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Side Friction in Conventional Consolidation Tests

by

Akio Nakase

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CORRIGENDA

Side Friction in Conventional Consolidation Tests •••• by Akio Nakase

- P·2 8th line from bottom · · · · Hansdo ----> Hansbo

Table 1 •••••••••••••G_s of Test No.R-1 should be <u>2.67</u>

P-6 2nd line from bottom $\cdot \cdot \cdot Q_z = \underline{p} \longrightarrow Q_z = \underline{P}$

- P-7 Caption of Fig.4 should be 'Relationship between coefficient of side friction f and consolidation pressure p '
- P-8 and 9 Caption of Figs.5(a) (d) should be 'Relationship between \overline{Q}/P and consolidation pressure p, where h_0 is the initial height of specimen '

P.18 equation (8a)

$$-Q_{zf} = \underline{\Delta p} \frac{2}{a} \left\{ \frac{R_z}{2af} - \cdots \right\} \rightarrow Q_{zf} = \underline{\Delta P} \frac{2}{a} \left\{ \frac{Rz}{2af} - \cdots \right\}$$

equation (9)
$$F = Q_z \rightarrow F = Q_{z'}$$

equation (10) $P = Q_z \rightarrow P = Q_{z'}$
P.19 bottom line \cdots For the later of -- For
the later state of
P.20 equation (15) $\cdots \rightarrow P = e^{-b} \rightarrow P = e^{-b}$
10th line from top $\cdots Q'_{2H, f} = P_{n-1} \cdot e^{-2f}_{a}$
equation (18) $\cdots e^{-2f}_{R} \rightarrow e^{-2f}_{a}$ (16)
equation (18) $\cdots e^{-2f}_{R} \rightarrow e^{-2f}_{a}$
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Side Friction in Conventional Consolidation Tests

By Akio Nakase*

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* Chief Soil Test and Investigation Section, Soil and Structure Division

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Abstract

In conventional consolidation tests, an applied consolidation pressure is reduced by the side friction set up between a specimen and a consolidation ring. The side friction was measured with specially deviced consolidometer. And the concept of side friction was extended to the partially consolidated state by combining the Taylor's analysis and the Terzaghi's approximate solution of consolidation.

As the result of measurements and analyses of the side friction, it is concluded as follows;

- (a) The coefficient of side friction in conventional consolidation tests is between 0.30 and 0.15 according to the consolidation pressure at completion of 24 hours loading.
- (b) The mean effective consolidation pressure in the commonly used size of specimen is about 75 to 90 % of an applied pressure at completion of 24 hours loading.
- (c) The existence of side friction does not affect the m_v vs p and k vs p relationships in such a representation as specified in the JIS A 1217, in which logarithms of the coefficients are to be plotted against logarithms of mean consolidation pressure.
- (d) Within a single process of consolidation, the coefficient of side friction decreases with increasing degree of consolidation.

I Introduction

In conventional consolidation tests, a specimen is compressed vertically and the lateral displacement is restrained by a consolidation ring, so the frictional stress is set up between the specimen and the ring wall. Set up of side friction causes the decrease in consolidation pressure. In natural soil strata, such a friction does not set up. Then the possible error due to the side friction in conventional consolidation tests would be in the unsafe side.

Experimental studies on this problem had been carried out by D. W. Taylor¹⁾, H. Muhs and M. Kany²⁾ and S. Hansdo³⁾. Taylor had analytically obtained the distribution of the side friction and measured the actual magnitude of the friction using the specially deviced floating ring type consolidometer. He did not ascertain the distribution of the friction because he used specimens with a constant height. He found that the value of the coefficient of side friction was between 0.15 and 0.30. Muhs and Kany used the floating ring type consolidometer as well as Taylor's, and it was deviced that the specimen height could be changed. They assumed the linear distribution of the side friction and found that the value of the coefficient of side

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friction was between 0.15 and 0.30 according to the specimen height. Hansbo also used a kind of floating ring type consolidometer. He measured the critical force which is necessary to jack up or jack down the consolidation ring. And the difference of these two forces gave the double amount of the friction. He reported that the friction value varied from 17 to 29 % of the consolidation load.

In the soil mechanics laboratory of the Port and Harbour Technical Research Institute, two kinds of the studies on this problem had been carried out⁴). Both of them were the indirect measurements, however, the values of the coefficient of side friction obtained were far different from the value obtained by the above mentioned direct measurements. Therefore a special type of consolidometer for the measurement of side friction was made and the direct measurement was carried out.

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1. Consolidometer

The consolidometer for the side friction measurement is principally of fixed ring type as shown in Fig. 1. The specimen height can be changed from 1 cm to 4 cm.





Fig. 1-(a) Consolidometer for side friction measurement.



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Since the lower porous disc is supported on a proving ring, a little settlement occurs Strictly speaking, therefore, this consolidometer is not of at the specimen bottom. fixed ring type on account of the relative movement between the specimen and the ring wall. Initially it was aimed to maintain the elevation of the lower porous disc by means of abjusting jack. However, it was found to be difficult to follow rather large, sudden movement of the disc at the starting of test. And also this following by adjusting jack caused a fluctuation of a proving ring reading. Then the use of the adjusting jack was not made. With this consolidometer the side friction is obtained as the difference between a known consolidation load and the force transmitted to the lower porous disc which is measured by a proving ring. The loading lever ratio can be changed either 4 or 20.

2. Soil

Four kinds of clay sample were used in the measurement. Index properties of clay samples are shown in Table 1. Tests Nos. 1 to 4 refer to the tests on the undisturbed clay and Tests R-1 and R-2 refer to the tests on the remoulded clay.

		Intial water Density	Specific	ecific Initial	Liquid	Plastic	Plasticity	Texture			Unconfined	
Test	Sample	content	Density	gravity	ratio	limit	limit	index	Sand	Silt	Clay	strength
No.	-	w	r	Gs	е	wL	w _P	1 _P	(00)	100	100	q _ц
		(%)	(g/cm ³)			(%)	(%)		(%)	(%)	(%)	(kg/cm²)
1	Kinkai (-13.5m)	116.9	1.37	2.73	3.30	122.2	44.1	78.1	3.0	20.0	77.0	0.6
2	Kinkai (-17.5m)	115.7	1.38	2.72	3.24	120.7	47.2	73.5	2.0	31.0	67.0	0.5
3	Shimizu (-22.4m)	32.1	1.89	2.76	0.93	50.3	24.7	25.6	7.7	35.3	57.0	2.5
4	Kamatsujima (-19.3m)	42.9	1.78	2.75	1.22	52.2	28.9	23.3	4.0	58.2	37.8	1.4
4	Komatsujima (-17.0m)	46.6	1.73	2.70	1.29	52.4	29.1	23.3	2.1	61.9	36.0	1.0
R1	Yokkaichi (-21.3m)	71.8	1.56	2.62	1.94	68.1	34.3	33.8	0.8	39.6	59.6	
R-2	Yokkaichi $(-15.7m)$	55.5	1.66	2.72	1.55	45.6	26.7	18.9	15.0	46.0	39.0	

Table. 1 INDEX PROPERTIES OF CLAY SAMPLES

Heights of specimen were 1, 2, 3 and 4 cm and diameter was 6.5 cm. The specimen was prepared as well as in conventional tests. Since the lower porous disc intruded into the consolidation ring by 4 mm as shown in Fig. 1, the 4 mm height spacer was used to keep the clearance.

3. Test procedures

Consolidation tests were carried out according to the specification of the Japan Industrial Standard (JIS A 1217-1960). Load increment ratio was unity for the consolidation load from 12.5 kg to 200 kg, then the load was mounted to 300 kg and

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then to 400 kg. Each consolidation load was maintained for 24 hours. In some cases, however, the loading duration was extended to 7 days in order to investigate the effect of secondary compression.

4. Test results

Typical examples of the measurement are shown in Fig. 2. In this figure Q is the



Fig. 2-(a) Typical results of measurement.





load transmitted to the lower porous disc which is equal to the difference between the applied consolidation load and the total side friction. Compression of the specimen was taken as the difference between the settlements of upper and lower porous discs. Fig. 3 illustrates a typical relationship between Q and the specimen height at completion of 24 hours loading.

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Fig. 3 Typical plot af Q_h and h.

The primary compression in general finished within an hour, the change in Q during the secondary compression was smaller than that during the primary compression. During the secondary compression the relationship between Q and logarithm of time was found to be approximately linear even in the case of 7 days loading.

5. Analysis of the side friction at completion of consolidation

Analysis of the test results was carried out in the same way as made by Taylor. Taylor's analysis is confined to the state of complete consolidation, at which no excess pore water pressure exists.

The side friction over height dz is

$$\frac{\mathbf{Q}_{\mathbf{z}}}{\pi \mathbf{R}^2} \cdot f \cdot 2\pi \mathbf{R} \cdot d\mathbf{z} = \frac{2f}{\mathbf{R}} \cdot \mathbf{Q}_{\mathbf{z}} \cdot d\mathbf{z}$$
(1)

where Q_z : the actual intergranular load in a sample at depth z from the upper end, R : the radius of a specimen,

f: the constant of proportionality which is called the coefficient of side friction.

The total side friction above any depth z is obtained by integrating equation (1), and the load acting across any plane at depth z is

$$Q_{z} = P - \frac{2f}{R} \int_{0}^{z} Q_{z} dz \qquad (2)$$

in which P is an applied load on the upper end of specimen. Under the boundary condition that $Q_x = p$ at z=0, solution of equation (1) becomes

Then
$$f = \frac{R}{2z} \ln \frac{P}{Q_z}$$
(4)

The coefficient of side friction f can be obtained by substituting a sample height h and a measured intergranular load Q_h on the lower end of specimen into equation (4), and we get

With measured values of h and Q_h , the value of f can be obtained by the method of the least square using equation (4'). In Fig. 3 measured intergranular loads Q_h are plotted against specimen heights h. Thick line in the figure represents the value of Q calculated by equation (3) using the f value obtained from test results. For each series of test, it was found that the relationship between Q_h and h can be well represented by equation (3) for the consolidation stage in which the consolidation pressure was larger than the preconsolidation pressure.



Fig. 4 Relationship between \overline{Q}/P and consolidation pressure P. where h_0 is the initial height of specimen.

Fig. 4 shows the values of f obtained from each series of test. The preconsolidation pressure of clays used was between 0.8 and 1.0 kg/cm^2 , and the value of f was between 0.15 and 0.30 for consolidation pressure larger than the preconsolidation pressure.

Mean effective consolidation load \overline{Q} at complete consolidation is expressed from equation (3) as

$$\overline{\mathbf{Q}} = \frac{1}{h} \int_0^h \mathbf{Q}_x \cdot d_x = \frac{\mathbf{P} \mathbf{R}}{2f h} (1 - e^{-\frac{2f}{\mathbf{R}}h}) \dots (5)$$

In Fig.5, values of \overline{Q}/P are plotted against consolidation pressure p for each series of test in which \overline{Q} was calculated by equation (5) using corresponding f value in Fig. 4.









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Fig. 6 shows the relationship between \overline{Q}/P and h/R in the conventional consolidation test at completion of 24 hours loading. In this figure \overline{Q} was calculated by equation (5) using the mean value of f corresponding to each consolidation pressure shown in Fig. 4. In Japan, specimen size of R=3.25 cm and h=2 cm is commonly used. For this size of specimen it is shown in Fig. 6 that the mean effective consolidation load at completion of 24 hours loading is about 75 to 90 % of an applied load.



Fig. 6 Relationship between \overline{Q}/P and h/R.

Correction for the side friction can be made by using the mean effective consolidation pressure calculated by equation (5). Fig. 7 shows the relationship between



void ratio e and consolidation pressurd at completion of 24 hours loading. In this figure, p is an applied consolidation pressure and \overline{p} is the mean effective consolidation pressure corrected for the side friction.





In Fig. 8 and 9, the coefficient of volume compressibility m_v and the coefficient of permeability k are plotted against p and \overline{p} . In these two figures together with Fig. 10, the coefficients are plotted against the mean value of successive consolidation pressures as specified in the JIS A 1217.



Fig. 8-(a) Coefficient of volume compressibility $m_{\rm V}$.

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Since the value of c_v has no direct connection with the pressure, the correction for the side friction need not be considered for c_v . And the effect of the side friction is to be considered only in the plotting of values against consolidation pressures, i. e. c_v value should be plotted against \overline{p} .

Values of m_v and k become larger by correcting them for the side friction, however, the pressure corresponding to the corrected m_v and k values is to be reduced to the mean effective pressure \overline{p} . From Figs. 8 and 9 it would be concluded that the side friction has no appreciable effect on the m_v vs p and k vs p relationship in such a way of representation.

6. Analysis of the side friction in partially consolidated state

In this section the concept of side friction shall be extended to the partially consolidated state. Taylor had also carried out an analysis on the partially consolidated state using an average intergranular stress claculated by an average degree of consolidation. In this present paper, the following assumptions are made;

- (a) The side friction depends solely on the intergranular stress in clay.
- (b) The distribution of the side friction is characterized by equation (2).
- (c) The process of consolidation is defined by the Terzaghi's approximate solution of consolidation.

In the Terzaghi's approximate solution of consolidation, the isochrone is assumed to be a parabola and an intergranular load in a sample at depth z is expressed⁵)

$$Q_{z} = \Delta P \frac{(\sqrt{12TH^{2}} - z)}{12 T H^{2}}, \quad T \leq \frac{1}{12}$$
(6a)
$$Q_{z} = \Delta P \left\{ 1 - e^{-\left(3T - \frac{1}{4}\right)} \frac{z}{H^{2}} (2H - z) \right\}, \quad T \geq \frac{1}{12}$$
(6b)

where Q_{z} : intergranular load in a sample at depth z,

- T : time factor,
- H : length of drain path,
- z : depth from the drainage surface, and
- ΔP : consolidation load.

As shown in equations (6a) and (6b), the intergranular load has the different expressions depending on the time factor. It is due to the fact that, in the Terzaghi's approximate solution, the principal axis of parabola is at z=H when $T\geq 1/12$, and at z<H when T<1/12.

When the side friction is taken into consideration, the intergranular load Q_{zf} in partially consolidated state is obtained by substituting equation (6a) or (6b) into equation (2), as

$$Q_{zf} = Q_z - \frac{2f}{R} \int_0^z Q_{zf} dz \qquad (7)$$

Integration of equation (7) under the condition that $Q_{zf} = \Delta P$ at z = 0, gives

$$Q_{zf} = \Delta p \frac{2}{a} \Big[\Big\{ \frac{R_z}{2af} - \frac{R^2}{4af^2} - \frac{R}{2f} \Big\} + e^{-\frac{2f}{R}z} \Big\{ \frac{a}{2} + \frac{R^2}{4af^2} + \frac{R}{2f} \Big\} \Big], \quad T \leq \frac{1}{12} \dots (8a)$$

$$Q_{zf} = \Delta P \frac{2e^{-b}}{H^2} \Big[\Big\{ \frac{R_z}{2f} - \frac{R^2}{4f^2} - \frac{RH}{2f} \Big\} + e^{-\frac{2f}{R}z} \Big\{ \frac{H^2}{2} + \frac{R^2}{4f^2} + \frac{RH}{2f} \Big\} \Big], \quad T \geq \frac{1}{12} \dots (8b)$$

where $a = \sqrt{12 \text{ T} \text{ H}^2}$ and b = (3T - 1/4). In the case of no side friction, i. e. f=0, Q_{zf} of equation (8a) or (8b) becomes identical with Q_z of equation (6a) or (6b).

In the early state of consolidation, i. e. T < 1/12, the intergranular load Q_z vanishes at z=a if the side friction is not considered. On the other hand, when the side friction is taken into consideration, Q_{zf} vanishes at z < a. This situation leads to the concept that, in the case of single drainage consolidation, the total side friction F is

where z' is the depth at which Q_{zf} of equation (8a) vanishes. Even at T > 1/12 O , can wanish depending on the value of f

Even at T>1/12, Q_{zf} can vanish depending on the value of f. If Q_{zf} vanishes at z < H in the case of T>1/12, concept of the total side friction may be the same with that mentioned above for the case of T<1/12.

In the case of double drainage consolidation, if the side friction is not considered, the intergranular loads at both drainage surfaces are equal to an applied consolidation load throughout the test. When the side friction is taken into consideration, let it be assumed that the intergranular load at the lower drainage surface is equal to

where F is the total side friction in the case of single drainage consolidation expressed by equation (9).

In the later state of consolidation, i. e. T>1/12, the total side friction F up to depth z is equal to





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then the total side friction in the case of double drainage consolidation is

in which Q_{2H} and $Q_{2H,f}$ are the intergranular loads expressed by equations (6b) and (8b) respectively.

Above stated situations are illustrated in Fig. 11. As an example, the isochrones at T=0.02, 0.4 and 1.0 with f=0.4 are shown and compared with that given by the Terzaghi's approximate solution in Fig. 12.



Fig. 12 Comparison of isochrones with and without side friction.

For the first step of loading the procedures of the analysis for T < 1/12 are as follows. The depth z', at which the intergranular load Q_{zf} of equation (8a) vanishes, is obtained as

Rewriting the above expression using the parameters $\alpha' = R/H$ and n = z'/H, we get the relationship between T and f as

After obtaining the values of n for varions combinations of parameters f and α' the total side friction F is given as

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$$F = \Delta P - Q_{nH} = \Delta P - \frac{(\sqrt{12TH^2} - nH)^2}{12TH^2} = \Delta P \left(1 - \frac{n}{\sqrt{12T}}\right)^2 \qquad (14)$$

Therefore, if values of F and $\alpha' = R/H$ are known, the magnitude of the coefficient of side friction f can be obtained.

For the later of consolidation at T > 1/12, the intergranular load at the lower end,

 $Q_{2H,f}$, is given from equation (8b)

where $\alpha = R/2H$ and b = (3T-1/4). Then the value of the coefficient f can be obtained from equation (15) by substituting the values of b, α and $Q_{2H.f.}$.

For the *n*-th step of loading, the applied load is $P_n = P_{n-1} + \Delta P_n$, in which P_{n-1} is the consolidation load in the (n-1) th step and ΔP_n is the load increment in the *n*-th step. And the side friction is assumed to be separated into two parts, one part is corresponding to P_{n-1} and the other to ΔP_n . The side friction corresponding to P_{n-1} is assumed, regardless of T, to be

$$Q'_{2H,f} = P_{n-1} \cdot e^{-\frac{2f}{\alpha}} \qquad \dots \qquad (1r)$$

which is identical with the expression by Taylor at the completion of consolidation. And the side friction corresponding to ΔP_n is assumed, according to T, to be

 $Q''_{2H.f} = \Delta P_n \cdot \phi(\alpha', f)$ or $\Delta P_n \cdot \phi(\alpha, f)$ (17) where the functions $\phi(\alpha', f)$ and $\phi(\alpha, f)$ represent the ratio $Q_{2H.f}/\Delta P$ defined by equations (14) and (15) respectively. Then the total intergranular load at the lower end of specimen can be expressed as

As stated before, the load increment ratio in the test was unity up to the consolidation load of 200 kg, then equation (18) reduces to

Calculations were carried out at following consolidation states;

Т:	0.004	0.07	0.25	0.70	infinite
\overline{U} :	0.073	0.30	0.60	0.90	1.00

in which the $T-\overline{U}$ relationship was that defined by the Terzaghi's approximate solution. T of 0.004 corresponded, on an average, to the initial reading in the practical measurements.

Fig. 13 shows the values of the coefficient f in the partially consolidated state obtained by the above stated analysis. The f value at complete consolidation in each loading step in this figure correspond to the f values shown in Fig. 4. in the case of Tests Nos. 1 and 2, the value of f at completion of 24 hours loading is larger than that at 90 % consolidation, as shown Fig. 13. This is due to the fact that in thes cases the secondary compression was for greater than that in other tests.



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Fig 13-(d)

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Ⅲ Considerations

It is noted that the value of the coefficient of side friction does not show as much difference as expected by the deviation in the stiffness and consistency of clays. It might be qualitatively said that the value of f increases with increasing values of I_p , clay content and soil strength. And the value of f, regardless of the type of clay, seems to converge to the identical value of 0.15 for larger consolidation pressure as shown in Fig. 4. However, the inter-relationship between the value of fand the index properties of clay could not be found out from the limited number of tests.

In the preceding sections, the analysis of side friction is based on the Taylor's analysis. In his analysis, as well as in that by Muhs and Kany, the side friction was considered entirely frictional in its nature. On the other hand, if the side friction is considered to be entirely cohesive in its nature, it ts noted that fully mobilized cohesive resistance over all side surface of clay specimen is well larger than the measured total side friction in the present tests.

IV Conclusion

As the result of measurements and analyses of the side friction in consolidation tests, it is concluded as follows;

- (a) The coefficient of side friction in conventional consolidation tests is between
 0.30 and 0.15 according to the consolidation pressure at completion of 24 hours loading.
- (b) The mean effective consolidation pressure in the commonly used size of specimen is about 75 to 90 % of an applied pressure at completion of 24 hours loading.
- (c) The existence of side friction does not affect the m_v vs p and k vs p relationships in such a representation as specified in the JIS A 1217, in which logarithms of the coefficients are to be plotted against logarithms of mean consolidation pressure.
- (d) Within a single process of consolidation, the coefficient of side friction decreases with increasing degree of consolidation.

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