case we must use a different type of analysis considering a squeezing phenomena or the equation of earth pressure as done by Meyerhof[7] and Odenstad.[4]

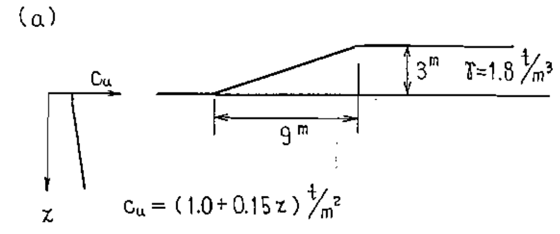
2-6. Examples

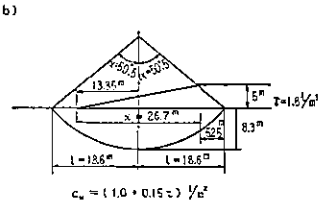
(A) Find the factor of safety of an embankment against a circular slip failure. Conditions are given in Fig. 15 (a).

$$\left.\begin{aligned} q &= (3^m)\times(1.8\,{}^t/_{m^3}) = 5.4\,{}^t/_{m^2} \\ H &= 1.0/0.15 = 6.67^m \\ x &= 9^m \end{aligned}\right\} x/H = 9/6.67 = 1.35$$

From Fig. 13, Fq/c_0 value corresponding to $x/H=1.35$ is given as 8.8. Therefore

$$F = 8.8\frac{c_0}{q} = 8.8\times1.0\times\frac{1}{5.4} = 1.63$$





(a) (b)

Fig.-15. Examples of using the design graph.

(B) Height of a proposed embankment is 5 m. The unit weight of embankment material is 1.8 t/m^3. Find the shape of side slope for an assigned factor of safety of 1.5. The shear strength of foundation soil is expressed as

$$c_u = (1.0 + 0.15\,z)^{t}/_{m^2}$$

Length of side slope:—

$$q = (1.8\,^{t}/_{m^3})\,(5^m) = 9.0\,^{t}/_{m^2}$$

$$H = 1.0/0.15 = 6.67^m$$

$$Fq/c_0 = 1.5 \times 9.0/1.0 = 13.5$$

From Fig. 13, x/H value corresponding to this value of Fq/c_0 is 4.0.

$$x/H = 4 \qquad x = 4H = 26.7^m$$

Position of the critical slip circle:—

Chord length $2l$ is given as $2n_0x$. From Fig. 12, n_0 value corresponding to $H/x = 6.67/26.7 = 0.25$ is 0.695. Then

$$2l = 2 \times 0.695 \times 26.7 = 2 \times 18.6 = 37.2^m$$

Angle α of the critical circle is given by Fig. 6, and it is found 0.88 rad. (50.5°) corresponding to the parameter $H/l = 6.67/18.6 = 0.358$.

Depth of the critical slip circle is given by Fig. 14 and D/H value corresponding to the $Fq/c_0 = 13.5$ is 1.25, then

$$D = 1.25H = 8.3^m$$

The centre of the critical circle is located above the middle of the side slope. The section of the embankment and the position of the critical slip circle are shown in Fig. 15 (b).

Now let us check an accuracy of the design charts. The restoring moment is given by equation (6) as

$$M_r = 2kl^3 \frac{\alpha}{\sin^2\alpha}\left[\frac{H}{l} + \frac{1}{\alpha} - \cot\alpha\right]$$

$$= 2 \times 0.15 \times 18.6^3 \times \frac{0.88}{\sin^2(0.88)}\left[\frac{6.67}{18.6} + \frac{1}{0.88} - \cot(0.88)\right]$$

$$= 1{,}911^{t-m}/_m$$

Referring to Fig. 15 (b) the disturbing moment is

$$M_a=9\times5.25\times\left(18.6-\frac{5.25}{2}\right)+\frac{26.7}{2}\times9\times\left(13.35-\frac{26.7}{3}\right)$$
$$=1,289^{t-m}/_m$$
$$\therefore\quad F=M_r/M_a=1,911/1,289=1.49$$

Therefore the error is 0.7% in the unsafe side.

Note: The results of numerical computations performed as a check showed that the psssible error was about ±3%.

§ 3. Comparison of Odenstad's method and the methed in the present paper

These two methods have been derived by the same sort of analysis, i. e. by the $\phi=0$ analysis using the circular slip surface. Only difference is in the assumed shape of the embankment shoulder. Odenstad's method gives the section of an embankment with the vertical cut at the shoulder, height of which corresponds to the load intensity of 5.52 c_0/F.

At first let us consider the case where c_0 is zero, and a load intensity of embankment is q.

In Odenstad's method, when q/c_0 tends to be infinite in the case of infinite depth of soil stratum, the parameter κ/σ tends to be 1. σ is the measure of the angle of a side slope and can be expressed as q/x in terms of the method described in the present paper. Then we get, using general expression of k/F intead of k,

$$k/\sigma F=1 \quad \therefore\quad \sigma=k/F$$

$$\sigma=q/x \quad \therefore\quad x=q/\sigma=\frac{q}{k}F$$

This expression for x is identical with equation (23).

Now numerical examples shall be shown. Let it be assumed that height of a proposed embankment is 5 m, the unit weight of embankment material is 1.8 t/m^3.

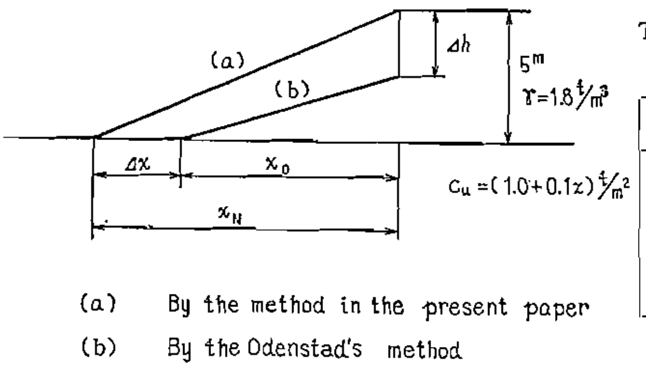


Fig. 16. Key sketch for the comparison of Odenstad's method and the method in the present paper.

Table-4 Comparison of Odenstad's method and the method in the present paper

F	1.0	1.5	2.0	4.0
Δh	3.1	2.1	1.5	0.8
x_0	13	35	62	195
x_N	14	40	76	243
Δx	1	5	14	48

The shear strength of the foundation soil is expressed as;

$$c_u=(1.0+0.1z)\,t/m^2$$

Sections of the embankment given by these two methods are shown in Table 4 for various factor of safety. The key sketch is shown on Fig. 16.

As seen in Table 4, Odenstad's method gives an economical section for the embankment, as far as the quantity of soil is concerned. In the case where the construction space is restricted, i. e. length of the side slope is limited, Odenstad's concept is of practical importance.

§ 4. Ultimate bearing capacity of long footing

The previous analysis may be applied to the problem of the ultimate bearing capacity of a long footing on the horizontal surface of a soil stratum. In the $\phi=0$ analysis the ultimate bearing capacity of a long footing is expressed as

$$q_{ult.}=N_c\,c_0$$

where c_0 is the shear strength which is uniform with depth and N_c is the bearing capacity factor. If the slip circle analysis is used, the value of N_c is 5.52.

Let us consider a footing of width $L=2b$ and carrying a uniform load intensity of p. The shear strength of the soil stratum is assumed to increase linearly with depth. It is assumed that the slip surface always starts at the edge of the footing. Also the footing is assumed to be sufficiently long to neglect the side resistance against sliding.

Referring to Fig. 17, the disturbing moment is

$$M_d=2bp(y-b) \tag{26}$$

where $y>b$ from symmetry. The restoring moment is written from equation (11),

$$M_r=2ky^3\left(A+B\frac{H}{y}\right) \tag{27}$$

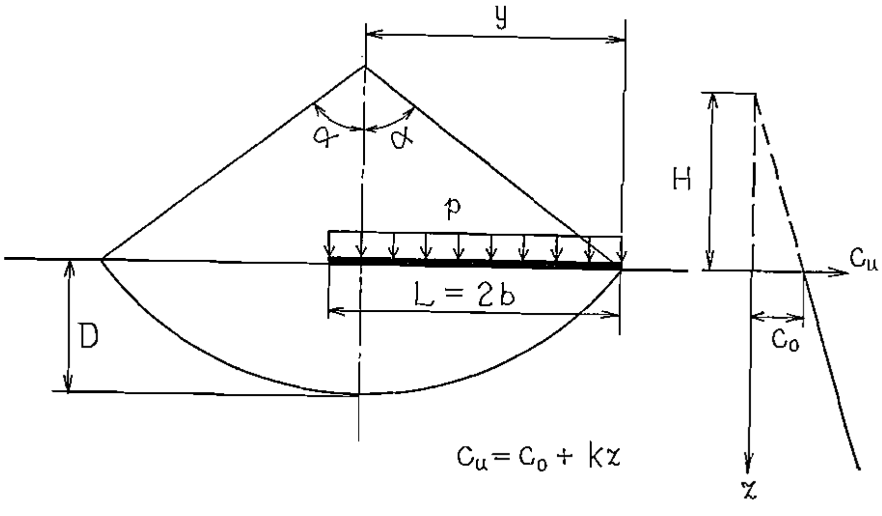


Fig.-17. Situation considered in the analysis of ultimate bearing capacity of long footing.

For the condition of critical equilibrium, $M_r = M_a$, then we have

$$2bp(y-b) = 2ky^3\left(A + B\frac{H}{y}\right)$$

$$p = \frac{k}{b}\frac{y^3}{y-b}\left(A + B\frac{H}{y}\right) \tag{28}$$

Introducing a new parameter $t = y/b$, equation (28) is written as,

$$p = k\cdot b\cdot\frac{t^2(At + BH/b)}{t-1}$$

$$= k\cdot b\cdot\frac{t^2(At + Bm)}{t-1} \tag{29}$$

where $m = H/b$ and $t > 1$.

The minimum value of p is obtained from the condition that

$$\frac{\partial p}{\partial t} = \frac{kbt}{(t-1)^2}[2At^2 + (Bm-3A)t - 2Bm] = 0 \tag{30}$$

For $t = 1^+$ and $t \to \infty$, the p value becomes infinite as seen from equation (29), then the p value becomes minimum for the t value which is given by

$$t = \frac{1}{4A}\left\{-(Bm-3A) \pm \sqrt{(Bm-3A)^2 + 16ABm}\right\}$$

$$= \frac{1}{4A}\left\{(3A-Bm) + \sqrt{(3A-Bm)^2 + 16ABm}\right\} \tag{31}$$

$$\because \quad t > 1$$

The calculation proceeds as follows;

(a) Evaluate equation (31) by substituting various values of $m = H/b$ for the three cases of the combination of the constants A and B.

(b) Sustitute the t value into equation (29), which gives the ultimate bearing capacity as a ratio to the value of kb.

These calculations are tabulated in Table 5.

Table-5 Values of t and parameter p/kb

$0 \leqq \frac{m}{t} \leqq 0.2$

m	t	$\frac{m}{t}$	$\frac{p}{kb}$
0	1.50	0	2.25
0.1	1.581	0.06	3.147
0.2	1.636	0.12	4.016
0.3	1.677	0.18	4.871

$0.2 \leqq \frac{m}{t} \leqq 0.8$

m	t	$\frac{m}{t}$	$\frac{p}{kb}$
0.4	1.644	0.24	5.562
0.6	1.686	0.36	6.796
0.8	1.718	0.47	8.017
1.0	1.744	0.57	9.230
1.2	1.765	0.68	10.438
1.4	1.783	0.79	11.641

$0.8 \leqq \frac{m}{t}$

m	t	$\frac{m}{t}$	$\frac{p}{kb}$
1.5	1.762	0.85	12.210
2	1.795	1.12	15.012
2.5	1.820	1.38	17.803
3	1.839	1.63	20.587
5	1.887	2.65	31.684
10	1.935	5.18	59.338
20	1.965	10.20	114.571
50	1.985	25.2	280.192
100	1.992	50.1	556.201
200	1.996	100	1108.203

$m = \frac{H}{b}$

$\frac{m}{t} = \frac{H}{y}$

Here it must be noted again that constants A and B depend on the value of $H/y=H/bt$.

As seen in Table 5, the t value lies between 1.5 and 2.0, hence the y value, which is the half length of the chord of the critical circle, lies between 1.5b and 2b. This leads to the parameter H/l, as defined in section 2-2, being written as

$$H/l=\frac{H}{y}=\frac{H}{bt}=\frac{1}{1.5}\cdot\frac{H}{b}\sim\frac{1}{2}\cdot\frac{H}{b}$$

As stated in section 2-2, the boundary values of H/l for which the constants A and B are specified are 0 to 0.2, 0.2 to 0.8 and 0.8 to infinity. When H/b is larger than 1.6, all values of H/y are larger than 0.8. And when H/b is smaller than 0.3, all values of H/y are smaller than 0.2.

In Fig. 18 the value of p/kb, which is appropriate to the values of constants A and B, is plotted against the value of H/b. As seen in the figure p/kb is an approximately linear function of H/b, and can be represented

$$p/kb=3.68+5.52H/b. \quad \text{for } H/b\geq 0.8$$

In Fig. 19 the relationship between p/kb and H/b is shown for the H/b value smaller than 1.

If we represent the non-linear part of the p/kb versus H/b relation by a straight line which connects points A and B (See Fig. 19), the error is about 7% at maximum on conservative side. However in the interest of simplicity this small error would be justifiable.

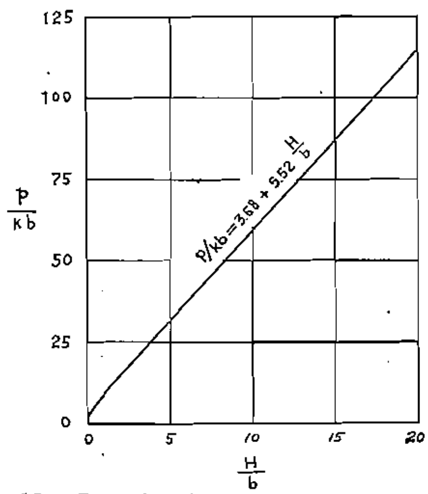


Fig.-18. Relationship between parameters p/kb and H/b at the condition of critical equilibrium.

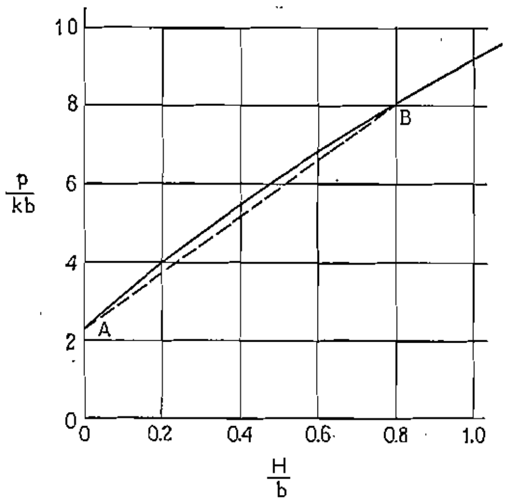


Fig.-19. Sectional approximation of p/kb vs H/b relationship.

Finally we may express the parameter p/kb by following linear equations;

$$\left.\begin{aligned} p/kb &= 3.68+5.52\frac{H}{b}, \quad H/b \geq 0.8 \\ &= 2.25+6.64\frac{H}{b}. \quad H/b \leq 0.8 \end{aligned}\right\} \qquad (32)$$

Therefore we have the expressions for the ultimate bearing capacity;

$$\begin{aligned} p &= 3.68kb+5.52kH \\ &= 1.84kL+5.52c_0 \qquad H/L \geq 0.4 \end{aligned} \qquad (33)$$

$$\begin{aligned} p &= 2.25kb+6.94kH \\ &= 1.13kL+6.94c_0 \qquad H/L \leq 0.4 \end{aligned} \qquad (34)$$

When the shear strength is constant with depth, i. e. $k=0$ and H becomes infinite, the ultimate bearing capacity is written from equations (33) as,

$$p=5.52c_0$$

This value of the bearing capacity factor coincides with the well known value.

When c_0 is zero, i. e. $H=0$, we have from equation (34)

$$p=1.13kL$$

Now let us consider the depth of a soil stratum for which equations (33) and (34) are valid.

Referring to Fig. 17, the depth of the slip circle D is

$$\begin{aligned} D &= R(1-\cos\alpha) \\ &= y\frac{1-\cos\alpha}{\sin\alpha} \end{aligned} \qquad (35)$$

and

$$D/L = y/2b \cdot \frac{1-\cos\alpha}{\sin\alpha} \qquad (36)$$

For the critical slip circle, the value of y/b is shown corresponding to the value of H/b in Table 5. On the other hand angle α for the critical slip circle

Table-6 Calculaticn of depth of the critical slip circle

$\frac{H}{b}$	$\frac{H}{L}$	$\frac{y}{b}$ (1)	$\frac{H}{y}$	α (2)	$\frac{D}{L}$ (3)
0	0	1.5	0	0	0
0.5	0.25	1.68	0.298	0.8	0.355
1	0.5	1.73	0.578	0.9	0.418
2	1	1.80	1.11	1.0	0.491
3	1.5	1.87	1.60	1.04	0.535
4	2	1.88	2.13	1.07	0.557
10	5	1.94	5.15	1.11	0.600
20	10	1.95	10.25	1.15	0.631
∞	∞	2.0	∞	1.165	0.650

(1) from Table 5
(2) from Fig. 6
(3) $\frac{D}{L}=\frac{y}{2b}\frac{1-\cos\alpha}{\sin\alpha}$

is plotted on Fig. 6 against the value $\bar{H}/l$, which is equal to $\bar{H}/y$ (or m/t) in this case. Therefore we can calculate the value of D/L by equation (36) for values of $H/L=H/2b$.

These calculations are tabulated in Table 6, and the depth of the critical slip circle, as a ratio to the footing width, is plotted against the parameter H/L on Fig. 20.

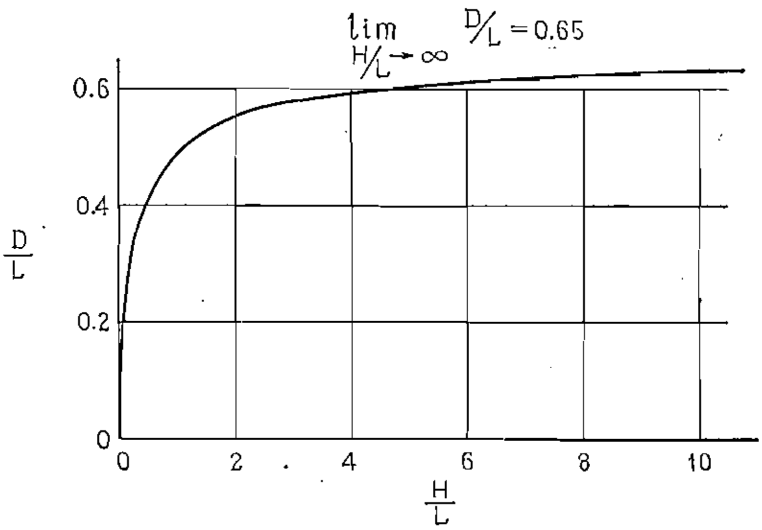


Fig.-20. Relationship between parameters H/L and D/L for the critical slip circle.

When a depth of soil stratum is smaller than D obtained from Fig. 20 corresponding to a given condition of c_0, k and L, the above mentioned analysis does not hold, and we must consider different type of failure such as squeezing failure.

Example

Load $Q=20\,t/m$ is requested to be supported by a footing. The shear strength of foundation soil is expressed as

$$c_u=(1.5+0.1z)\,\text{t/m}^2$$

Find the necessary width of a footing for an assigned factor of safety of 2.0.

As a first trial we use equation (33) for the condition $H/L\geq 0.4$.

$$p=Q/L=\frac{1}{F}(1.84kL+5.52c_0)$$

$$20/L=\frac{1}{2}(1.84\times 0.1\times L+5.52\times 1.5)$$

$$0.184L^2+8.28L-40=0 \qquad \therefore \quad L=4.4\,m$$

In this case,

$$H=1.5/0.1=15m,$$

$$H/L=15/4.4=3.4>0.4$$

Therefore above calculation is valid.

Depth of the critical circle is given by Fig. 20.

$$H/L=15/4.4=3.41, \quad D/L=0.59$$

$$D=0.59 \cdot L=0.59\times 4.4=2.6m$$

Therefore when the depth of soil stratum is greater than 2.6 m, above calculation is valid.

APPENDIX

Outline of Odenstad's method

In his analysis Odenstad considered a soil stratum of thickness D_1 underlain by firm base and exhibiting a linear increase in shear strength with depth, i. e. c_0 at the surface becoming (c_0+kD_1)* at the bottom of the stratum. He aimed to find out the shape of a side slope for a proposed vertical embankment BB' (See Fig. 21) at the critical equilibrium condition. The $\phi=0$ method is applied to a circular slip analysis.

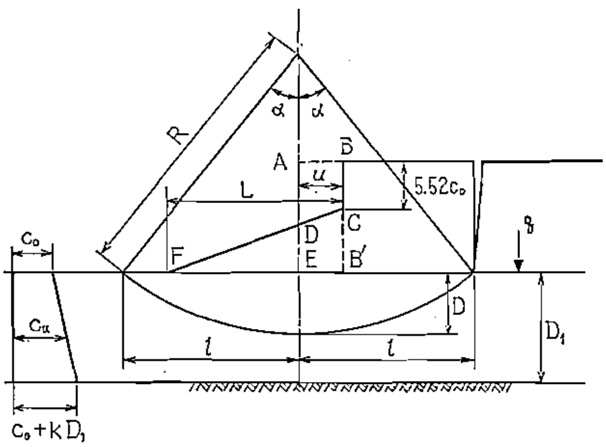


Fig.-21. Situation considered in the Odenstad's method.

The embankment considered has a vertical face BC at the shoulder, as shown in Fig. 21. The load corresponding to this vertical cut is 5.52 c_0, which is the ultimate bearing capacity on a soil stratum having a uniform strength of c_0. The problem now is to find out the shape of the side slope, i. e. the slope of FC, for the given embankment and the assumed soil strength characteristics.

* In this analysis soil strength parameters c_0 and k should be taken as that divided by factor of safety.

At first the slip circle is considered to extend to the bottom of the stratum. The circle is specified by the chord length $2l$ and the sector angle 2α. For geometrical reasons following condition must hold, that is,

$$l=D_1\frac{\sin\alpha}{1-\cos\alpha} \tag{37}$$

The restoring moment is expressed as

$$M_r=\int_{-\alpha}^{+\alpha}R\cdot(Rd\theta)c_u$$
$$=R\cdot(R\cdot 2\alpha)\bar{c}_u$$

where $\bar{c}_u$ is the average shear strength along the arc, and can be written

$$\bar{c}_u=c_0+kR\left(\frac{\sin\alpha}{\alpha}-\cos\alpha\right) \tag{38}$$

Using this expression for the average shear strength the restoring moment M_r is written as;

$$2\frac{M_r}{l^2c_0}=4\left(\frac{\alpha}{\sin^2\alpha}+\frac{1-\dfrac{\alpha}{\tan\alpha}}{\sin^2\alpha}\cdot\frac{kl}{c_0}\right) \tag{39}$$

Considering the disturbing moment, it is assumed that the embankment material has no shear strength, i. e. the vertical tension crack would develop in the fill above the point at which the failure surface emerges from the foundation.

The maximum disturbing moment for the given slip circle can be found out by shifting the centre position of the circle horizontally. In the case where slip circle intersects the ground surface beyond the toe F (See Fig. 21), it can be proved that the disturbing moment becomes a maximum when the area DEF and $ABCD$ are equal. For this situation the distance u between the shoulder and the centre of the circle is obtained from

$$u=\sigma L^2/2q \tag{40}$$

where σ is a measure of the slope FC and is written as,

$$\sigma=(q-5.52c_0)/L \tag{41}$$

The maximum disturbing moment M_d is determined by

$$2\frac{M_d}{l^2c_0}=\frac{q}{c_0}-\frac{1}{12}\left(1-\frac{5.52c_0}{q}\right)^3\cdot\left(1+\frac{16.56c_0}{q}\right)\cdot\left(\frac{q}{c_0}\right)^3\cdot\left(\frac{k}{\sigma}\right)^2\frac{1}{\left(\dfrac{kl}{c_0}\right)^2} \tag{42}$$

At the condition of critical equilibrium $M_r=M_d$, then from equations (39) and (42)

$$\frac{1}{12}\left(1-\frac{5.52c_0}{q}\right)^3\left(1+\frac{16.56c_0}{q}\right)\frac{\left(\dfrac{q}{c_0}\right)^3\cdot\left(\dfrac{k}{\sigma}\right)^2}{\left(\dfrac{kD_1}{c_0}\right)^2}$$
$$=\frac{q}{c_0}\left(\frac{\sin\alpha}{1-\cos\alpha}\right)^2-4\left[\frac{\alpha}{(1-\cos\alpha)^2}+\frac{\sin\alpha-\alpha\cos\alpha}{(1-\cos\alpha)^3}\cdot\frac{kD_1}{c_0}\right] \tag{43}$$

The maximum value of σ occurs when the right hand side of equation (43) becomes minimum. As seen it is a function of α only. Therefore from the condition,

$$\frac{\partial}{\partial\alpha}\left[\frac{q}{c_0}\left(\frac{\sin\alpha}{1-\cos\alpha}\right)^2-4\left\{\frac{\alpha}{(1-\cos\alpha)^2}+\frac{\sin\alpha-\alpha\cos\alpha}{(1-\cos\alpha)^3}\cdot\frac{kD_1}{c_0}\right\}\right]=0$$

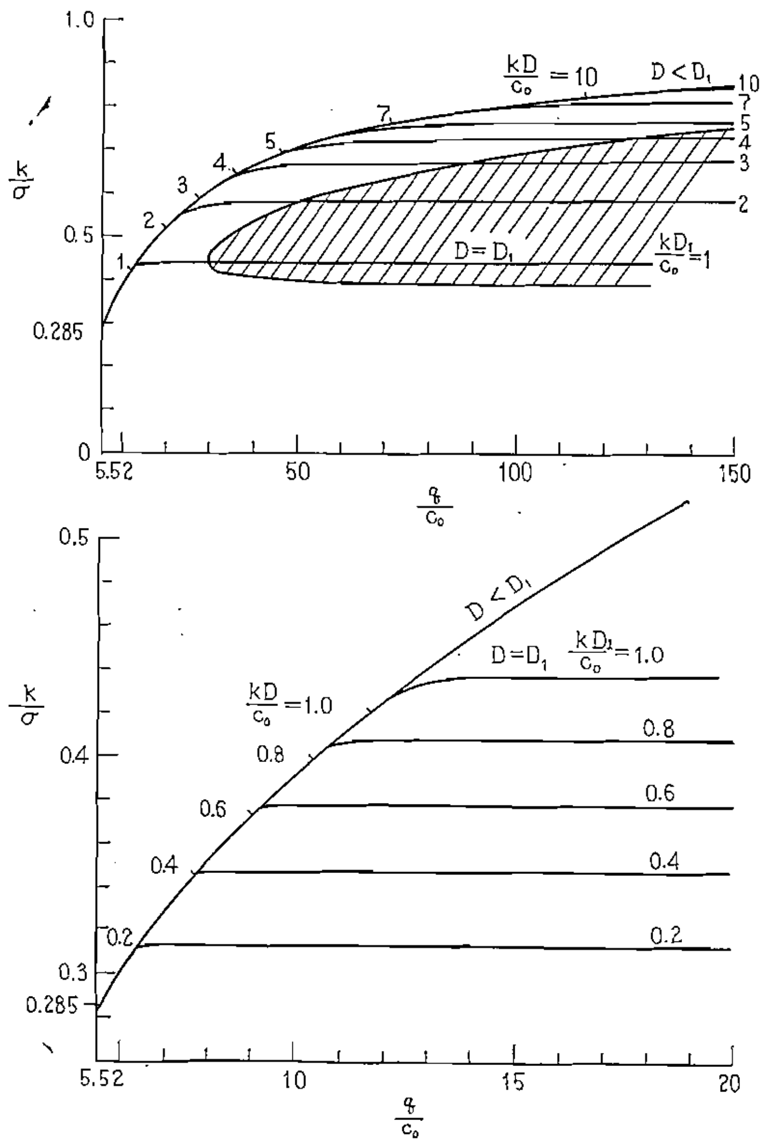


Fig.-22. Design graph of side slope of embankment—relationship between parameters k/σ and q/c_0 (by S. Odenstad).

the following expression is obtained,

$$\frac{q}{c_0}=2\frac{3\sin\alpha-\alpha(1+2\cos\alpha)}{(1-\cos\alpha)^2}\cdot\frac{kD_1}{c_0}-2\left(\frac{1}{\sin\alpha}-\frac{2\alpha}{1-\cos\alpha}\right) \tag{44}$$

By calculating the value of α for any given combination of D_1, q, k and c_0 from

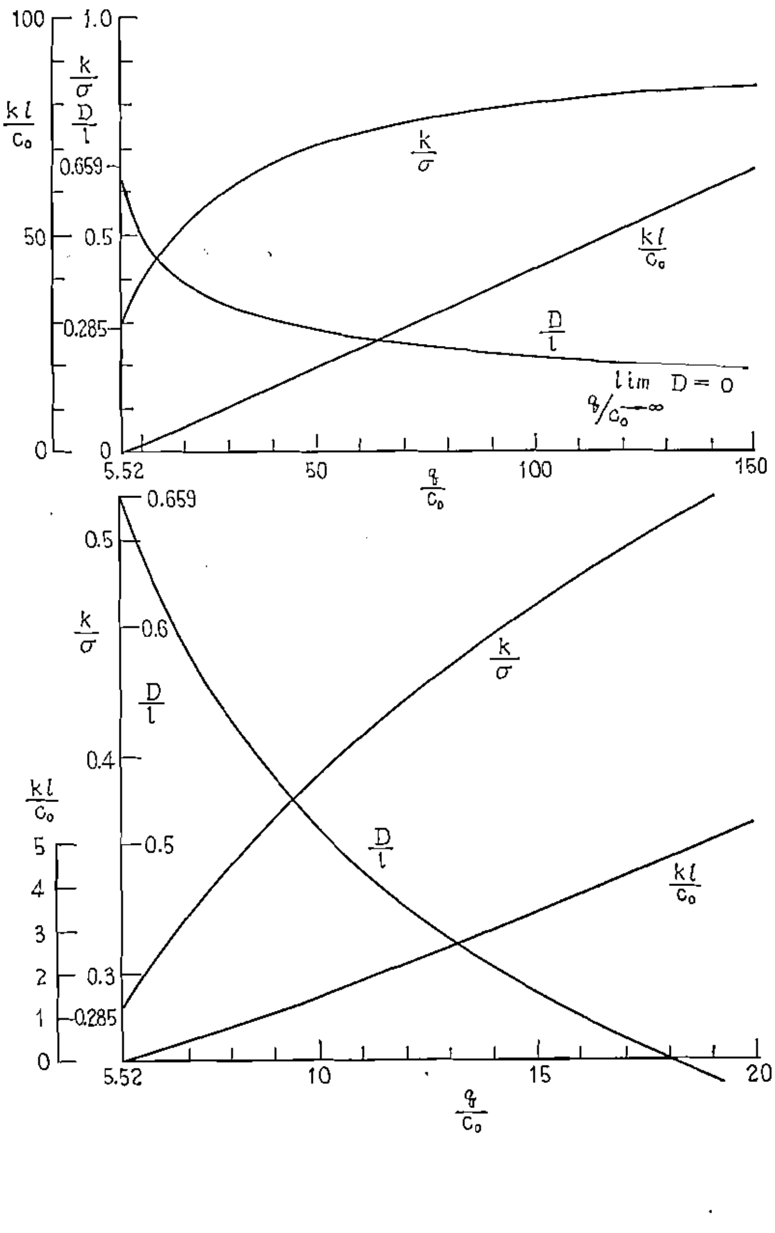


Fig.-23. Graph showing geometry of the slip circle with change of parameter q/c_0 (by S. Odenstad).

equation (44), and subsituting it into equation (43), the maximum value of σ is obtained corresponding to these values of independent variables. The results of repeating these calculations are shown on Fig. 22, where the parameter k/σ is plotted against the parameter q/c_0 for various values of kD_1/c_0. And the parameters kl/c_0 and D/l which specify the geometry of the critical slip circle are plotted against the parameter q/c_0 on Fig. 23.

If the load q is increased for a given combination of c_0, k and D_1, the slope of the side slope must become smaller for limiting equilibrium to be maintained, i. e. k/σ becomes larger, and finally the position of the slip circle coincides with the toe of the side slope. Beyond this state the further increment of load q must be applied over the whole range of the circle to maintain equilibrium. Therefore no further increase in the disturbing moment is permissible, hence no more change in the slope of the side slope.

On the other hand if the depth of the stratum becomes larger, the slip circle corresponding to the critical equilibrium does not touch the bottom of the stratum. This situation is also shown in Fig. 22, which is represented by the envelope of the curves for each parameter kD_1/c_0. When $c_0=0$, k/σ tends to 1.*

Odenstead also examined the possibility that a plane slip surface might be more critical than a slip circle. The equilibrium of the soil mass for the entire range of side slope dimension was studied for the condition of horizontal sliding by the $\phi=0$ method.

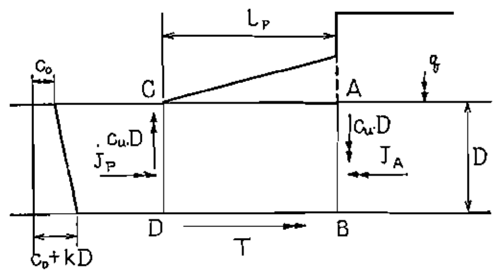


Fig.-24. Situation considered in the analysis of plane slip surface (by S. Odenstad).

As shown in Fig. 24 the forces considered are the active earth pressure J_A,

* σ can be expressed as q/x in terms of the analysis described in section 2. If the general expression of k/F is used instead of k,

$$k/\sigma F=1, \quad \sigma=k/F=q/x \quad x=qF/k$$

This expression is identical with equation (23).

the passive earth pressure J_P and the shearing resistance T along the bottom of the soil mass. These three forces are expressed as;

$$\left.\begin{aligned} J_A &= qD - 2\sqrt{2}\left(c_0 + \frac{1}{2}kD\right)D \\ J_P &= 2\sqrt{2}\left(c_0 + \frac{1}{2}kD\right)D \\ T &= (c_0 + kD)L_P \end{aligned}\right\} \qquad (45)$$

From the condition that $J_A = J_P + T$, we have for equilibrium,

$$\frac{kL_P}{c_0} = \frac{q/c_0 - 4\sqrt{2}\left(1 + \frac{1}{2}\cdot\frac{kD}{c_0}\right)}{1 + \frac{kD}{c_0}} \cdot \frac{kD}{c_0} \qquad (46)$$

Putting $\frac{\partial}{\partial D}\left\{\frac{kL_P}{c_0}\right\} = 0$, for minimum value of L_P, the depth D_f of the most critical slip surface is written as

$$\frac{k}{c_0}D_f = \sqrt{\frac{1}{2\sqrt{2}}\cdot\frac{q}{c_0} - 1} - 1 \qquad (47)$$

The corresponding minimum width of the side slope depends on the depth D_f. From equations (46) and (47), the minimum width is obtained from

$$\frac{kL_P}{c_0} = 2\sqrt{2}\left(\sqrt{\frac{1}{2\sqrt{2}}\cdot\frac{q}{c_0} - 1} - 1\right)^2 \qquad (48)$$

This expression for L_P is valid if the depth of the stratum D_1 is smaller than D_f. If $D_f > D_1$, the necessary width L_P is written by equation (46)

$$\frac{kL_P}{c_0} = \frac{\frac{q}{c_0} - 4\sqrt{2}\left(1 + \frac{1}{2}\frac{kD_1}{c_0}\right)}{1 + \frac{kD_1}{c_0}} \cdot \frac{kD_1}{c^0} \qquad (49)$$

In Fig. 22 the shaded area represents the range in which the plane slip surface is more critical than the slip circle. Therefore the relation between k/σ and q/c_0 is valid outside the shaded area.

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List of symbols

A, B: constants depending on value of H/l
A_f : pore pressure coefficient at failure
b : half width of footing
c' : cohesion intercept in terms of effective stress
$\bar{c}$: average shear strength
c_0 : undrained strength at ground surface
c_u : undrained strength
D : depth of slip circle below ground surface
D_1 : thickness of soil stratum
D_f : critical depth for plane slip surface
F : factor of safety
H : c_0/k
J_A : active earth pressure
J_P : passive earth pressure
K : principal stress ratio
k : rate of change of c_u with depth
L : width of footing
L_P : horizontal length of side slope
l : half length of chord of slip circle
m : H/b
M_d : disturbing moment
M_r : restoring moment
N_c : bearing capacity factor
n : l/x
p : consolidation pressure, intensity of uniform load on footing
q : intensity of embankment load
$q_{ult.}$: ultimate bearing capacity
R : radius of slip circle
T : shearing resistance along bottom of soil mass
t : y/b
u : distance between centre of slip circle and embankment shoulder
W : weight of soil mass
x : horizontal length of slope, horizontal distance
y : distance between slope toe and centre of slip circle, distance between edge of footing and centre of slip circle
z : depth below ground surface
α : half of sector angle of slip circle
β : H/x
θ : angle measured from vertical line
ϕ : angle of shearing resistance in terms of total stress
ϕ' : angle of shearing resistance in terms of effective stress
σ : inclination of side slope

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