August 18, 2017

How to generate a causal time history

Atsushi Nozu

In the site-effect substitution method (Hata *et al.*, 2011), Fourier amplitude and phase at a target site are evaluated separately and then combined to generate the time history of strong ground motions. The simplest equation to formulate this process would be as follows:

$$F(\omega) = |A(\omega)| \frac{O(\omega)}{|O(\omega)|} \qquad (1)$$

where $F(\omega)$ is the Fourier transform of the time history at the target site, $|A(\omega)|$ is the Fourier amplitude at the target site, $O(\omega)$ is the Fourier transform of the selected aftershock record at the target site and $|O(\omega)|$ is its amplitude. In the site-effect substitution method, $|A(\omega)|$ is evaluated by multiplying the Fourier amplitude observed at a nearby strong motion station during a large event, with the Fourier spectral ratio of aftershock records at the target site with respect to the nearby strong motion station. However, the inverse Fourier transform of $F(\omega)$ generally results in an acausal time history. This problem can be resolved by using a simple scheme proposed by Nozu et al. (2009). Here, the essence of the scheme will be explained.

Let us first understand the generation mechanism of acausal time histories. In the Fourier analysis, an arbitrary time history is expressed as a sum of sinusoidal waves with different frequencies. It should be noted that each of the sinusoidal waves is acausal. Therefore, superposing those components with arbitrary weights will just result in an acausal wave. To obtain a causal time history, the weights, *i.e.*, the Fourier coefficients should be determined elaborately in such a way that the neighboring frequency components cancel each other out at the beginning of the time history. In general, Fourier amplitude spectrum of a causal seismogram has small ripples. The ripples can be interpreted as expressing relations between neighboring frequency components, which are necessary for generating a causal time history. It should be noted that the relation is important only for “neighboring” frequencies. This can be seen in the following numerical example.

Figure 1 (top) shows a seismogram obtained in Japan and its Fourier amplitude spectrum. The seismogram satisfies causality and the Fourier amplitude spectrum has ripples. Let us apply a Parzen window with a band width of 0.5 Hz to the Fourier amplitude spectrum around 1 Hz (0.8 – 1.2 Hz). Taking the inverse Fourier transform of it results in an acausal time history as shown in Figure 1 (middle). The noise at the initial part of the time history has frequency components around 1 Hz as can be seen in the enlarged view, which exactly corresponds to the smoothed part of the spectrum. Similarly, smoothing the spectrum around 3 Hz (2.4 – 3.6 Hz) results in an acausal wave with a noise around 3 Hz at the initial part as shown in Figure 1 (bottom). Thus, smoothing a spectrum at a frequency band results in an acausal time history with a noise having that particular frequency at the initial part, because the balance between neighboring frequency components is lost. Therefore, the ripples included in the Fourier amplitude spectrum have a significant role in generating a causal time history.

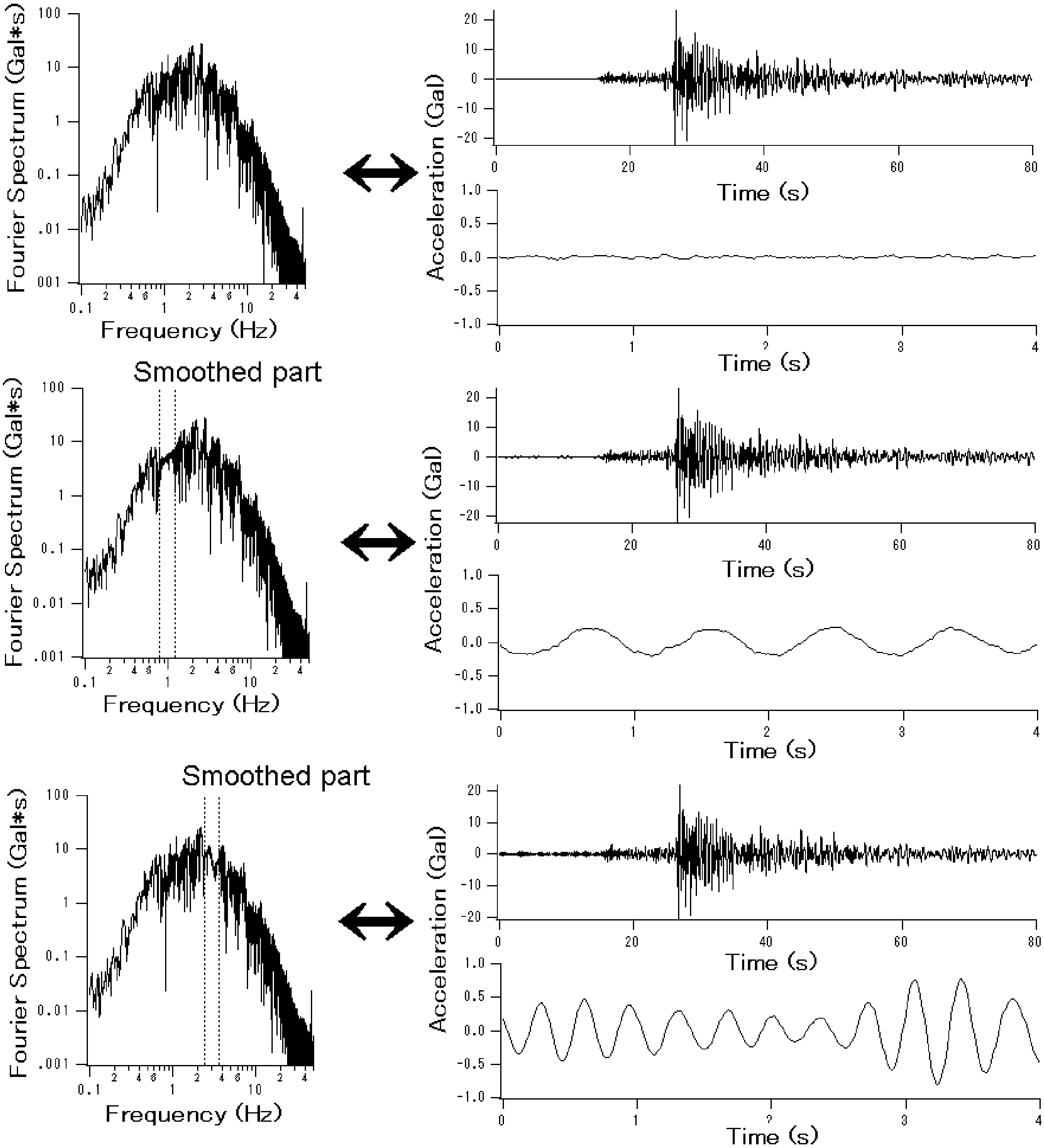

Figure 1 Generation mechanism of acausal time histories. The top panel shows a seismogram obtained in Japan (at KiK-net station TKCH07 during the September 26, 2003, 7:20 event; ground surface; NS component) and its Fourier amplitude spectrum. The seismogram satisfies causality and the Fourier amplitude spectrum has ripples. The middle panel shows the Fourier amplitude spectrum smoothed around 1 Hz (0.8 – 1.2 Hz) and corresponding time series. The time history is acausal and the noise at the initial part has frequency components around 1 Hz. The bottom panel shows the Fourier amplitude spectrum smoothed around 3 Hz (2.4 – 3.6 Hz) and corresponding time series. The time history is acausal and the noise at the initial part has frequency components around 3 Hz.

Having this in mind, it is easy to reformulate the site-effect substitution method for the purpose of obtaining a causal time history. In Equation (1), $O(\omega)$ is divided by $|O(\omega)|$. This process completely removes the ripples included in O(ω), resulting in an acausal time history. Therefore, instead of Equation (1), the following equation should be used in the site-effect substitution method (Hata *et al.*, 2011):

$$F(\omega) = |A(\omega)| \frac{O(\omega)}{|O(\omega)|_p} \qquad (2)$$

where $|O(\omega)|_p$ is the Parzen-windowed version of $|O(\omega)|$. Because $|O(\omega)|_p$ is a slowly-varying function of frequency, the balance between neighboring frequency components of $O(\omega)$ is maintained, resulting in a causal time history. In the applications of this method, a band width of 0.05 Hz has frequently been used. In addition to $|O(\omega)|_p$, $|A(\omega)|$ in Equation (2) should also be a slowly-varying function of frequency to maintain the balance between neighboring frequency components. Therefore, when $|A(\omega)|$ is evaluated from observed Fourier spectra, they should also be smoothed.

Acknowledgment

We thank the National Research Institute for Earth Science and Disaster Resilience for providing strong motion records.

References

Hata, Y., A. Nozu and K. Ichii (2011): A practical method to estimate strong ground motions after an earthquake based on site amplification and phase characteristics, *Bulletin of the Seismological Society of America*, Vol.101, No.2, pp.688–700, doi:10.1785/0120100142.

Nozu, A., T. Nagao and M. Yamada (2009): Simulation of strong ground motions using empirical site amplification and phase characteristics: Modification to incorporate causality, *Journal of Structural Engineering & Earthquake Engineering, JSCE*, Vol.65, pp.808–813, doi:10.2208/jsceja.65.808 (in Japanese with English abstract).